Crater formation in the laboratory: An introductory experiment in error analysis

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One of the primary objectives of the introductory physics laboratory is to teach error analysis. We describe a very simple and inexpensive experiment which exposes students to the central ideas of estimation and uncertainty, and to the evaluation of theory by graphical display of data. The task is to measure the diameter of a crater formed by dropping a small steel ball into a sand-filled container, and then to deduce the functional dependence of the crater diameter on the kinetic energy of the falling ball.

I. INTRODUCTION

One of the most commonly articulated goals of the introductory physics laboratory is to teach experimental strategy.[1,2] Quite often, the first few lab exercises encountered by students deal with estimation of uncertainty, error propagation, and comparison of data with theoretical predictions (curve fitting). At Colgate, our introductory calculus-based mechanics course is organized around a strong central theme: The laws of physics are the same everywhere in the universe.[3] Throughout the course, we extrapolate the physical laws (e.g., conservation of momentum, energy, and angular momentum) governing the behavior of familiar lab-sized systems to systems of astronomical size: planets, stars, galaxies, and the entire universe. For our first laboratory, we wanted an exercise which highlighted this theme and, at the same time, introduced students to the basics of data analysis. The experiment described below—impact cratering—is inherently interesting, blends well with the course theme, and clearly illustrates how multiple trials are used to estimate uncertainty and to test theory. (Error propagation is a more complicated topic, and it is addressed in a second lab exercise.) Furthermore, the cratering lab is very inexpensive and easy to implement, and it can be completed by beginning students with care and success within the typical 3-h lab period.

II. BACKGROUND

Craters are found on all planets (except Jupiter, which has no solid surface) and moons of the solar system, and an understanding of crater formation is important for examining the history of the solar system. On Earth, craters are formed by meteor impact as well as by large-scale underground explosions, e.g., nuclear weapons.[4] In either case, the energy: mass (or volume) ratio is very large (compared to chemical explosions, for example), and it is believed that crater formation by either process follows the same scaling laws. According to Gault *et al.*,[5] the kinetic energy of an impacting meteor is distributed among five processes: heating, comminution (the creation of new surface area), deformation, ejection of material, and seismic waves. If plastic deformation is the most important process, then the volume V of the crater must scale with the energy of the meteor. Since $V \propto D^3$, where D is the crater diameter (Fig. 1), then D scales as the cube root of the energy, or

$$\frac{D}{D_0} = \left(\frac{E}{E_0}\right)^{1/3}. \tag{1}$$

On the other hand, if material ejection absorbs most of the energy, then the kinetic energy is converted to gravitational potential energy needed to lift a volume $V \propto D^3$ to a height approximately equal to the crater depth. Since the depth is proportional to D (for a spherical crater), then

$$\frac{D}{D_0} = \left(\frac{E}{E_0}\right)^{1/4}. \tag{2}$$

We do not discuss the theory of crater formation with students, other than to pique their interest in the phenomenon and to motivate the exercise. From a pedagogical point of view, the cratering experiment is attractive because of its overlap with the course theme, because repeated trials and careful data analysis are so clearly needed to extract useful information, and most important, because the correct answer is not known *a priori*.[6] At the outset, it would seem unlikely that such a crude experiment could yield useful information.

III. APPARATUS AND PROCEDURE

The experimental apparatus is arranged as shown in Fig. 2. An 11-in. (28-cm) diam plastic food container[7] (available at almost any supermarket or discount store) is filled to a depth of about 8 cm with *dry* sand. (We use a mixture intended for sandblasting.[8]) The sand-filled container is placed on the floor close to a lab workbench. A standard table clamp and vertical pole support a 2-m stick and a ball launcher. The latter (a three-pronged spring-loaded lens holder works nicely[9]) is centered directly above the sand. To simulate cratering, a small steel ball of known mass m and diameter d is dropped from a known height h into the sand, creating a well-defined circular crater of diameter D. The crater diameter is measured to the nearest 0.1 mm using inexpensive plastic calipers.[10] (A flexible clear plastic ruler, with resolution 0.5 mm, would work nearly as well.) Some students also use a floor-level lamp to cast a shadow over the sand surface, to better define the outline of the crater. Between trials, the sand is ''sifted'' by plunging a 10×15-cm piece of stiff hardware mesh (mesh spacing 1/4 in.) into the sand repeatedly. Following this, the sand surface is leveled by gently shaking the container horizontally. (The sand must *not* be compacted.) This simple procedure takes only about 20 s per trial, and yields surprisingly reproducible data. For example,

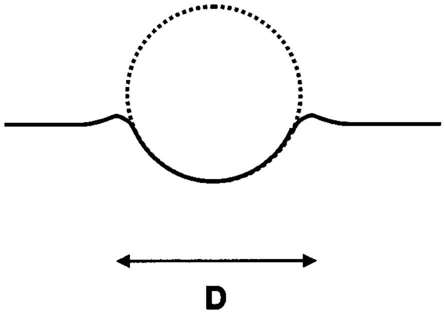

Fig. 1. Cross section of the sand crater created by the falling ball. The position of the peak of the crater wall is used to estimate the diameter D.

a 1-in.-diam ball ($m = 67.4$ g) dropped from a height $h = 100$ cm produces a crater of average diameter $\langle D \rangle = 90.4$ mm with a standard deviation $\sigma = 1.2$ mm.

The immediate task of the lab is to determine the crater diameter as a function of the energy $E = mgh$ of the falling ball. (We assume that students have prior knowledge, from secondary school, of gravitational potential energy in its simplest form.) Using balls ranging in diameter from 1/2 in. (1.3 cm) to 1 1/4 in. (3.2 cm), and using four equally spaced values of h from 50 to 200 cm, the value of E can be varied over almost two orders of magnitude. For each ball and height chosen, students must take a sufficient number of measurements to establish meaningful estimates of σ and σ_m, the standard deviation of the mean.

How many measurements are sufficient? There is not a physics teacher alive who has not had to answer this question. Although it is sorely tempting to yield to expediency and provide an explicit answer (e.g., ten), we believe that by doing so, the instructor may be missing a golden opportunity

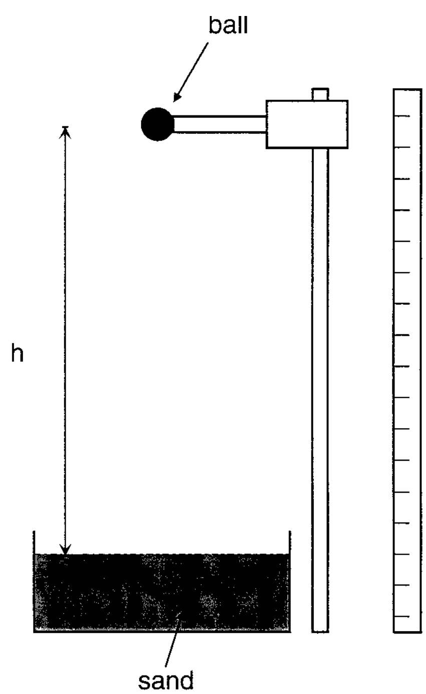

Fig. 2. Apparatus: A tall vertical rod supports a 2-m rule and a ball launcher. The latter is positioned directly above the center of the sand-filled container.

to teach the essence of intelligent experimental design. Recognizing that we are dealing with first-term physics students, we believe that it would also be a mistake to respond to such questions by appealing to formal mathematical analysis. In other words, we believe that some concepts, such as experimental uncertainty, are best introduced to beginning students by *telling* them the principal ideas rather than by *deriving* those ideas. Our highest teaching priority is to enable students to plan experiments effectively and to report and interpret experimental results correctly.

Accordingly, we discuss—but do not derive—the principal ideas of data analysis: Given an infinite set of measurements $\{x_i\}$ subject to random error, (1) that data set is distributed symmetrically about the ''true'' value x_{true}, and (2) 68% of the measurements fall within $\pm\sigma$, the standard deviation, of x_{true}. A real set of n measurements $\{x_1, x_2, \ldots, x_n\}$ can be interpreted as a subset of the infinite data set; the purpose of repeated readings is to obtain sufficiently accurate estimates of x_{true} and σ. The best estimate of x_{true} is the average value, and the best estimate of σ is the *sample* standard deviation, defined in the usual way. Our ability to estimate σ improves with increasing number of readings n, with $\Delta\sigma$ given by[11]

$$\frac{\Delta\sigma}{\sigma} = \frac{1}{\sqrt{2(n-1)}}, \tag{3}$$

by which we mean that for n measurements, the calculated sample standard deviation is within $\Delta\sigma$ of the true σ with a 68% level of confidence. Hence, for ten readings, one can estimate the true experimental uncertainty σ to within about 25%.[12] Finally, we introduce the standard deviation of the mean, $\sigma_m = \sigma/\sqrt{n}$, as the best measure of the overall uncertainty of the experiment.

IV. RESULTS AND DISCUSSION

Students record their raw data directly in their laboratory notebooks, and then construct a table listing the energy of the falling ball, and the mean, standard deviation, and standard deviation of the mean (standard error) of the resulting crater. Using a common computer spreadsheet, they then plot their data to determine if the theory discussed earlier is consistent with their measurements. Of course, they cannot do this using a linear plot. But the spreadsheet allows them to convert their data almost instantly to a log–log plot; after doing so, they are surprised and pleased to see a clear straight line dependence. Although log–log plots are bewildering to many beginning students, this spreadsheet approach alerts students to the power of graphical analysis, and motivates them to understand graphing with nonlinear axes. We conclude the experiment by determining which exponent, $p = 1/3$ or $p = 1/4$, best describes their data.

Figure 3 is a summary of data using four ball sizes ($d = 1/2$, 3/4, 1, and 1 1/4 in.) and four heights ($h = 50$, 100, 150, and 200 cm). The data unmistakingly follow a 1/4 power law. Moreover, there is no explicit dependence on the ball diameter (apart from energy). Almost all student groups obtain results similar to Fig. 3 with little special supervision.

At the conclusion of the laboratory, we expect that students will have a clear understanding of: (1) the need for repeated trials; (2) the meaning and significance of the mean, standard deviation, and standard error; (3) how to report the results of a measurement. Furthermore, we hope that they

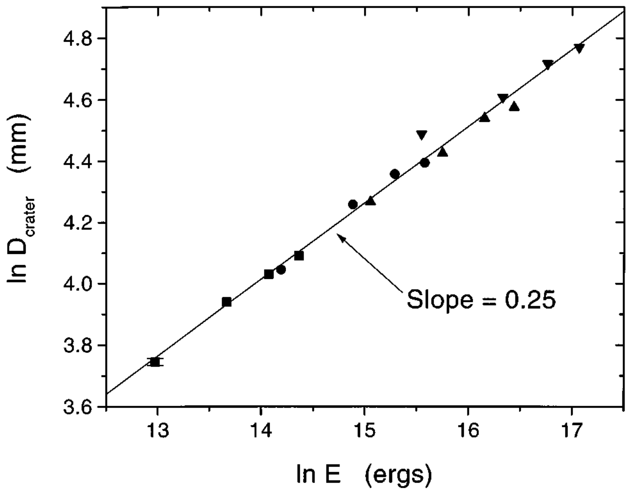

Fig. 3. Crater diameter versus energy of the falling ball. The largest error brackets are roughly the size of the square symbol. (■) 1/2-in. ball diameter; (●) 3/4 in.; (▲) 1 in.; (▼) 1 1/4 in.

now have an appreciation of the power of graphical analysis for comparing theory with experiment. Students interested in a better appreciation of Eq. (3) are encouraged to take an extended set of readings (20 or more) for one ball at one height. By plotting the running value of the standard deviation σ_n (for $n=3,4,...,$ readings), it is easy to verify that σ_n is stable to within 25% for $n \geqslant 10$.

There is, of course, no way for a single lab group to acquire and analyze 16 or more data sets in the time span of a single lab period. Accordingly, we have designed a cooperative strategy in which the data from each group is collected and made available for use by the entire class. At Colgate, we work with a laboratory class size of 16, or eight groups of two students. For this exercise, we use four ball sizes and four heights, for a total of 16 experimental conditions. Each group is given two balls and asked to take crater measurements for two ball heights, so that each set of experimental parameters (d,h) is sampled twice. This works well if and only if the instructor is alert to simple errors. In particular, the instructor must ensure that groups are measuring h from the sand surface rather than from the floor, and that all groups are using the same crater feature (e.g., the peak sand height, Fig. 1) to estimate D. In practice, the results of each laboratory group ($\langle D \rangle$ and σ_m) are recorded and displayed on the class blackboard in a two-dimensional table. This allows the instructor to discuss differences between overlapping results in terms of statistical significance. Any *significant* disagreements between data taken by separate groups must, of course, be resolved before further work is done. We have used this cooperative strategy only once so far, but have observed that it allows students to work in a less harried manner, while inspiring them to assume greater responsibility for their measurements and calculations. Overall, they seem to enjoy and appreciate the lab experience more fully.

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[1]J. G. Potter and J. Burns, ''Alternative justification for introductory physics laboratory courses,'' Am. J. Phys. **52**, 12–13 (1984).

[2]See Introductory University Physics Project, Newsletter No. 4, April 1990, the American Institute of Physics, New York; see also Final Report of the IUPP Laboratory Working Group (unpublished).

[3]C. H. Holbrow and J. C. Amato, ''Inward bound/outward bound: Modern Introductory Physics at Colgate,'' in *Changing Roles of Physics Departments in Modern Physics: Proceedings of the International Conference for Undergraduate Physics Education*, edited by E. F. Redish and J. S. Rigden (American Institute of Physics, Woodbury, NY, 1997).

[4]H. J. Mydosh, *Impact Cratering* (Oxford U.P., New York, 1989), Chap. 7.

[5]Donald E. Gault *et al.*, ''Some comparisons of impact craters on Mercury and the Moon,'' J. Geophys. Res. **80**, 2444–2460 (1975).

[6]Jan Beyea and Edward F. Kennedy, ''Motivating error analysis,'' Am. J. Phys. **43**, 267–268 (1975).

[7]Rubbermaid® Servin' saver™ 22 cups (5.2 ℓ) with lid (to keep the sand dry and clean).

[8]This is important. Ordinary playground sand will not work well. Our mixture is called ''white silica sand'' and was purchased from a local masonry supply outlet.

[9]Central Scientific Co., Cat. No. 72288.

[10]Frey Scientific Co., ''economy vernier calipers,'' Cat. No. F990927.

[11]*Experimentation: An Introduction to Measurement Theory and Experiment Design*, edited by D. C. Baird (Prentice-Hall, Englewood Cliffs, NJ, 1962).

[12]See also J. Higbie, ''Uncertainty in the measured width of a random distribution,'' Am. J. Phys. **44**, 706–707 (1976); Granvil C. Kyker, Jr., ''The range of a data set: A quick estimator of the standard distribution,'' *ibid.* **51**, 852–852 (1983).

EXPERIMENTATION IN THE ARTS

What is generally called ''experimentation'' in the arts more nearly resembles my ignorant and youthful self-indulgent mess-making [in high school chemistry]. I was acting out a fantasy, not learning anything about chemistry, and while every smelly substance I concocted had to have been made according to chemistry's laws, I did not know those laws, nor could I have learned them from anything I was doing. And how many botches have been excused by calling them the results of the experimental spirit? We have to imagine an artist wondering what would happen if she were to do this, try that, perform a play in silence, omit the letter ''E'' in three pages of French prose, construct a world of clothes hanger wire, color walls with cow manure. Having found out, though, then what?

William H. Gass, ''On Experimental Writing: Some Clues for the Clueless,'' New York Times Book Review, 21 August 1994.