Planck's constant determination from blackbody radiation

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for their valuable ideas and contributions: Clifford Fairchild, John Gardner, Kenneth Krane, Michael Lampert, Gary McCown, Larry Silva, Robert Simpson, Darren Stevens, and Wayne Swenson.

[1] Alternative approaches have been presented in this Journal: G. P. Lietz, "Microcomputer Interfacing: A Course for Science Majors," Am. J. Phys. **55**, 796–799 (1987); T. Karcher, T. J. Burch, J. J. Ruddick, J. Backman, and J. Caravella, "The Microcomputer in the Undergraduate Physics Laboratory—System, Hardware, Student Reaction, Evaluation," Am. J. Phys. **55**, 545–548 (1987).

[2] Mail-order sources, with competitive prices, are listed monthly in *Computer Shopper* (Ziff-Davis, Boulder, CO).

[3] Single-bit input is described by R. Wisman and K. Forinash, "Discount Interfacing with the IBM Parallel Printer Port," Am. J. Phys. **57**, 561–562 (1989).

[4] For example, J. N. Demas and S. E. Demas, *Interfacing and Scientific Computing on Personal Computers* (Allyn and Bacon, Needham Heights, MA, 1990); S. C. Gates with G. Becker, *Laboratory Automation Using the IBM PC* (Prentice-Hall, Englewood Cliffs, NJ, 1989); B. G. Thompson and A. F. Kuckes, *IBM-PC in the Laboratory* (Cambridge U.P., Cambridge, 1989); J. W. Snider and J. Priest, *Electronics for Physics Experiments Using the Apple II Computer* (Addison-Wesley, Reading, MA, 1989); C. D. Spencer, *Digital Design for Computer Data Acquisition* (Cambridge U.P., Cambridge, 1990).

[5] C. A. Kocher, "Computer Interfacing and Instrumentation," laboratory manual for Physics 415, Oregon State University (1991), 130 pp. Copies are available from the author, at cost, through 1992.

[6] For a recent description of an *analog* lock-in amplifier, see R. Wolfson, "The Lock-In Amplifier: A Student Experiment," Am. J. Phys. **59**, 569–572 (1991), and references cited therein.

Planck's constant determination from black-body radiation

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A method originally described by Crandall and Delord has been improved to measure the ratio of the Planck to the Boltzmann constant with low cost apparatus. Such a system can be easily implemented in high school or college laboratories. These improvements enable one to attain accuracies of 2% for Planck's constant.

I. INTRODUCTION

There are many methods that able us to measure more or less accurately Planck's constant. The most useful, for a college or high school physics laboratory, are based on application of the photoelectric effect[1] or black-body radiation.[2] Those are not the most accurate, but are the easiest to implement under the above-mentioned conditions. The first method relies upon Einstein's formula describing the energy of electrons emitted from an illuminated surface for different frequencies of incoming photons (see Ref. 1). This method is quite often used in introductory laboratories. The second method seems more interesting as its implementation requires more knowledge about basics of quantum and statistical physics. While implementing the method of Crandall and Delord,[2] we noted various improvements that greatly enhance the overall accuracy. Our improvements enable one to attain accuracies of 2% for Planck's constant. The cost of the required apparatus is quite modest.

II. THEORY

Radiation from a hot surface is emitted as energy quanta according to Planck's law. Each energy quantum (photon) carries an energy:

$$\epsilon = \hbar\omega, \tag{1}$$

where $\hbar = h/(2\pi)$ and ω stands for an angular frequency of a quantum. The symbol h denotes Planck's constant which is recognized as a basic and fundamental physical constant. It is equal to $6.6261762(3)\times10^{-34}$ J s. Planck's law leads to the following frequency distribution for the radiation intensity emitted by a black-body:

$$B(\omega,T) = \frac{N\omega^3}{\exp(\hbar\omega/k_B T) - 1}, \tag{2}$$

where N is a constant, T is the absolute temperature of the emitter, and k_B is Boltzmann's constant. The constant N is independent of frequency and temperature as long as the radiator approximates a black-body. Hence, the intensity ratio measured at the same frequency and different temperatures has the following form:

$$\frac{B_1}{B_2} = \frac{B(\omega,T_1)}{B(\omega,T_2)} = \frac{\exp(\hbar\omega/k_B T_2) - 1}{\exp(\hbar\omega/k_B T_1) - 1} \approx \frac{\exp(\hbar\omega/k_B T_2)}{\exp(\hbar\omega/k_B T_1)}. \tag{3}$$

The approximate formula describes Wien's law and it is well satisfied for $\omega > 6.504\times10^{11}\ T$, where ω is in rad s^{-1} and T in k. Such conditions are relatively well satisfied within visible range of radiation and for temperatures accessible in solids. Therefore, the ratio $\hbar/k_B$ can be expressed as follows:

$$\frac{\hbar}{k_B} = \frac{1}{\omega\cdot(1/T_2 - 1/T_1)} \ln\left(\frac{B_1}{B_2}\right). \tag{4}$$

The last expression can be used for determination of either Planck's constant or Boltzmann's constant, provided the other is known.

III. EXPERIMENTAL SETUP

In order to exploit relationship (4) it is necessary to select a narrow and well-known range of frequencies, to measure an absolute temperature, and to measure a quantity proportional to the intensity emitted around frequency ω.

We have found that a low-voltage bulb having a spiral tungsten filament heated by a dc current is the best compromise for a source of "black-body" radiation. We have used a car stop-light bulb (12 V/21 W). Such a source is stable within reasonable time periods and does not fluctuate at short time intervals. A dc power supply with a variable output voltage and equipped with the ammeter and voltmeter in order to measure voltage U and current I across the filament is required.

Any single-color optical filter is suitable for the purpose of selecting a frequency. Very dark filters should be avoided. Unfortunately, all low cost and reasonably transparent optical filters are unable to cut off infrared radiation, the latter being dominant in the spectrum. Hence, it is critically important to use light detectors having sufficiently large activation energies. We were unable to obtain reasonable results either with photoresistors or with photodiodes as they were too sensitive to the infrared radiation. A gas-filled external photocell has been found to work best. It is important to optimize the voltage across the photocell to achieve sufficient sensitivity and to stay within the linear response range. Vacuum photocells are more linear, however their sensitivities seem to be insufficient without applying elaborate electronics. The photocell current has to be measured as a voltage across a resistor (about 200 kΩ)—see Fig. 1—with the help of a standard digital multimeter. The multimeter output is proportional to $B(\omega,T)$ and can be used instead of B in the expression (4). An experimental setup is shown in Fig. 1.

The last important point is to measure temperature of the filament. The simplest and the most reliable method is to rely upon dependence of the filament resistance versus temperature. A filament resistance $R(T) = U/I$ can be expressed as follows:

$$R(T) = R_0[1 + \alpha(T - T_0)], \qquad (5)$$

where R_0 stands for the resistance at temperature T_0, the latter being an ambient temperature, and α for the temperature coefficient. The parameters R_0 and α are reasonably stable against thermal cycling for a given bulb. A linear dependence is well satisfied for temperatures exceeding the Grüneisen temperature and lower than the onset of anharmonicity (2/3 of the melting temperature), i.e., practically from room temperature to about 2500 K in the case of tungsten. Relationship (5) can be used to read the temperature T provided R_0 and α had been determined previously. An ambient temperature does not need to be measured precisely. It is dangerous to rely upon published temperature coefficients due to the uncontrollable impurities in commercial tungsten. Thus the above-mentioned parameters have to be obtained as an outcome of the linear regression fit to the resistance data obtained at various temperatures T. Such a calibration can be conveniently performed with the help of any optical pyrometer. Typical results plotted in the form $T(R)$ are shown in Fig. 2. The

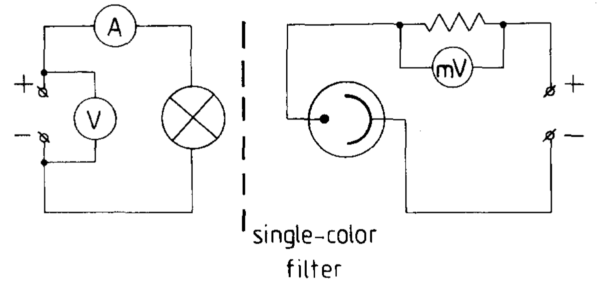

Fig. 1. A schematic diagram of the experimental setup. The circumscribed X denotes the lightbulb.

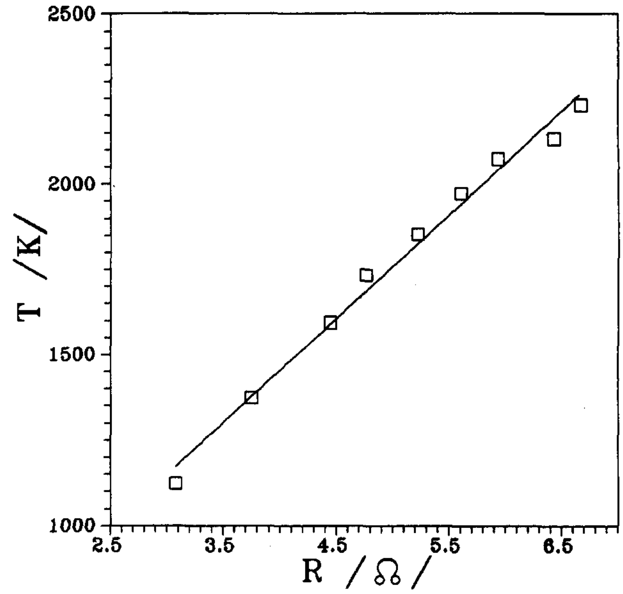

Fig. 2. The filament temperature plotted versus bulb resistance. A linear regression fit to the data is used to determine the absolute temperature of the filament during the actual experiment.

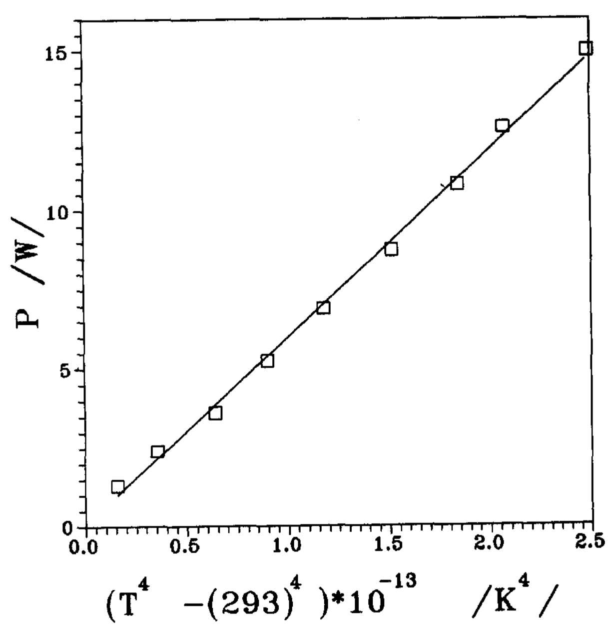

Fig. 3. Plot of the supplied electrical power versus $(T^4 - T_0^4)$. Its linearity indicates that the filament approximates a grey-body relatively well. The temperature T has been measured by a calibrated optical pyrometer with a tungsten filament.

Table I. Results obtained for Planck's constant using our setup with different filters and calibration parameters of the filament resistance versus temperature. Blue—$\omega = 4.266\times 10^{15}$; green—$\omega = 3.689\times 10^{15}$; red—$\omega = 3.027\times 10^{15}$ (rad/s). Filters have been calibrated against Ne emission lines applying a standard prism spectroscope. An external effect photocell had a partly oxidized Cs–Sb cathode and was filled with Ar. It had been operated at 20 V applied to the whole circuit sensing light intensity. A multimeter input resistance was about 10 MΩ. Weighted mean value: $h = (6.51 \pm 0.06)\times 10^{-34}$ (J s). $T(K) = 304.78R[(\Omega)] + 234.16$.

Filter	$\hbar/k_B$ (K s)	h (J s)
Blue	$(7.21 \pm 0.20)\times 10^{-12}$	$(6.25 \pm 0.18)\times 10^{-34}$
Green (dark)	$(7.69 \pm 0.75)\times 10^{-12}$	$(6.67 \pm 0.65)\times 10^{-34}$
Red (bright)	$(7.54 \pm 0.08)\times 10^{-12}$	$(6.54 \pm 0.07)\times 10^{-34}$

last form does not require knowledge of the ambient temperature either during calibration or during the actual measurement of the Planck's constant.

A real filament is not a black-body. However, expression (4) yields correct results as long as filament emissivity $E(\omega,T)$ does not depend significantly upon temperature. For all practical cases it can be assumed that emissivity is constant as long as emitted power is proportional to T^4 (Stefan's law). We have checked this assumption for our filament by plotting an electrical power input $P = UI$ vs $(T^4 - T_0^4)$, and, in fact, the emissivity of the filament is constant—see Fig. 3. However, a filament is not a black-body, but a grey-body as its area calculated from the slope (Fig. 3) and Stefan's constant σ is about 50% of the geometrical area determined by the visible projection method.[2]

Typical results obtained for Planck's constant h are summarized in Table I; also indicated are the parameters of our calibration curve $T(K) = aR[(\Omega)] + b$.

IV. DISCUSSION

It has been shown that a primitive setup is able to yield the ratio of Planck's constant to Boltzmann's constant to within about 2% accuracy. Such a setup is easily accessible even for a high school physics laboratory.

Our improvements to the treatment of Crandall and Delord[2] are indicated by the following summarizing remarks. First of all, ac power supplies generate too much fluctuation in the power emitted by a filament. Also, light sensors relying upon internal photoeffect are too sensitive to the infrared radiation, the latter being very hard to filter out with low cost filters of sufficient transparency in the desired frequency range. An estimate of the filament temperature from the supplied electrical power is uncertain because an emitter is a grey-body having emissivity significantly less than 100%. There are also difficulties in estimating the effective area of the filament even by the visible projection method.[2] Finally, the expression for h/c^2 [Eq. (7) of Ref. 2] relies upon the fourth power of the logarithm of B_1/B_2, which leads to a magnification of relative errors in B_1 and/or B_2.

[1] R. L. Bobst and E. A. Karlow, "A direct potential measurement in the photoelectric effect experiment," Am. J. Phys. **53**, 911–912 (1985); J. Dean Barnett and Harold T. Stokes, "Improved student laboratory on the measurement of Planck's constant using the photoelectric effect," Am. J. Phys. **56**, 86–87 (1988).

[2] R. E. Crandall and J. F. Delord, "Minimal apparatus for determination of Planck's constant," Am. J. Phys. **51**, 90–91 (1983).