Magnetic Pendulum (2.12)

Introduction:

This experiment is aimed at exploring the idea of chaotic dynamics using a magnetic pendulum. There will be certain objectives discovered in this experiment as followed:

- How simple systems can be highly non-linear and exhibit complex behaviour under certain condition,
- How non-linearity can be made more prominent using simple methods and hardware modification,
- Discovering the simplistic richness of the mathematical and physical structure of dynamic systems.
- The conditions and consequences of the notion of super-sensitivity and its relationship with chaos,
- How to tell chaos apart from statistical indeterminancy
- Constructing and interpreting phase portraits and Poincare Maps for different types of responses of a system
- How fractals are associated with attractors and are manifest in the graphical data of such systems.

Simple Equation of a pendulum is as followed:

$$\tau = I\alpha => \ -mgSin\theta L = mL^2\frac{d^2\theta}{dt^2}$$

$$\frac{d^2\theta}{dt^2} + \frac{g}{L}Sin\theta = 0$$

$$Sin\theta \approx 0$$

$$\frac{d^2\theta}{dt^2} + \frac{g}{L}\theta = 0$$

Non linearity may exist in the equation due to the presence of $Sin\theta$ but if take small amplitudes in considerations that $Sin\theta$ maybe eliminated and linearity in equation may exist.

Apparatus and Method: Student Manual. [1]

Background Theory:

Determinism: This concept states that for everything that will happen, there are conditions given, without them this process may not happen. A deterministic system is whereby no randomness is involved in the development of future states of the system. [2]

Chaos Theory: Usually chaos is used to define an aperiodic time behaviour of a system in terms of a nonlinear dynamical system and is sometimes reffered as “noisy” or “random” behavior. Chaotic behavior occurs in a system that is free from noise and has few degrees of freedom. This tells us that chaos will occur in a system where the system is sensitive to initial conditions, it is topologically mixing and the periodic orbits are dense. Small changes in initial conditions lead to chaos in a system even if the system is deterministic, which means that the future behaviour is fully determined by the initial conditions and no randomness is involved.[3]

Phase Potrait: It is a graphical representations of trajectories in a dynamical system in the phase plane. The initial conditions are represented with different curve or a point. Phase portrait is a graphical representation of the trajectories of a dynamical system in a phase plane. It is the plot of Angular Speed against Phase angle. This plot consists of typical trajectories in the state space. We may find out information an attractor or repeller being present in the chosen parameter value. [4]

Poincare Map: It is the intersection of a periodic orbit in the state space of a continuous dynamical system with a certain lower dimensional subspace, called the Poincaré section, transversal to the flow of the system. It involves considering a periodic orbit with some initial conditions within a section of space, which leaves the section afterwards, and observes when the orbit returning to a point in that section. Then a map is formed to send first point to second. Poincaré maps can be interpreted as a discrete dynamical system. The stability of a periodic orbit of the original system is closely related to the stability of the fixed point of the corresponding Poincaré map. [5]

Attractors: An attractor is a set towards which a dynamical system evolves over time. Points thats close to this set remain close to this even if it is disturbed. It maybe a point, a curve, a manifold or even a complicated set with a fractal structure known as a “strange attractor”. [6] A fractal is "a rough or fragmented geometric shape that can be split into parts, each of which is (at least approximately) a reduced-size copy of the whole,"[1] a property called self-similarity. [7]

Self-similarity: A self –similar object maybe described as exactly or approximately similar to a part of itself. [8]

Resonance: Resonance is the tendency of an object to resonate with a larger amplitude at some frequencies as compared to others.

Results and Discussion:

The advantage of using a flywheel in the apparatus is that it maintains energy in the system as rotational energy. Also it was necessary to break the connecting rod into two to allow movement of the pulley and flywheel system. If we had used a smaller magnet instead of a large ring magnet, the magnetic field will be smaller and chaos could not be seen as prominently.

4.1: Measuring Nonlinearities

- Increasing the amplitude and measuring the Time Period.

Angle of displacement (°) (approximate)	No. of oscillations	Time (s)	Time Period (s)
10	5	4.18	0.836
15	10	7.38	0.738
25	15	11.35	0.756667
30	15	11.47	0.764667
45	15	10.75	0.716667

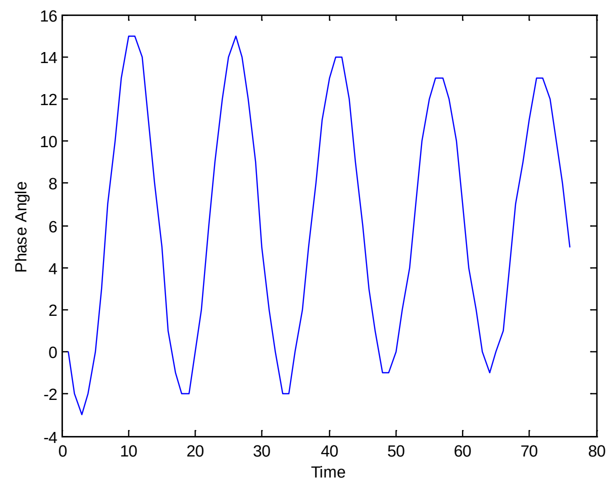
- Placing ring magnet and measuring Time Period for different amplitude.

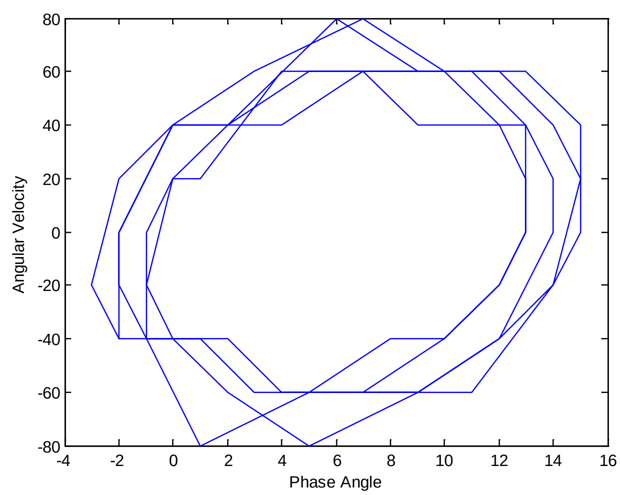
Angle of displacement (°) (approximate)	No of oscillations	Time (s)	Time Period (s)
10	8	6.62	0.8275
15	8	6.81	0.85125
25	4	3.09	0.7725
30	10	9.53	0.953
45	15	15.09	1.006

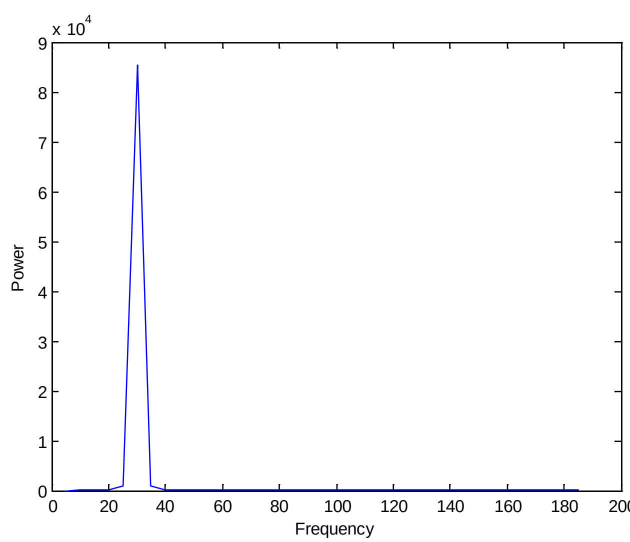
It can be seen when the amplitude is increased, the time period shows a general trend of increase without the ring magnet being placed under the pendulum. However, when the ring magnet is placed, the time period is seen to generally increase but the increase is greater as compared to when there was no ring magnet placed. The reading show when the amplitude of oscillations is small, the trend of time period is linear as $Sin\theta$ maybe ignored. However, when oscillation amplitude is increased, non-linearity can be observed as the $Sin\theta$ may not be neglected in the equation anymore.

Following graphs show graphs (time series, phase plots and fourier spectra) collected from MATLAB for different amplitude oscillations:

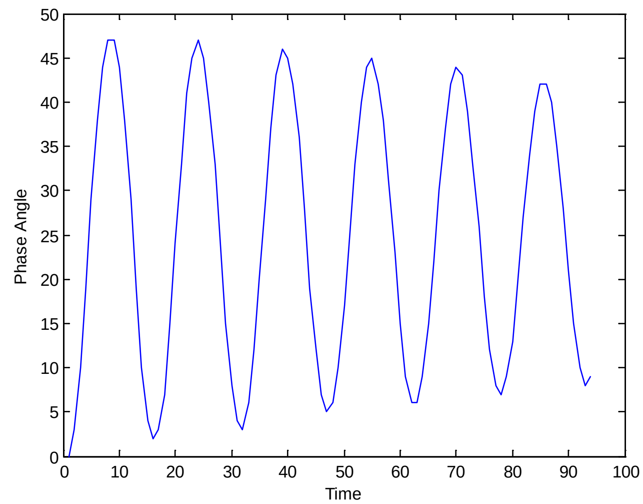
- Small amplitude

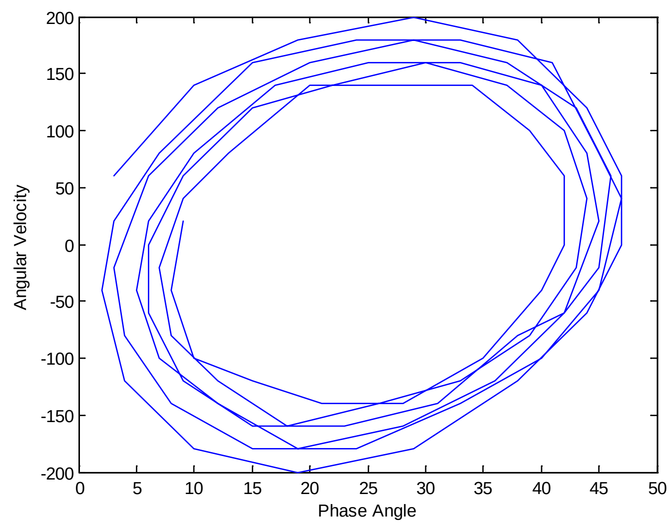


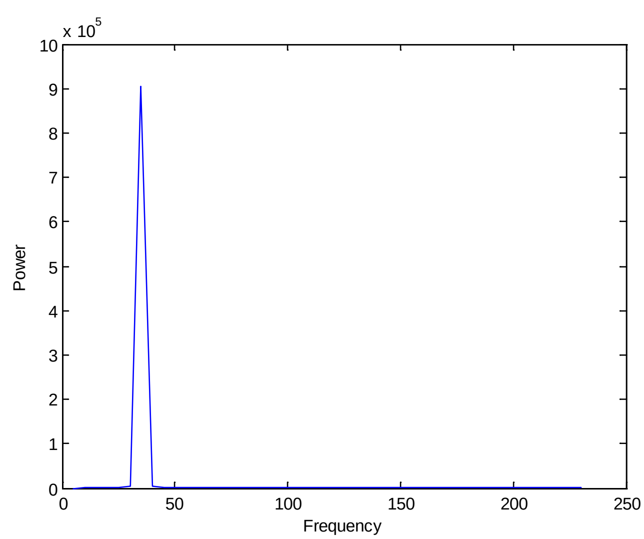




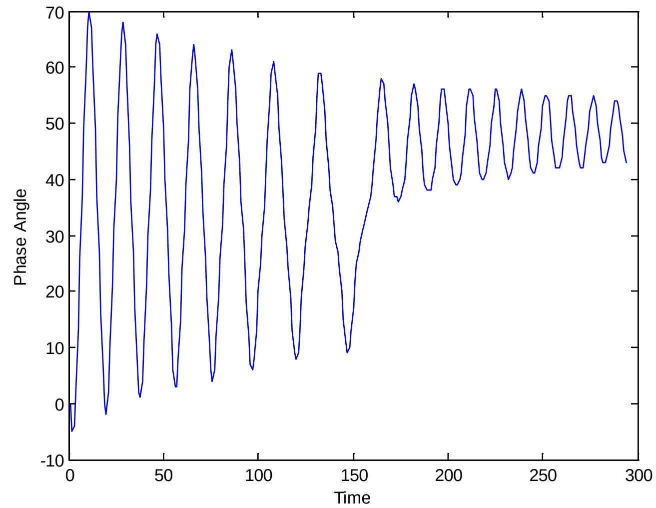
- Larger amplitude

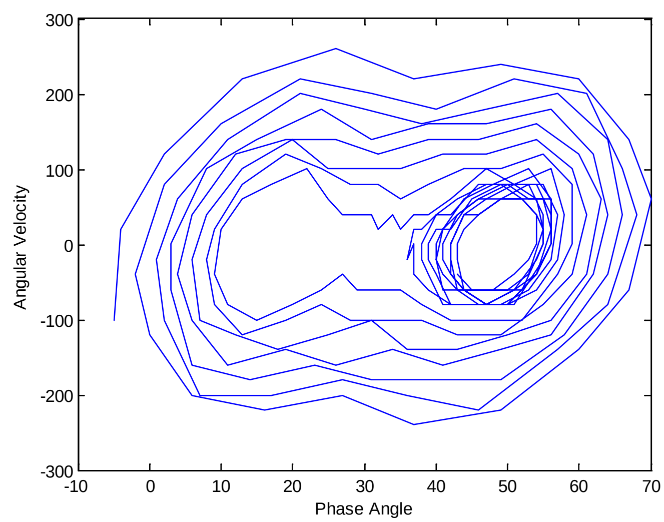


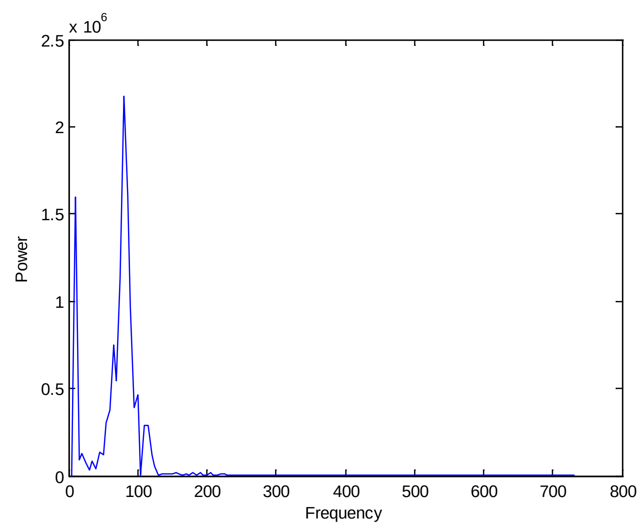




- Magnet placed



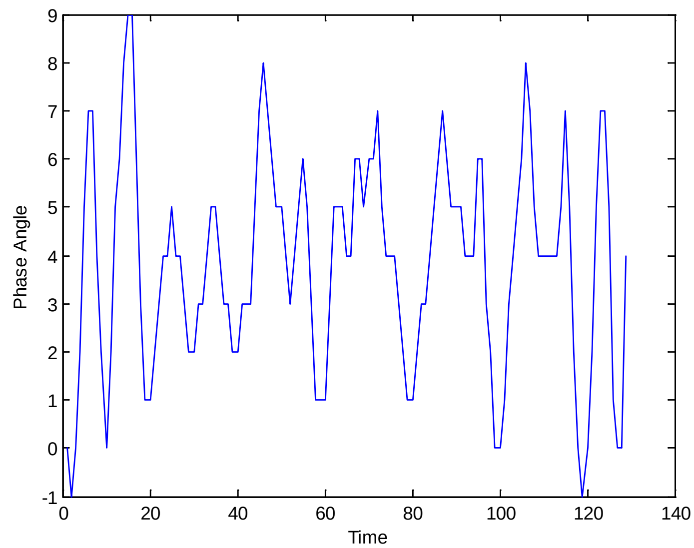


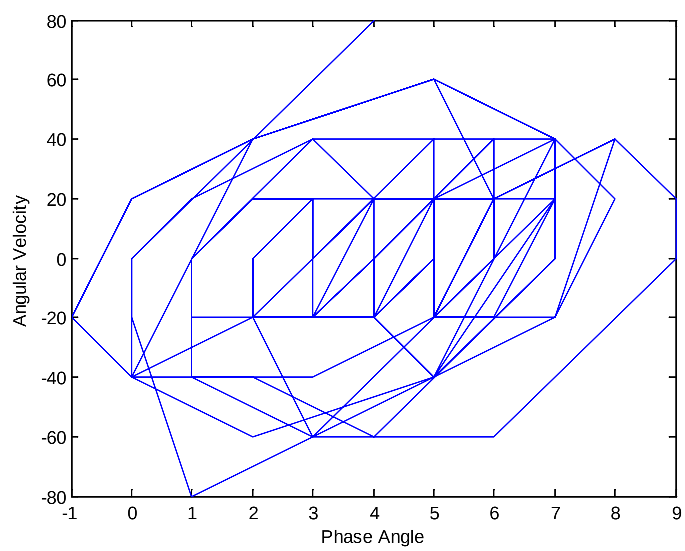


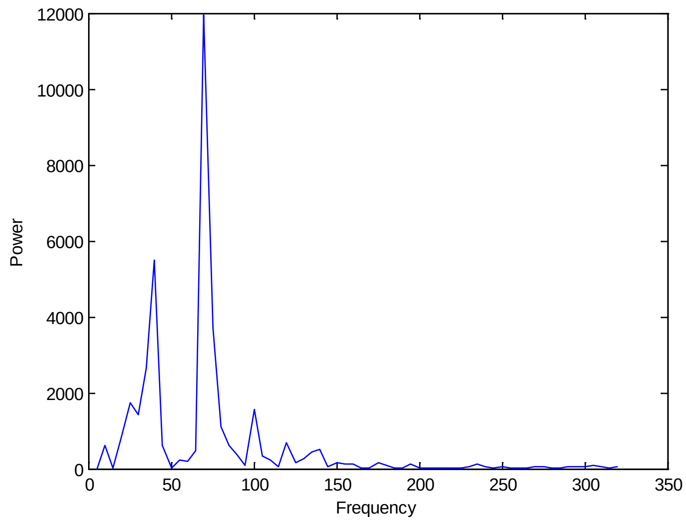
Increased non-linearity can be observed in the graphical data when the frequency of the time series starts to differ and does not remain constant. When the frequency changes considerably, degree of non-linearity is higher.

4.2: Stepping into Chaos

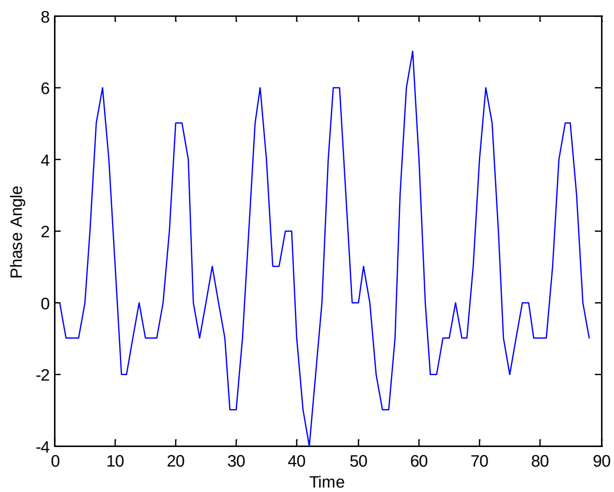
- Placing ring at 7cm, 0.9 cycles/s

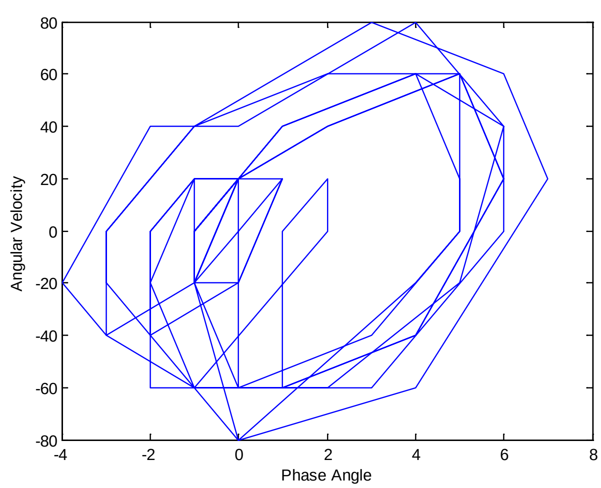


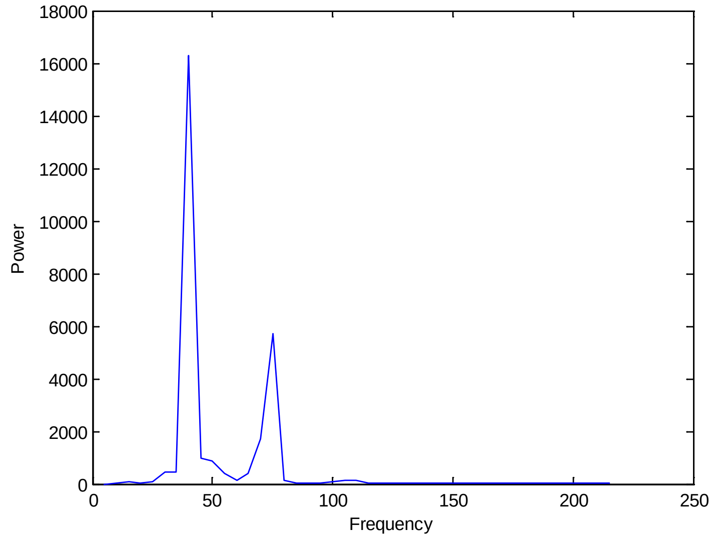




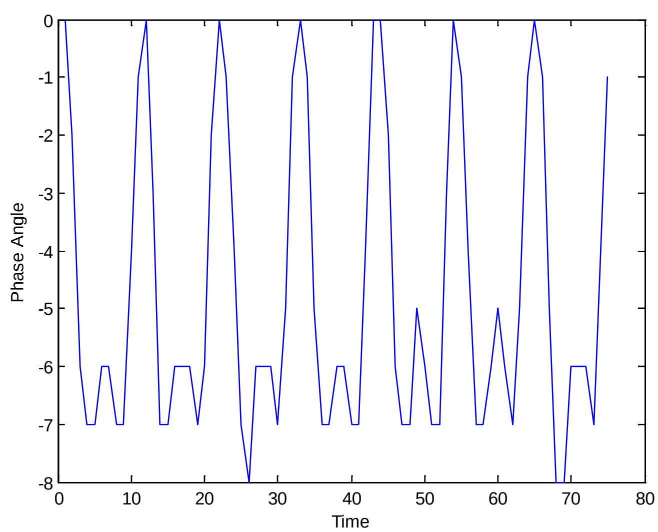
- Ring at 4 cm, 1.5 cyles/s

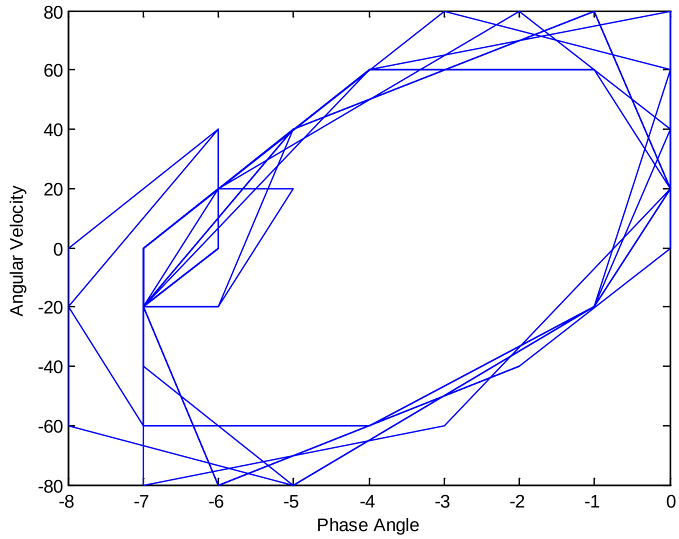


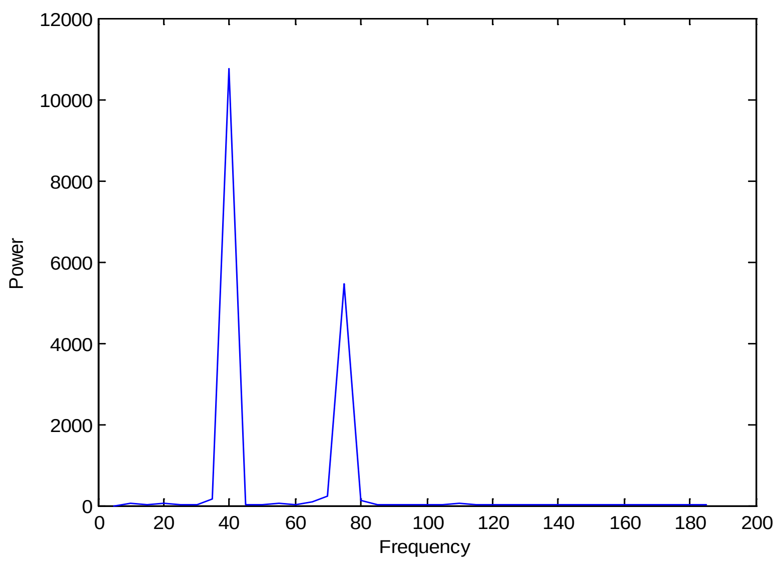




- Ring at 4 cm, 1.8 cycles/s

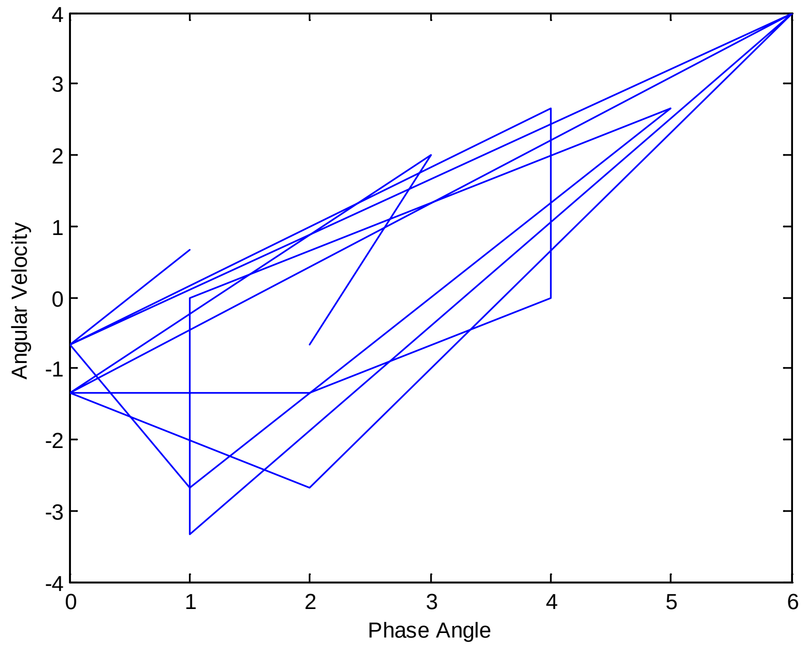






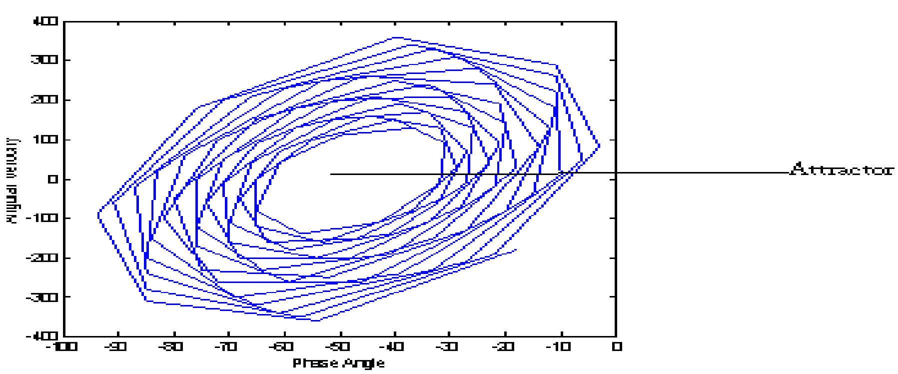
For a chaotic response, the fourier spectra observed shows that there are higher energy level at higher frequencies as compared to the fourier spectra of a periodic response. The time series and the phase portraits of a chaotic response are more random and disturbed as compared to the plots of a periodic response. It has been observed that for a lesser nonlinear response, the frequency remains almost the same but for a greater nonlinear response, it has been observed that the frequency changes a considerable amount.

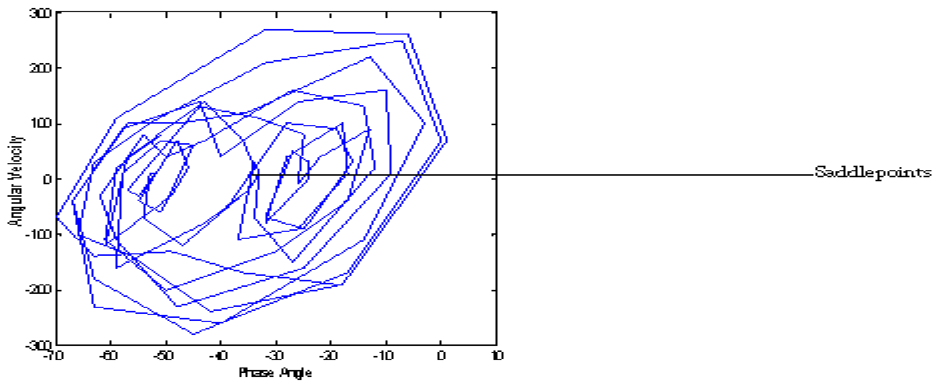
- Sampling time=1.5 samples/s, speed=1.5 cycles/s



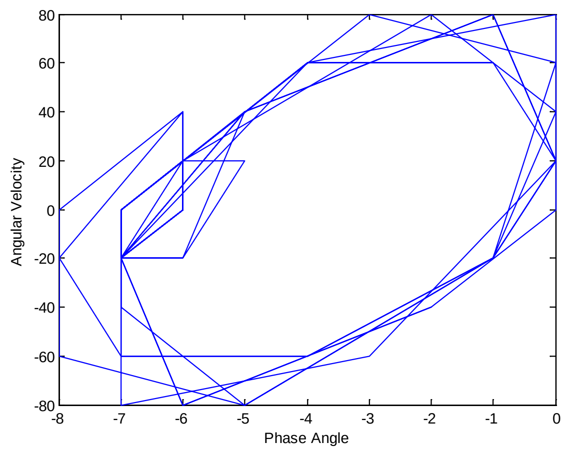
When the sampling time is made same as the speed of the pulley, the graph becomes more random and non-linear which proves that chaos has occurred.

If the pattern of the graph diverges from its normal pattern and becomes non-linear, it can be said the system has reached a state of chaos. We can identify a chaotic phase portrait by looking at the shape of an attractor. If the shape of an attractor looks random or strange, it can be said that system has reached a chaotic state.





Fractals can be observed in this phase portrait, as the attractor becomes "strange":



<u>Conclusion:</u>

To conclude, it can be said that the experiment successfully showed us chaotic dynamics using a magnetic pendulum. Time series, phase portraits and fourier spectra helped us gain an insight to the chaotic behavior of this system and identify different aspects of a phase portrait.

Bibliography:

[1]- Student Manual
[2]- Wikipedia
[3]-wikipediaHasselblatt, Boris; Anatole Katok (2003). A First Course in Dynamics: With a Panorama of Recent Developments. Cambridge University Press. ISBN 0521587506.]
[4]-Jordan, D. W.; Smith, P. (2007). Nonlinear Ordinary Differential Equations (fourth ed.). Oxford Univeresity Press. ISBN 978-0-19-9208241. Chapter 1.
[5]- Shivakumar JOLAD, Poincare Map and its application to 'Spinning Magnet' problem, (2005) Retrieved from "http://en.wikipedia.org/wiki/Poincar%C3%A9_map"
[6]- Attractor at Scholarpedia, curated by John Milnor.]

[7]- Mandelbrot, B.B. (1982). The Fractal Geometry of Nature. W.H. Freeman and Company.. ISBN 0-7167-1186-9.]
[8]- Benoît Mandelbrot, How Long Is the Coast of Britain? Statistical Self-Similarity and Fractional Dimension]