
Noise Reduction by Signal Accumulation

Yaakov Kraftmakher, Department of Physics, Bar-Ilan University, Israel

The aim of this paper is to show how the noise reduction by signal accumulation can be accomplished with a data acquisition system. This topic can be used for student projects. In many cases, the noise reduction is an unavoidable part of experimentation. Several techniques are known for this purpose, and among them the signal accumulation is the simplest one. It is based on exactly knowing the period of a signal to be expected. This is possible when the signal is a response to periodic changes of some external parameters set by the experimenter. For instance, the sample may be subjected to periodic changes of temperature, pressure, irradiation, magnetic or electric field, etc. Therefore, the response of the sample is of exactly the same periodicity as the external influence. A source of this influence provides a signal strictly defining the period of the signal.

The accumulation technique rests on two very simple and commonly accepted assumptions: (i) during a sufficient number of periods, the signal exactly reproduces itself, and (ii) the sum of errors accompanying the measurement is zero in any phase of the signal. For random errors, this assumption is quite natural and accepted in measurements of one quantity. When measuring a periodic signal, the same assumption is applicable to any of its parts. The period of the signal is divided into a definite number of equal parts (channels). For each such channel, the correct value of the signal is an average of all measurements falling into the channel. With a data-acquisition system, the same internal clock defines the period of the signal and the sampling rate, and the system exactly divides the

period into channels.

When using the accumulation technique, the signal is presented as a function of time or of a periodically changing external parameter. During many periods, the instantaneous signal values inside each channel are accumulated separately. The expected signal does not change during the measurements, so that the accumulation in each channel means summation of the same values of the signal. This sum is thus proportional to the number of the periods. At the same time, the noise or signals of other periods make irregular contributions, positive and negative. The accumulation of noise follows the law of summation of random values: the sum is proportional to the square root of the number of the periods. This means that the accumulation technique improves the signal-to-noise ratio proportionally to the square root of the number of periods involved in the accumulation. This simple rule shows the noise reduction attainable with this technique. The signal accumulation technique is valid even when the noise has components in the same frequency range as the signal, so that a low-pass filter cannot be applied to reduce the noise.

When the signal is presented in digital form, the accumulation technique becomes extremely simple. To perform the necessary operations, one should prepare special software. This is a task for students familiar with programming; therefore, the simple software that we use is not given here. The algorithm for preparing it is quite obvious: all data X_i and Y_i falling into one channel are considered to belong to one measured quantity and are replaced by their mean values.

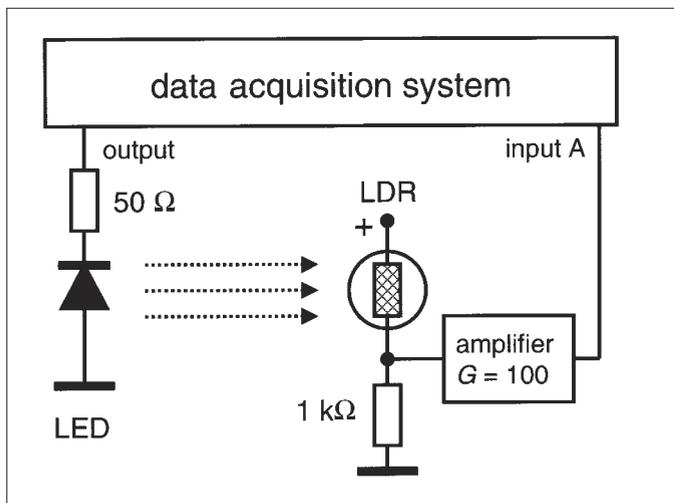


Fig. 1. Diagram of setup to demonstrate the accumulation technique.

The data stored by the data-acquisition system represent a set of instantaneous values of X (time or an external parameter) and of periodically repeated Y (the signal + noise). When the data are obtained as a function of time, the software puts all X values into one signal period set by the experimenter according to the period of the external influence. The software divides the period into a definite number of channels, which is also set by the experimenter. Then mean values of all X_i belonging to each channel are stored as X_k values. The mean values of corresponding Y_i are stored as Y_k values. Here i is the number of a point inside a channel, and k is the channel number.

Now it remains to plot a graph of Y_k versus X_k . To maintain good resolution, the number of the channels should be sufficiently large, but this requirement contradicts the necessity to have many data points inside each channel for the averaging operation. A compromise should thus be found between the two requirements.

To demonstrate the accumulation technique, we use the ScienceWorkshop data acquisition system with DataStudio software from PASCO scientific (Fig. 1). A light emitting diode (LED) is fed by the *Square wave* voltage from the *Signal generator* incorporated into the ScienceWorkshop 750 Interface. The frequency of this voltage is 200 Hz. The *Signal generator* operates in the *Auto* regime: it starts to produce the output voltage after starting the measurements. A 50-Ω resistor limits the current through the LED. The

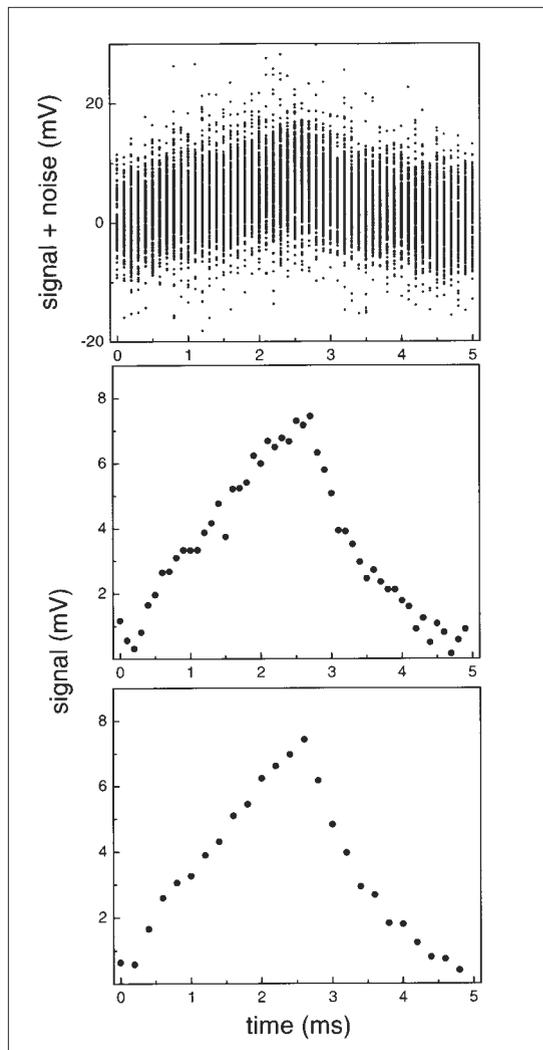


Fig. 2. Data accumulated during 200 periods (10,000 experimental points) and signal recovered by the accumulation technique, with 50 and 25 subdivisions of the period. Before amplification, the signal was of the order of 0.1 mV.

rectangular light pulses illuminate a light dependent resistor (LDR). The LDR is connected in series with a dc source and a 1-kΩ load resistor (the resistance of the load is much smaller than that of the LED). The voltage on the load is amplified ($G = 100$) and then measured by the *Voltage sensor* (PASCO, CI-6503). The data are displayed on the screen of a computer.

In our case, random errors are added to the signal by the measuring system, and this case is typical for many real measurements. Therefore, the signal-to-noise ratio depends on the signal level. It is easy to increase the latter by increasing the dc voltage applied

to the LDR + load combination. The signal increases while the noise remains the same.

The CdS light dependent resistor used here is very sensitive, but its electrical conductance does not immediately follow the illumination. The conductance gradually increases under illumination and gradually decays in the dark. The frequency of the rectangular light pulses was chosen to observe this behavior of the LDR. The time of the measurements and the *Sample rate* are set to obtain a sufficient number of data points in each channel.

In the example presented here (Fig. 2), the time of the measurements is 1 s, and the *Sample rate* is 10 kHz. The total number of data points accumulated during 200 periods thus equals 10,000. The shape of the signal is quite unclear from the data obtained, but the accumulation technique solves the problem. The processing of the data was performed twice, with 50

and 25 subdivisions of the period. Therefore, 200 or 400 data points were accumulated in each channel. The improvement of the signal-to-noise ratio thus is expected to be 14 and 20, respectively. The demonstration clearly shows the principle and power of the accumulation technique.

I would like to thank my son Alex for preparing the software used here.

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Yaakov Kraftmakher is the author of *Lecture Notes on Equilibrium Point Defects and Thermophysical Properties of Metals* (World Scientific, 2000), *Modulation Calorimetry* (Springer, 2004), and of more than 100 scientific papers. His interests include teaching methods in physics.

Department of Physics, Bar-Ilan University, Ramat-Gan 52900, Israel; krafty@mail.biu.ac.il

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"In an effort to explain her theoretical work in an accessible way, Randall has published *Warped Passages: Unraveling the Mysteries of the Universe's Hidden Dimensions*. The book imparts her excitement about the conundrums that pervade the study of physics, and helps the lay reader understand difficult concepts through engaging illustrations, pop song lyrics, and other familiar reference points."¹

1. "Invisible Dimensions," *The Yard* (Faculty of Arts & Sciences Harvard University, Fall 2005), p. 3.

Editor's Note: Lisa Randall was the recipient of AAPT's Klopsteg Award and delivered the award lecture at the 2006 National Summer Meeting in Syracuse, NY.