Data Analysis with a Capacitor

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There are numerous natural processes in which the rate of change of a quantity is proportional to that quantity. An example from biology is population growth of a species that is proportional to the number present. In a radioactive sample the decrease of the number of nuclei is proportional to the number of nuclei present, resulting in a decrease in the number of atom. Another example that is relevant to the present experiment is the charging and discharging of a capacitor through a resistor. This experiment deals with this phenomenon and involves careful data processing to obtain quantitative information about an exponential process.

KEYWORDS

capacitor · data acquisition · square wave · frequency · sampling rate · time constant · curve fitting

1 Conceptual Objectives

In this experiment, we will,

1. learn about charging and discharging of a capacitor,
2. practice data acquisition,
3. understand the relation between mathematical expressions, through curve fitting techniques
4. practice loading data into Matlab, extracting useful data and
5. finding area under a graph.

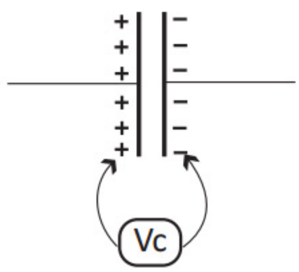
2 Experimental Objectives

The main objective of this experiment is to investigate the time needed to discharge a capacitor and determine the RC time constant of an RC series circuit. We will also practice how to generate and acquire a signal using the data acquisition system along the path. We will measure the voltages across the resistor and the capacitor to determine the RC time constant and the value of the unknown capacitor.

3 Introduction

3.1 Mathematical Model of a Capacitor

The simplest capacitor comprises two conductive plates in very close proximity to each other, such that no electrical path may exist between them.



Two oppositely charged conductive plates of a capacitor.

Both plates have equal and opposite electrical charge, the net charge is zero but the capacitor is charged. The relation between the charge stored Q and the voltage across the capacitor terminals V_c is,

$$Q = CV_c, \tag{1}$$

Where C is the constant of proportionality and is called the capacitance of the capacitor. The capacitance describes how many Coulombs of charge the capacitor can store for each volt applied across its terminal. If the voltage and charge change over time, the equation is rewritten as

$$Q(t) = CV_c(t), \tag{2}$$

The electrical current is the flow of charge per change in time,

$$i(t) = \frac{dQ(t)}{dt}, \tag{3}$$

and by integration the charge stored on the capacitor is,

$$Q(t) = \int_0^t i(\tau)d\tau + Q_o, \tag{4}$$

where Q_o is the charge stored on the capacitor at time $t = 0$. This is called the initial value of the charge. Therefore, by finding the area under the current-time graph, we can determine the charge stored on the capacitor. Similarly from (2) we obtain,

$$V_c(t) = \frac{1}{C}\int_0^t i(\tau)d\tau + V_c(0). \tag{5}$$

Again, $V_c(0)$ refers to the initial voltage on the capacitor.

3.2 Charging and discharging a capacitor

A simple RC circuit consists of a resistor and capacitor connected in series. We charge the capacitor by applying a source of voltage across it and discharge by allowing the stored charge to flow out through a conductive path.

Consider the circuit shown in figure 1(a), with an initially uncharged capacitor.

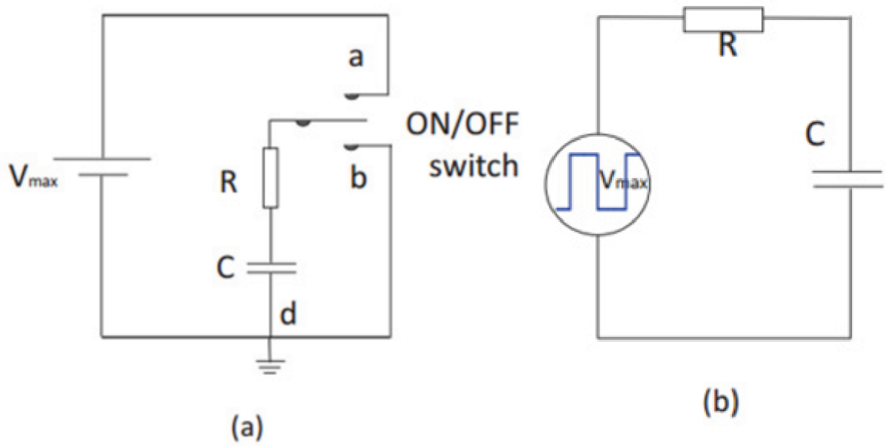


Figure 1: (a): Charging and discharging of a capacitor using a manual switch; (b): Automatic charging and discharging of a capacitor using a square wave.)

Suppose the switch is in position a. The DC voltage source V_{max} is introduced into the circuit. Initially the current i is maximum but as the capacitor builds up charges, i reduces. If we wait long enough the capacitor can charge up to V_{max}, at which point i goes to zero. This situation is represented by,

$$i(t = 0) = \frac{V_{max}}{R} \tag{6}$$

$$i(t = \infty) = 0 \tag{7}$$

Now if the switch is moved to position b, a conducting path bd is available for the charge to flow out, establishing current. The current decays till the charge on the capacitor has been fully depleted.

The manual toggle of switch *a* and *b* can be automated by applying a square wave with a non-zero mean as shown in figure 1(b). This approach will be used in the current experiment.

Suppose that the capacitor is initially uncharged. During the charging part of the cycle, the charge on the capacitor changes as,

$$Q(t) = Q_{max}\left(1 - \exp\left(-\frac{t}{RC}\right)\right), \tag{8}$$

where, $Q_{max} = CV_{max}$,

and during the discharge period it decays as,

$$Q(t) = Q_o \exp\left(\frac{-t}{RC}\right), \tag{9}$$

Q_o, is the charge stored at the start of the discharge cycle.

Q 1 Based on equations (8) and equation (9), write down expression for the voltage Vc across the capacitor.

Q 2 Show that the current through the capacitor during the charging and discharging is given by,

$$i(t) = \frac{Q_{max}}{RC} \exp\left(-\frac{t}{RC}\right), \tag{10}$$

$$i(t) = -\frac{Q_o}{RC} \exp\left(-\frac{t}{RC}\right), \tag{11}$$

Q 3 Hand sketch the exponential curves for the voltage V_c and current $i(t)$

Q 4 Air (an insulator) is between the plates of the capacitor. How can current flow through the capacitor?

The combination RC is called the time constant τ of the circuit and has dimensions of time.

4 The experiment

The goal of this experiment is to find an unknown capacitance by analyzing the charging and discharging curves. We use a DAQ card (National Instrument, PC1 6221) to generate a square wave of known frequency of peak voltage V_{max}, baseline zero, and input this voltage across a series of a capacitor and resistor. The voltage across the resistor and capacitor is acquired by the computer.

The routing is done through the routing breadboard (National Instruments, SCC-68). The applied square waves frequency should be adjusted between 0 to 50 Hz and its amplitude between 0 and 10 V. The OFF voltage is 0 V, reproducing the scenario depicted by the position b of the switch in figure 1(a). A Labview file named rccircuitv4.vi has already been prepared. A screen view of the the front panel is shown in Figure 2

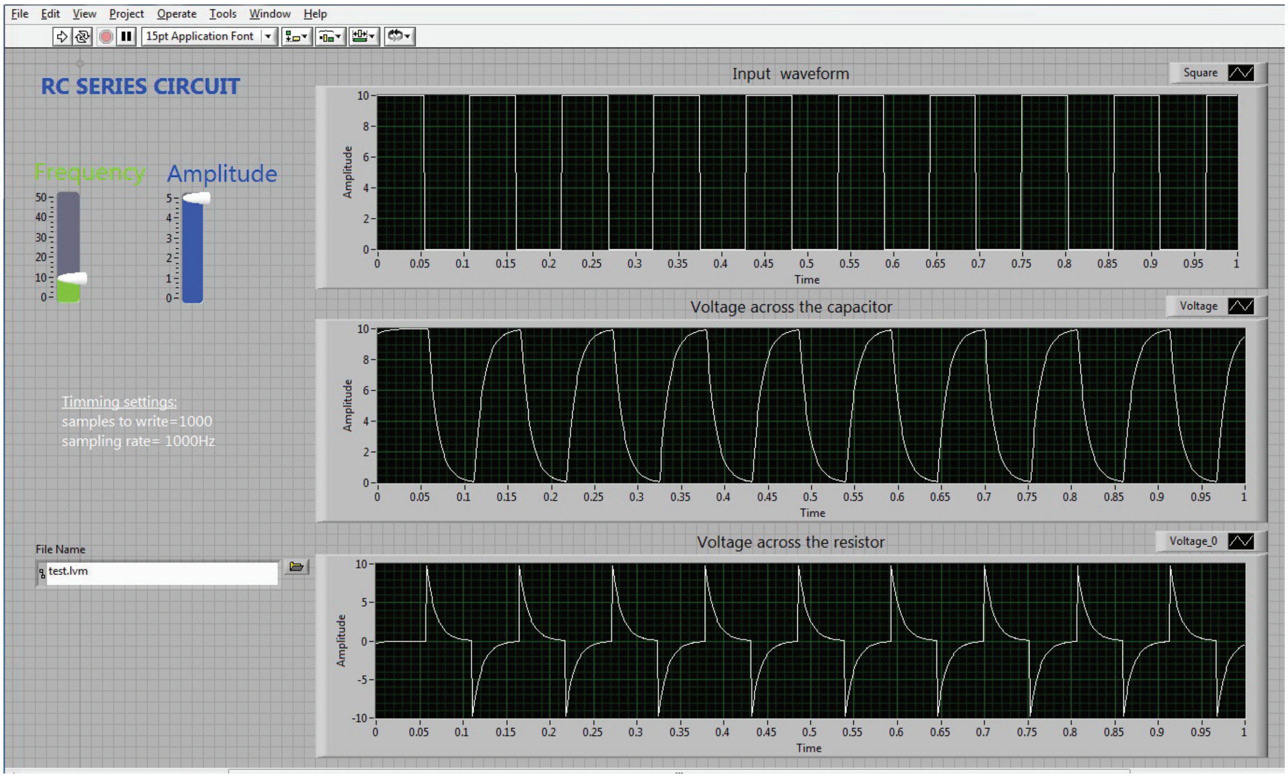


Figure 2: Front panel of labview file rccircuitv4.vi.

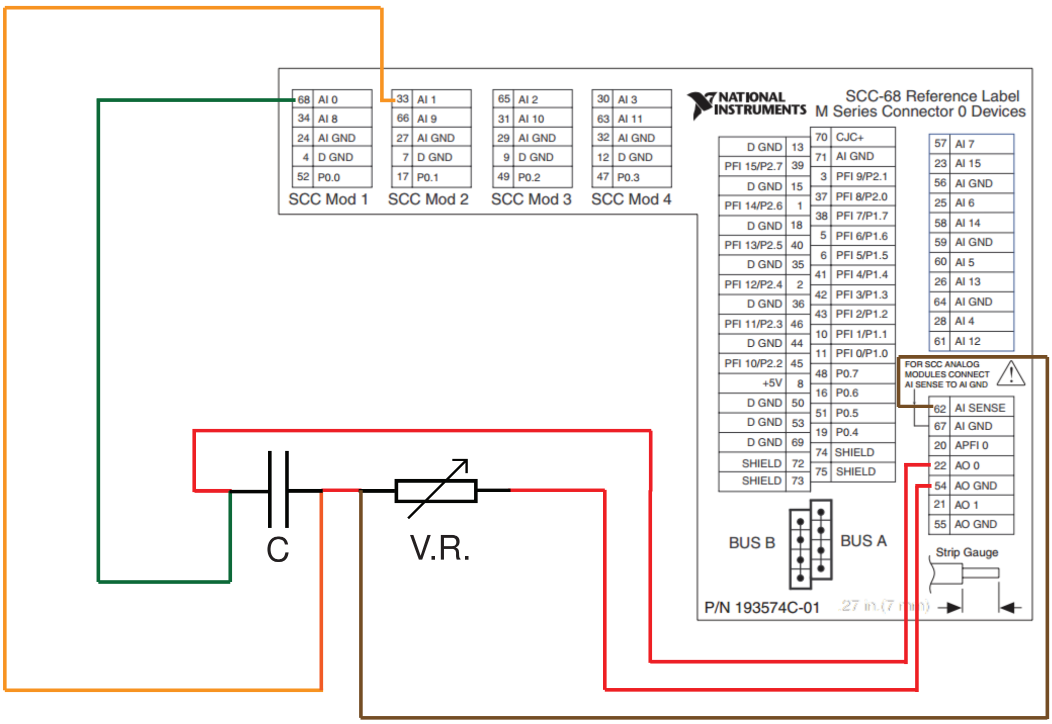


Figure 3: Typical connection diagram.

For a given pair of capacitor and resistor, perform the following steps:

1 Connect the circuit on the breadboard. Refer to Figure 3 for assistance.

2 Open the Labview file adjust the frequency and amplitude and run the file by pressing the $\Longrightarrow$ button. Describe your observations on your notebook by varying the frequency and amplitude.

3 Save your data. Load into Matlab. The files you will create will have three columns, one for the time, one for V_C, and one for V_R. In this manuscript, these files are also called the data files.

4 Extract charging and discharging parts of the V_c curves. Plot them separately and obtain the time constant through curve fitting, measure the resistance using the voltmeter and find the best estimate of the capacitor. Note down the curve fitting parameters in your notebook as well as estimate of the capacitance.

5 Linearize V_R during the discharge period. Use the linear data to estimate the time constant RC. Find C and its uncertainty.

6 From the data obtained during the charging of the capacitor, find the the charge stored on the capacitor. (HINT: Use the area under the V_R curve)