Frictional Damping of Rotational Motion

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The task was to improve the precision in height measurements of the Rotational Mechanics experiment performed in the freshmen physics lab by using the photogatein order to calculate frictional losses. We also analyzed the damping effect by observing the the heights reached in each successive maximums. This document consists of the following:

1. The calculation of the frictional losses.
2. Analysis of the damping of oscillations.

1 Preview and our Target

In the previous experiment on Rotational Mechanics, students were required to measure the Moment of Inertia of the **Main Platter**. This was done in two steps:

1. The photogate, connected with the Smart Timer was set across the Main Platter, with one strip of paper taped to block the photogateonce in every revolution. The photogate was set in the **Time - Fence**[1]. The mass of the Main Platter M(kg) and radius of the Main Platter and its pulleys were measured with mass balance and vernier callipers. Then a string, with one end tied to a hanger with a 100 g mass, was wound around the second pulley on the Main Platter (but not tied). The mass was then allowed to fall[2] with its initial height measured with a meter rule, and the Smart Timer was started as soon as the mass hit the ground. So the potential energy of the 150 g mass and its hanger was converted to the frictional losses (friction between the thread and the Main Platter pulley, so these depended on the number of revolutions that the string completed.) and the kinetic energy of the Main Platter. The relation is:

   Potential Energy of mass+hanger = Kinetic Energy of Main Platter+ Frictional losses

   From the time of the first revolution recorded by the Smart Timer, we calculated an approximation of ω, the angular velocity of the Main Platter at the instant when the mass hits the ground. Now, with frictional losses ignored in this step, we find the Moment of Inertia I (kg m^{-2}) from the following equation (using conservation of energy):

   $$E = mgh = \frac{I\omega^2}{2} \qquad (1)$$

2. Next, we wanted to find how much energy was lost to friction, so that it can be included in the above equation. For this, we need to tie the string to the pulley on the Main Platter (so that the mass doesn't fall to the ground), let the mass fall, and measure the height (using meter rule) of the first rebound. This would give us the total energy loss E_f due to friction. We will also find the total number of revolutions (n) of the Main Platter from the time we let the mass fall to the first rebound. Hence, we calculate the *frictional loss per revolution*, and this, multiplied with the number of revolutions in the first step, gives us the total energy loss due to friction in the first step. By including this in the equation above, we can get a better result for the Moment of Inertia (I) of the Main Platter.

This was an inaccurate setup for several reasons. Firstly, the height measurements from the meter rule required a judgement of the maxima from the student, and further with the tilted mass hanger, this had an uncertainty of about 1.5 cm. Secondly, with only one strip on the Main Platter to block the photogate, we only knew the revolutions correct to the next integer number of revolutions[3], and this causes large uncertainty in the frictional loss per revolution[4]. So my job was to find a method, making good use of the photogate , for recording both the height and the number of revolutions, to make a better measurement of the frictional losses per revolution.

[1] In this mode, the Smart Timer records the (cumulative) times for the first ten times that the photogate light is blocked

[2] Before we proceed further, as discussed later, we will let the mass fall (slowing it with our fingers) till the string gets loose, and find the number of revolutions using the photogate.

[3] For example, if 4.6 revolutions were made by the Main Platter, our photogatewill only record 4, as the fifth one is not complete.

[4] There is one further cause of error that we are still ignoring: In the energy equation above, we also have the kinetic energy lost by the hanger mass at the moment it falls to the ground.

Furthermore, the old experiment required measurements on only the first rebound. But we now want to analyze the damping effects of friction by observing all the vertical oscillations made by the hanger mass until it comes to rest. We need to calculate the potential energy at each maxima, and use Matlab's least square curve fitting routines to demonstrate the exponential decay in potential energy, along with the value of the decay constant. This report gives details of the steps taken to carry out this experiment and analysis.

2 Calculating Frictional losses

The aim was to improve our method of calculating *frictional loss per revolution*, to be included in our Moment of Inertia calculations.

The main losses that we have to improve are:

1. The height of the suspended mass was measured by a meter rule. With an additional judgement required by the user in order to measure the rebound height, this proved to be a very inaccurate method with an uncertainty of 1.0 cm.

2. In order to incorporate frictional losses in the Moment of Inertia calculation, we needed to find the frictional loss per revolution, and this required the measurement of revolutions made during fall and rebound by the Main Platter to a high precision. With only one paper strip on the Main Platter to block the photogate light, this was an extremely inaccurate method and needed to be improved.

I started the experiment by attaching four paper strips to the Main Platter, so that now the number of revolutions could be recorded up to the nearest 0.25 revolutions completed. However, their were significant variations in the number of counts recorded: By using the meter rule, I found that the number of revolutions for the hanger mass to fall to the minimum height was approximately 2.25 (i.e, 9 counts should be recorded by the Smart Timer), but in three sets of readings, 7, 9 and 10 counts were recorded. Also, the reaction time error (in stopping the timer when the minimum height was reached) was a very important contributor to this imprecision.

So, in order to deal with the reaction time error, a new procedure was followed: The Timer was started, and the hanger mass was slowly lowered by hand till the minimum height was reached, and the number of counts n_0 recorded. Now, the mass was reset and allowed to fall, this time stopping the Timer when the maximum height was reached on rebound. By subtracting n_0 from this count, we can find the number of counts for rebound alone.

Secondly, in order to further improve precision, the photogate was put across the Small Pulley, so that we get ten counts per every complete revolution of the Main Platter, increasing our accuracy to an appropriate 0.1 revolutions.

In order to test this, I used a longer thread and redid the experiment. In the first case, when the string in not tied and the mass is allowed to fall to the ground, 4.8 revolutions (48 counts) were made. The circumference of the second pulley on the Main Platter, around which the string is winded, is 12.8 cm, meaning that this is the height lost by the mass in every complete revolution of the Main Platter. Converting it to potential energy: $E_p = mgh = (0.10462)(9.81)(12.8) \approx 0.131$ J. When the string was tied in order to calculate friction, a total of 59 counts were recorded, with 3.9 revolutions completed in falling to the ground, so there were 2 revolutions lost due to friction on the first rebound. Hence, potential energy lost due to friction was: $E_p = mgh = (0.10462)(9.81)(12.8\times 2) = 0.263$ J. This energy was lost due to friction over a total of 5.9 revolutions, hence frictional loss per revolution is: $E_f = \frac{0.2627}{5.9} = 0.0445$ J. In our first case, 4.8 revolutions were made, hence total frictional loss is:

$$E_f = 0.04453 \times 4.8 = 0.2096 \text{ J.} \tag{2}$$

Without considering friction, the Moment of Inertia was calculated to be 0.0119kgm^{-2}. Theoretically, using the Moment of Inertia of a disc, it should be close to 0.0079kgm^{-2}.Now when we include frictional losses by this method, the new calculated Moment of Inertia was: $I = 0.0084\text{kgm}^{-2}$. This is a much closer and a better calculation, and concluded the improvement of frictional losses calculation.

Now we move on to the details of analyzing the damping of oscillations:

3 First Attempt: Using the small pulley

The photogate, set in the counts mode, was placed across the small pulley, to record ten counts per revolution of the small pulley[5].

The thread was wound around the second pulley of the main platter. Using the **vernier callipers**, the radius (r/m), the circumference d(m) and potential energy per count E(J) [6], was:

$$r = 20.4 \times 10^{-3}\text{m} \tag{3}$$

$$d = 2\pi r \approx 128 \times 10^{-3}\text{m} \tag{4}$$

[5]The small pulley rotates once for every rotation of the main platter, so in effect we are measuring the number of rotations of the main platter.

[6]calculated as $E = 0.1 \times mgd$, because d is the height that the mass looses for every revolution of the Main Platter

$$E = 0.1 \times m \times g \times d = 0.0136\text{J}. \quad (5)$$

I used a 150 g mass, as this better illustrates damping by giving greater number of oscillations. The **Smart Timer** was first used to record the number of counts for the mass to reach its minimum height. This came out to be 37. Then, the mass was let go from its maximum height and the Smart Timer reading at every maxima[7] was noted. A set of three readings ($x1$, $x2$ and $x3$) were taken, and their mean was computed. The computations done on Matlab are shown below:

3.1 Collecting the Data

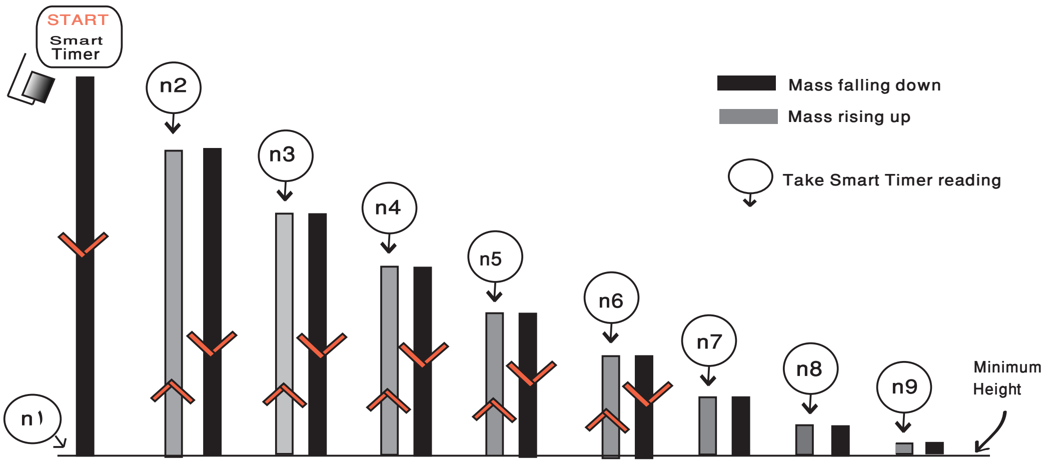

Figure 1: Collecting the data for the counts at each rewind.

We use the Smart Timer in the (Manual[8]) counts mode. The procedure is shown in Figure 1.

1. Ensure that the photogate is positioned correctly by checking that Smart Timer gives the correct number of counts (in this case 10) for one revolution.

2. Start the Smart Timer and lower the mass slowly till it reaches its minimum height, and record the Smart Timer reading (n_1). We can covert the number of counts to height traveled, so, taking the minimum potential energy to be 0J, this will tell us the maximum height.

3. Now rewind the mass again[9], start the Smart Timer and let go. As shown in the Figure 1, note the Smart Timer reading at every maxima, till the oscillation dies out. The Timer clicks momentarily stop at each maxima, so it is easier to record the readings.

4. The Smart Timer gives cumulative counts. Now from this information we need to find the height, or the counts (call these N_1, N_2, N_3 etc.) for each rewind. Let's consider the first case. We know that N_1, or the initial counts, equals n_1 recorded. For the first rewind, n_2 gives the total number of counts from the start. So the number of counts to rise up again, or N_2, are given by $n_2 - n_1$. Similarly, for the second rewind, $n_3 - n_2$ gives the counts to fall from the first rebound height and climb up to the second. So in order to find the counts for rising up to the second maxima alone, we need to subtract N_2 from this. Hence, N_3 is given by: $(n_3 - n_2) - N_2$.

5. Calculate all the values till the oscillations die out. The array $[N_1 N_2 N_3 ...]$ is one set of data. Repeat this procedure three or four times, take the average and use it to plot and curve fit an energy vs N graph to show the exponential damping and find the value of the decay constant.

3.2 The plots and Matlab

The Matlab code that I used to generate the damping plot is given below.

```
>>Energypercount = 0.019389;

>>x1 = [37 20 12 6 5 1 0];

>>x3 = [37 21 13 7 3 2 1];

>>x2 = [37 22 10 9 3 2 0];
```

[7]The timer counts momentarily stop when a maximum height is reached, enabling the observer to take the reading.
[8]In the Manual mode, it *ticks* on each count.
[9]It was observed that different counts were recorded for clockwise and anticlockwise rewinds, so decide before hand which way to rewind and be consistent with it.

```
>>x = (x1+x2+x3)/3;

>>Energy = x.*Energypercount;

>>n = [1:0.5:7];

>>figure;plot(n,Energy,'ro');

>>hold on;

>>plot(n,Energy); %we want only the plots and the curve fitting

>>xlabel('rebound number/Nth maxima (1 indicates initial height, 2 first rebound etc.)')

>>ylabel('Potential Energy/J (taking E=0 @ minimum height)')

>>title('Damping in Rotational Mechanics')
```

The resulting graph is shown in Figure 2.

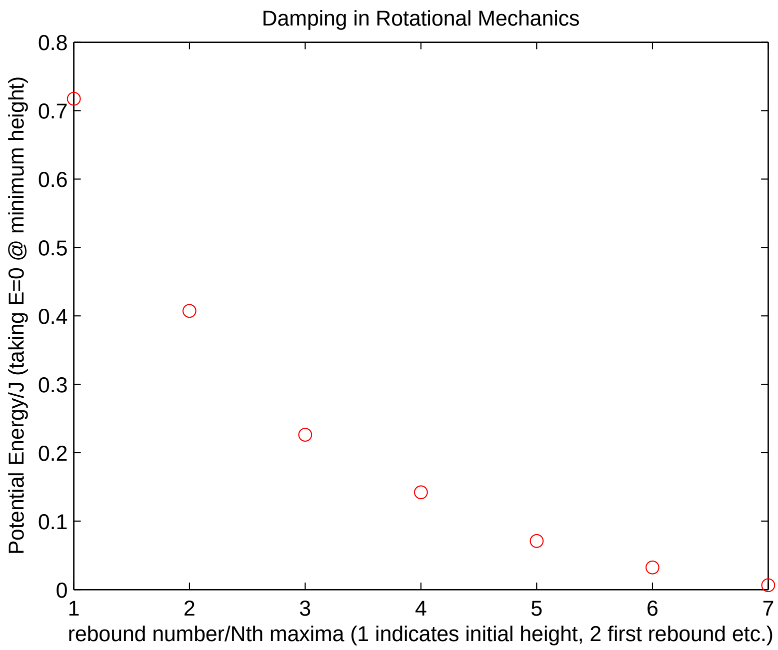

Figure 2: Graph generated without curve fitting.

So now we have a graph showing the rebound energies. It indicates exponential relationship, probably like the equation given:

$$E = A \exp(-\alpha N), \tag{6}$$

where E is potential Energy at the maxima, N is the number of Maxima[10], α is the decay constant and A is the amplitude. Therefore we want to fit it with the least square curvefitting function. The result is shown in Figure 3. The curve fitting also gives us the values of the amplitude A and decay constant $alpha$.

4 Second Attempt: Calibrating the Main Platter (manually)

Now, it was noted that, even with the use of a photogate across the small pulley, with only ten counts per revolution the uncertainty in the measurement of height was still very high. It was calculated to be 1.28 cm[11], which is not a significantly lower than the uncertainty from using meter rule. So although the use of photogate has improved the accuracy by enabling us to measure up to 0.1 revolutions (This is needed in calculating energy loss per revolution), we still need a better method for height measurement.

My next task, therefore, was to use a marked thread to make further calibrations on the Main Platter. As it has a larger circumference, we will calibrate it to get up to 32 counts per revolution, i.e, each division was set to 12.5^o, and repeat the experiment with the photogate being placed across the Main Platter.

[10] $N = 1$ means initial Potential Energy, $N = 2$ means Potential Energy after first rebound etc.

[11] $\Delta = 0.1 \times d$

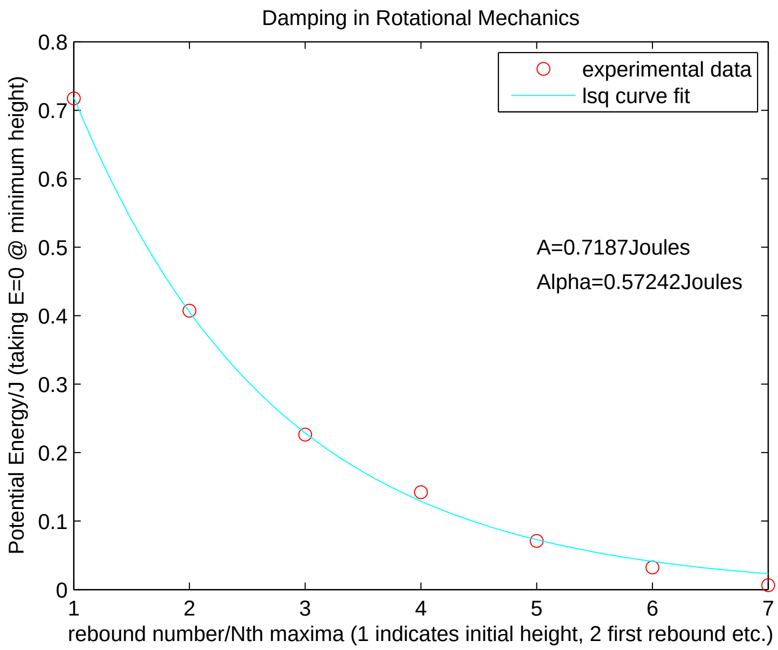

Figure 3: Graph generated with least square curve fitting

4.1 The Procedure and Setup

The setup is shown in Figure 4.

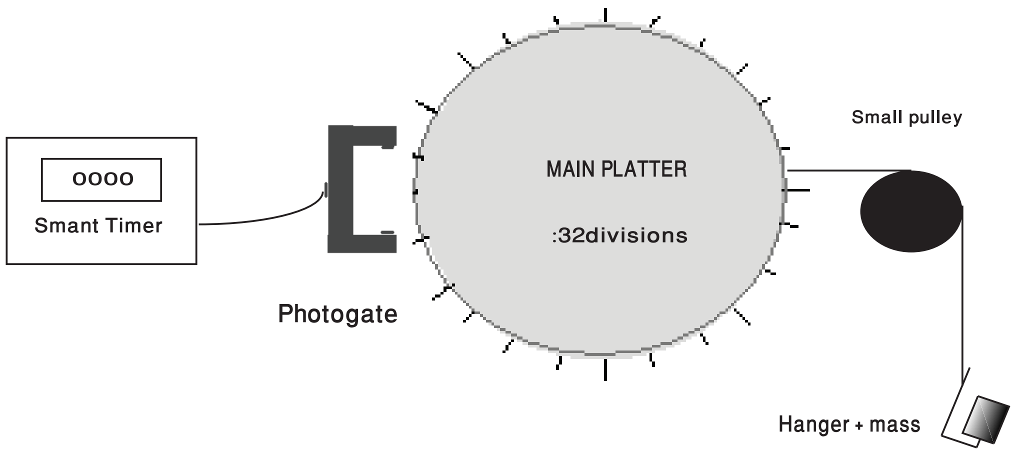

Figure 4: The setup, with the Main Platter calibrated

1. We measure the diameter of the main platter, and use it to calculate the circumference. This turned out to be 799.6 mm.

2. In order to calibrate the Main Platter with 32 equally spaced markings, we cut out s piece of thread of this circumference, by using a meter rule[12]. Then, by doubling the thread each time and marking, we make 32 markings on the thread. Check with a meter rule that the markings are of a reasonable accuracy (they should be 5 cm apart).

3. Now, using tape, paste the thread onto the side surface of the Main Platter.

4. Using the markings on the thread, stick pieces of wire taped to paper on the edge of the main platter. These will serve as markings. Use the retord stand to station the photogate across the Main Platter as shown above. Set the Smart Timer to the Counts - Manual Mode. After all 32 markings are done, slowly rotate the Main Platter once and check to see if you get 32 rotations.

5. After ensuring that photogate clicks on all of the markings, gradually lower the 150 g mass from the initial height, slowing it down with your hand, and note the Smart Timer Counts. You should check that this reading agrees with the number of revolutions you should get from dividing the change in height (as measured from meter rule) by the circumference of the small pulley.

[12]Take the thread of a length about $4 - 5$cm longer in order to make knots at both ends.

6. Rewind the mass again, start the Smart Timer and let go. Note the Smart Timer readings at every maxima and use them to find the number of counts (N) for each rewind as illustrated in section 3.2.

4.2 Observations and Result

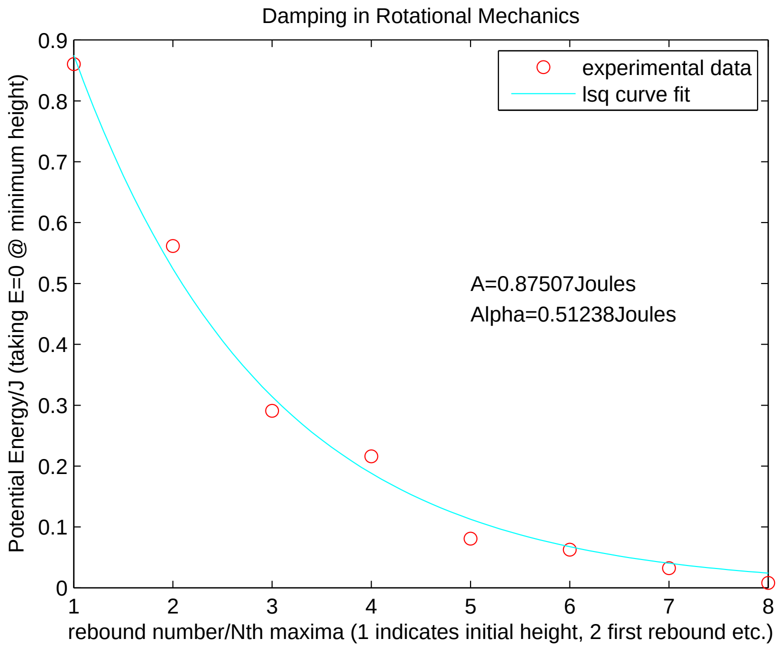

Figure 5: Graph generated and the curve fitting.

The resulting data, with least square curve fitting, is shown in Figure 5.

It can be seen from the curve fit that this data is not very precise. In fact, the experiment needed to be carried out more than ten times in order to get three suitable readings for the average, due to the fact that some of the markings often failed to block the photogate light, despite repeated checks. Furthermore, the manual calibration of the Main Platter using marked thread and pieces of wire taped to paper strips is a very time consuming task, possibly not suitable for the students in the given time frame. Hence, a further improvement is needed in calibrating the Main Platter.

5 Further Improvement

Mr.Wasif suggested an easier method of calibrating the Main Platter. With the diameter of the Main Platter(25.2 cm) and the separation between the markings (12.5^o) noted, we could make a cardboard drawing on **AutoCAD**[13] of a circle, the size of the Main Platter surface, and with 32 equally spaced markings on its edges. The markings have to be 4mm thick, as this thickness is required to block the photogate light. Then we simply paste it on the surface of the Main Platter. It has the advantage of both being accurate and quick (easier calibration, which the students can easily do in their given time frame). Then we check with the photogate that it gives 32 counts in one revolution, and repeat the procedure in Section 3.1. The setup is shown in Figure 6.

This was observed to be a much faster and easier approach. All we had to do was to keep checking that the paper markings remained straight and blocked the photogate. In all, four sets of reading were taken, and they gave a good precision.

The resulting plot, along with the least square curve fit and the values of the amplitude A and decay constant α, is shown in Figure 7.

6 Uncertainties

We have the uncertainties in the measurements of the height (h/cm), the number of revolutions (n):

$$\delta n = \frac{1}{32} = 0.031 \tag{7}$$

$$\delta h = \frac{1}{32} \times 2\pi r = 0.401 \tag{8}$$

To find the uncertainty in the potential energy E, we have two uncertainties to look at. From $E = mgh$,

[13] A commercial application for computer-aided-design (CAD).

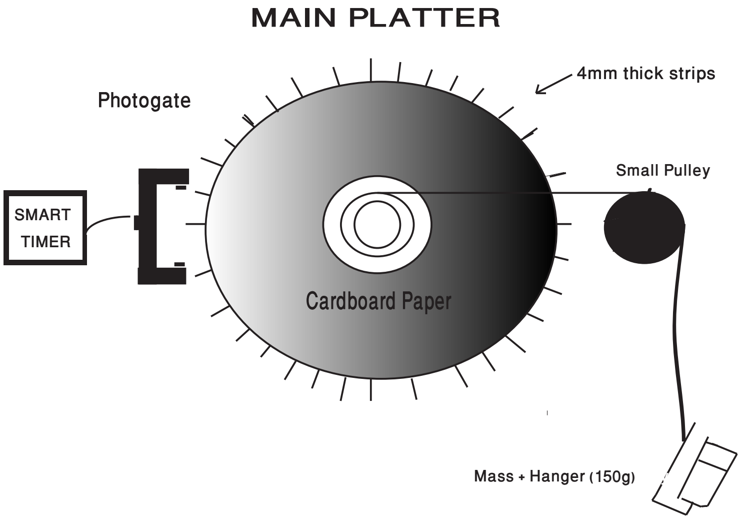

Figure 6: The new setup, with the marked cardboard pasted on the Main Platter.

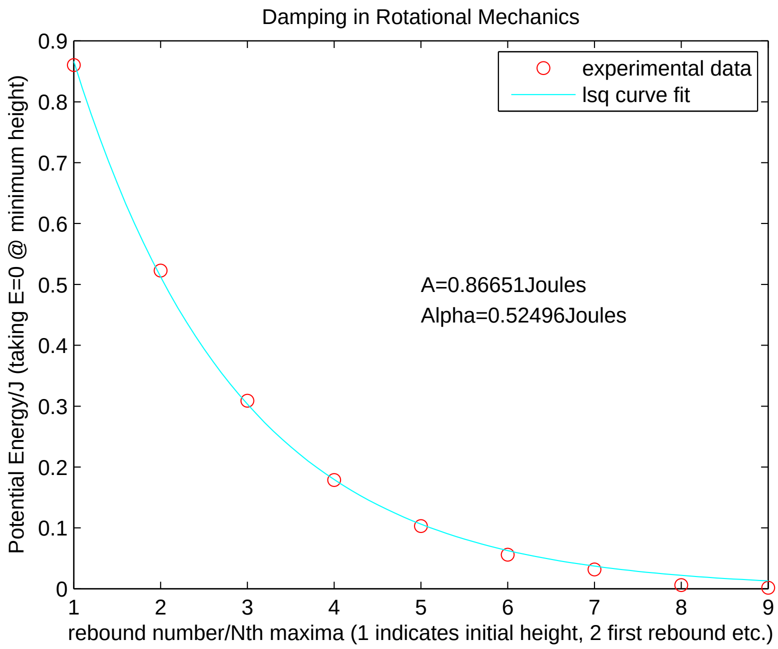

Figure 7: Final graph generated with curve fitting.

we have to take into account δh and δm. But we can ignore δm, as it is very small (±0.00001kg). So the uncertainty is:

$$\delta E = mg\delta h = 0.15420kg \times 0.9.81ms^{-2} \times 0.00401m = 0.006 \text{ J}. \quad (9)$$

7 Conclusions

The values of the decay constant in all three cases agree to the first decimal place. However. the curve fit, the convenience of the last method, along with the greater precision in the data collected (before the average was taken to be used in Matlab), suggests to me that the third method is by far the most effective for analyzing the damping.

In my opinion, this improvement can also be used in the calculation of the moment of inertia. We need the value of *instantaneous* ω, i.e, ω at the moment when the mass hits the ground. What our previous method gives us is the *average value* of ω for the first revolution that the Main Platter covers after the mass hits the ground, and this is significantly less than the instantaneous ω required. So, by making, say, 8 markings on the Main Platter, we can measure the angular velocity of the first 1/8th of the revolution, which is much closer to the instantaneous ω. [14]

[14] We will not need 32 markings for this part: the calculation of ω includes the reaction time error in starting the Timer when the mass hits the ground, and so further divisions will not decrease the uncertainty.