PHYSLAB 300/500 (2016)
Homework on uncertainties and measurements
Due date: Thursday October 13th, 9:30 am

In the questions below, show appropriate and refined workings on your favorite computer programs. These questions all refer to the book "Measurements and their uncertainties" by I. G. Hughes and T. P. A. Hase.

1. *Angular dependence of the reflection of coefficient of light*
   The intensity reflection coefficient, R, for the component of the field parallel to the plane of incidence is
   $$R = \frac{tan^2(\theta_i - \theta_t)}{tan^2(\theta_i + \theta_t)},$$
   where θ_i and θ_t are the angles of incidence and transmission, respectively. Calculate R and its associated uncertainty if $\theta_i = (45.0 \pm 0.1)^\circ$ and $\theta_t = (34.5 \pm 0.2)^\circ$.

2. *Best-fit straight line—an unweighted fit*
   The data listed below comes from an experiment to verify Ohm's law. The voltage across a resistor (the dependent variable) was measured as a function of the current flowing (the independent variable). The precision of the voltmeter was 0.01 mV, and the uncertainty in the current was negligible.

Current (μ A)	10	20	30	40	50	60	70	80	90
Voltage (mV)	0.98	1.98	2.98	3.97	4.95	5.95	6.93	7.93	8.91

   (a) Calculate the unweighted best-fit gradient and intercept, and their uncertainties.

   (b) Calculate the standard uncertainty and compare the value with the experimental uncertainty derived from a standard deviation calculation.

   (c) Plot a graph of the data and add the best-fit straight line.

   (d) Calculate the residuals, and comment on their magnitudes.

3. *Linear regression—unweighted fit*
   A dataset showing voltage versus frequency is tabulated below and is also shown in the accompanying plot.

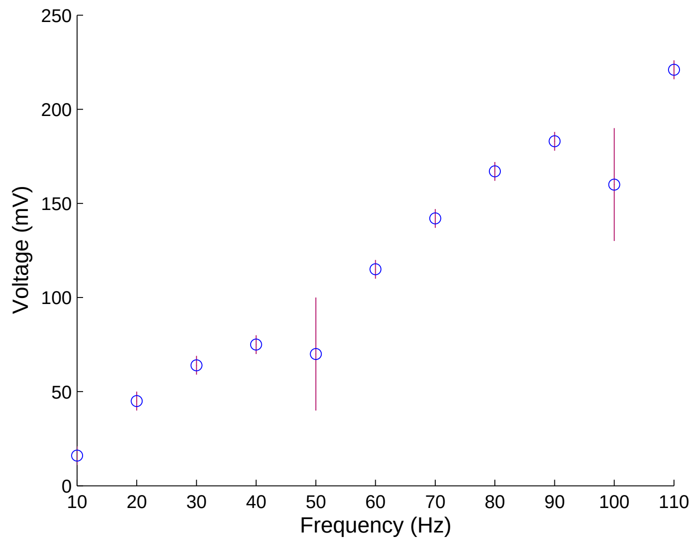

Figure 1: The signal to noise ratio from a frequency to voltage converter diminishes near harmonics of the main frequency.

Frequency (Hz)	10	20	30	40	50	60	70	80	90	100	110
Voltage (mV)	16	45	64	75	70	115	142	167	183	160	221
Uncertainty (mV)	5	5	5	5	30	5	5	5	5	30	5

Determine the best-fit gradient and intercept using an unweighted fit.

4. *Linear regression—unweighted fit*
   Now for the data set of question (3), calculate the best-fit gradient and intercept using a weighted fit. Draw the log plot and calculate the Durbin–Watson *statistic D*.

5. *Error bars from a* χ^2 *minimization*

   (a) For the data set of question (3), write a computer program to perform a χ^2 minimization procedure. Verify that χ^2_{min} is obtained for the same values of the parameters as are listed in Table 6.1 (given on p. 71 of the book).

   (b) By following the procedure of $\chi^2 \rightarrow \chi^2_{min} + 1$ outlined in section 6.5, check that the error bars for m and c are in agreement with Table 6.1. Let the parameters (a) increase from their optimal values to find the extremum for each parameter, and (b) decrease from their optimal values to find the extremum for each parameter. Are the uncertainties determined in (a) and (b) the same?

(c) Calculate the uncertainties on m and c by finding the extrema of the $\chi^2 \to \chi^2_{min}+4$ and $\chi^2 \to \chi^2_{min}+9$ contours. Are your results in agreement with Table 6.2 (given of p. 76 of the book)?