Statistical properties of White Noise
(Electronics & Signal Processing)

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The experiment gives you an introduction to noise and its analysis. Our main objective in this experiment is to investigate many statistical properties of white noise. This experiment is divided into sections, such that each section introduces one of the key concepts, and finally this culminates to our final objective.

KEYWORDS

Noise · Autocorrelation · White Noise · Power Spectral density · Probability density function · Standard Deviation · Effective Noise Power density

APPROXIMATE PERFORMANCE TIME 1 week

1 Experimental Objectives

The experimental objectives include:

1. understanding the concept of noise
2. understanding correlation and autocorrelation
3. filtering the white noise
4. understanding probability density function, and
5. measuring effective noise power density.

References and Essential Reading

[1] J. L. Passmore, B. C. Collings and P. J. Collings, "Autocorrelation of electrical noise: an undergraduate experiment", Amer. J. Phys. **63**, 592 (1995).

[2] P. Horowitz, The Art of Electronics, (Cambridge University Press, 1989).

[3] B. P. Lathi, Modern Analog and Digital Communication Systems, (Oxford University Press, 1998).

[4] Y. Kraftmakher, "Noise reduction by signal accumulation", Phys. Teach. **44**, 528 (2006).

[5] Y. Kraftmakher, "Two student experiments on electrical fluctuations", Amer. J. Phys. **63**, 932 (1995).

[6] A. Thompson and A. Trolestra, "Signal enhancement in the presence of high noise levels—the electrocardiogram", Phys. Teach. **34**, 418 (1996).

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2 Filtering White Noise

2.1 Objective

This section introduces the basic concepts of *white noise*, *correlation*, *autocorrelation*, and the effect of low pass filtering on white noise. The major impetus of this section comes from reference [1]. The circuit that generates the noise from pseudo-random bit sequences was designed in-house. We also designed the PCB that was fabricated from the hobbyist market. The complete details (including the circuit diagram) are provided on our website whereas the functional block diagram for the experiment is shown in Figure 1.

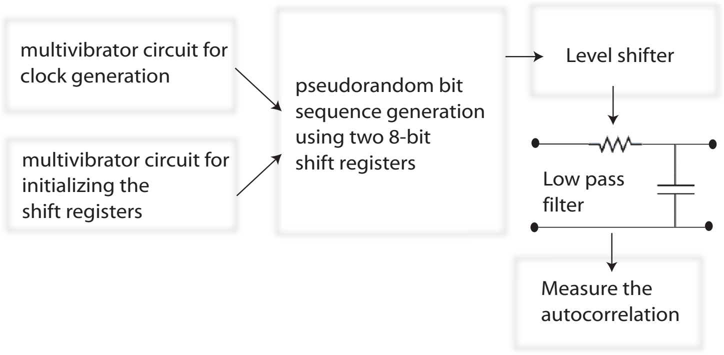

Figure 1: Functional block diagram for the low pass filtering of white noise.

2.2 Noise and its types

Anything that is mostly undesirable or unwanted is referred to as noise. Noise is inevitable in electric circuits.Noise can block, distort, or change the meaning of a message in both human and electronic communication. One form of electronic noise exists in all circuits and devices and is a result of *thermal noise*, also referred to as *Johnson noise*. In any electronic circuit, there exist random variations in current or voltage caused by the random movement of the electrons. *Pink noise* or *1/f noise* is noise whose graph between the noise power density versus frequency is proportional to the reciprocal of the frequency. It is sometimes also referred as *flicker noise*. The pink noise spectrum is shown in Figure 2(a).

2.3 White Noise

This is the type of noise in which all frequency components, ranging from zero frequency (DC) to infinite frequencies, are present. The graph between the noise power density versus frequency would be a constant line, and this continues to infinite frequencies. Thus it is safe to say that **white noise has all frequency components**. The white noise spectrum is shown in Figure 2(b).

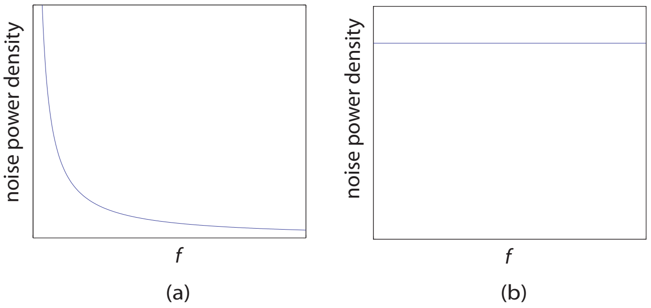

Figure 2: Power spectral density (noise power per unit frequency) versus frequency for (a) pink noise and (b) white noise.

In the present experiment, we will:

1. generate a pseudo-random binary signal (with logic levels +5 V and 0 V),
2. convert it into a symmmetric bipolar signal (with logic levels +5 V and −5 V), and
3. finally, low pass filter the signal and observe the autocorrelation of the output. (The autocorrelation will be discussed shortly.)

2.4 Correlation between signals

Correlation between two signals is the measure of the similarity or dissimilarity between them. Finding correlation between two signals is extremely important. This concept is widely used for signal processing applications in radar, sonar, and digital communications.

2.5 Autocorrelation

Autocorrelation is the correlation of the signal with itself. Informally, it is a measure of how well a signal matches a time-shifted version of itself, as a function of the amount of time shift.The autocorrelation of any signal is computed as follows,

1. Shift the signal in time.
2. Multiply each point of the original signal with the corresponding point in the time shifted signal.
3. Find the sum of all these products.
4. Divide by the total number of points. The answer is the value of the autocorrelation.

A high value of correlation (close to 1) indicates that the time-shifted and original signals are highly correlated. This means that the signal retains some kind of "memory"—the future samples are correlated with the present and past samples. A small value for the autocorrelation indicates that the signal *quickly* "forgets" its present values, suggestive of the presence of high frequency components, forcing the signal to change rapidly.

2.6 Autocorrelation of ideal white noise

White noise is composed of all possible frequency components. The existence of the high frequency components ensures that the values of the noise at *any* two instances are independent of each other. The noise values are changing rapidly, effectively they represent a random variable. Therefore, the autocorrelation of ideal white noise is zero for all non-zero time shifts. The autocorrelation function for ideal white noise is shown in Figure 3 and the concept is sketched out in Figure 4.

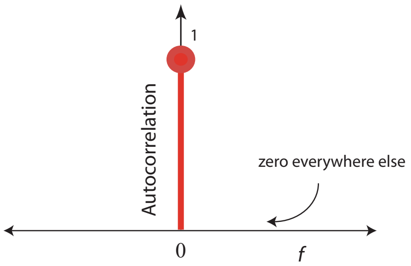

Figure 3: Autocorrelation function for ideal white noise.

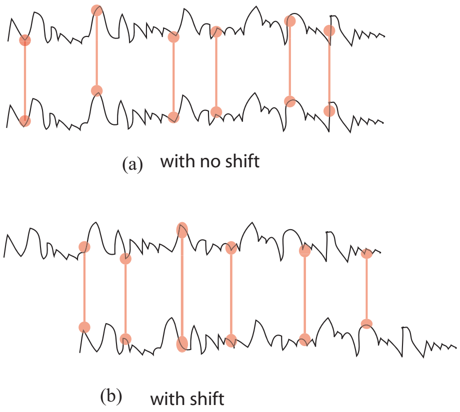

Figure 4: Autocorrelation of Ideal White Noise represented as the pointwise multiplication of noise with its time shifted signals. If the noise has a mean of zero, there will be as many positive products as there are negative products. Hence they will all sum up to zero.

2.7 Probability Density Function

White noise has many interesting statistical properties in addition to autocorrelation. A histogram is a good approximation to a probability density function (pdf) for a large number of data points. If we empirically sample numerous enough values of a continuous random variable, and make a histogram showing the relative frequencies of output ranges, the histogram will approximate the random variable's probability density.

2.8 Procedure

★ **Q** 1. The Pseudo Random Bit Sequence Generator PCB is provided to you. Connect it to the powers supply with $V_{cc} = 5$ Volts.

★ **Q** 2. Carry out the following experimental procedure to investigate the various statistical properties of white noise.

1. Run the Labview file **noise.vi**.
2. In the filename option, type in a filename of your choice such as **n1.txt**.
3. In the front panel window click the **Run** button (shown by the arrow key).
4. Now observe the following signals,
   (a) At **1**, the output from the first multivibrator
   (b) At **2**, the output from the second multivibrator,
   Now change the state of a switch 3 of a 4 way DIP switch (red color) on PCB. This will provide the feedback to the shift registers necessary to produce the pseudo random bit sequence.
   (c) At **3**, the digital noise appearing as a pseudo-random sequence of logic levels
   (d) At **4**, the level shifter output, once again a pseudo-random sequence of bipolar logic levels.
5. Stop data acquisition using the **Stop** button.
6. Design a low pass *RC* filter with a cut-off frequency of approximately 200 Hz.
7. Connect the output **4** to the filter.
8. Now, run the circuit again, type in your favourite filename (such as **n1.txt**) and observe the output across the capacitor.
9. On the front panel, click the button **Write 1000 data points**. This will save 1000 points from the filtered output in the file **n1.txt**.
10. After a delay of a few seconds, click the **Write 1000 data points** button again and repeat about ten times, thereby saving 1000 × 10 points, all of them successively arranged in the file **n1.txt**.
11. Stop the data acquisition.
12. Start Matlab and load the data saved from Labview using the command,

    ≫ **load('n1.txt')**;
13. Write the Matlab code to find the autocorrelation of the white noise.
14. Now design low pass filters for the following cut-off frequencies and repeat the above procedure. 10 Hz, 50 Hz, 100 Hz, 500 Hz, 1000 Hz.

★ **Q** 3. Plot the autocorrelation functions for different cut-off frequencies and compare between these plots.

★ **Q** 4. How do the autocorrelation functions depend on the cut-off frequencies? Come up with a suitable strategy to plot the relationship between autocorrelation and cut-off frequency. Discuss with your demonstrator.

★ **Q** 5. What is the advantage of saving 1000 × 10 data points for each value of the cut-off frequency instead of a 1000 points?

★ **Q** 6. Plot the probability density function at the above cut off frequencies. Also calculate the mean and standard deviation as well.