

# Investigations of Light with a Michelson Interferometer

Jeremy Ong\*

*Applied & Engineering Physics, Cornell University*

(Dated: April 5, 2010)

In this paper, we present several applications of the Michelson Interferometer in analyzing monochromatic, dichromatic, and white light. First, the wavelength of the mercury green line is determined to be  $540 \pm 36_{\text{theo}} \pm 10_{\text{sys}}$  nm by measuring the fringe separation. An estimate was made by counting fringes passing a reference point as a platform mounted mirror was moved translationally with a precision micrometer. Next, the two mercury yellow lines at  $576.99 \pm 0.07_{\text{theo}} \pm 0.04_{\text{sys}}$  nm and  $579.04 \pm 0.07_{\text{theo}} \pm 0.04_{\text{sys}}$  nm were determined by measuring the spacing between in phase and out of phase fringe segments. Finally, we report the observation of white light fringes that occur when the path length ratio approaches unity.

Keywords: Optics; Michelson Interferometer; Mercury Spectrum; White light fringes

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## I. INTRODUCTION

Introduced by Albert Michelson in 1881, the Michelson Interferometer was instrumental in ushering in the era of modern physics; most notably, it validated Einstein's theory of special relativity and dismissed the omnipresence of an æther through which light was thought to have propagated. It's applications, however, are varied and can be used to discern wavelengths given precise measurements of distance or vice versa.[2] In addition, the Michelson Interferometer can determine the separation of wavelengths of non-monochromatic light. If two wavelengths are present, they will "beat" in the same way that two closely separated audio waves beat. Intuitively, the interference patterns for each wavelength present in the extended source superimposes and will thus rotate in phase and out of phase as the path lengths are altered.

Superposition in this manner is true for arbitrarily contiguous light sources. The final application demonstrated in this study is the present of white light fringes which should exist in theory since there is a point where all waves interfere, irrespective of wavelength.[2]

## II. THEORETICAL BACKGROUND

Interference is one of the many significant consequences of the wave nature of light. Fortunately, these effects are also accessible, many of them are presented in an introductory physics course. The Michelson Interferometer is a practical way of superimposing two light sources to observe interference. A schematic from Guenther's *Modern Optics* is shown in figure 1.[1] After passage through a

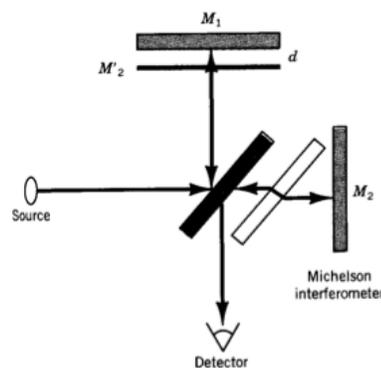


FIG. 1. Schematic of the Michelson Interferometer[1] -  $M_1$  denotes the moveable mirror and  $M_2$  denotes the tiltable mirror. The clear rectangle is made of the same material as the beam splitter (the solid rectangle) and is called the compensating plate. Its function is to equalize the optical path length along each arm. Without it, the light traversing the right arm travels a shorter distance in glass compared to air.

beam splitter the primary ray of light moves along two orthogonal paths (the arms of the interferometer) before rejoining to produce an interference pattern observed at the detector.  $M_2'$  is the location of the virtual image

\*jco34@cornell.edu; jeremyong.com

of  $M_2$  seen through the beam splitter.  $M_2$  and  $M'_2$  are equidistant from the beam splitter. The beams on each arm reflect from  $M_1$  and  $M_2$  identically so that both beams experience the same  $\pi$  phase change. Also, we assume that the entire apparatus is encased in air which has a dielectric constant approximately equal to unity. The difference in path lengths for parallel rays traversing each arm is given by  $|2d|$  since the light must travel to and back. Unfortunately, the rays incident on the detector are not all parallel; the rays in general are reflected at an angle  $\theta$  away from the center. Thus, the optical path length difference between the two arms is  $2d \cos \theta$  and modern optics dictate that maxima in the interference pattern occur when

$$2d \cos \theta = m\lambda \quad (1)$$

where  $m$  is an integer and  $\lambda$  is the wavelength.[2] Note that in equation 1, we have assumed that the light source is monochromatic. Also, we note that given a constant  $d$ ,  $m$ , and  $\lambda$ ,  $\cos \theta$  is constant, thus, these maxima and hence, the fringes between the maxima, are spherically symmetric. As we vary  $d$  linearly,  $m$  varies linearly as well (albeit discretely) so that a ring of maximal intensity disappears when  $2d$  decreases by  $\lambda$  and appears when  $2d$  is increased by  $\lambda$ . In this way, we can measure distances on the order of the wavelength of the light source or measure the wavelength of the light source by varying the position of  $M_1$  by a known quantity and counting fringes that appear or disappear, depending on the direction moved. If we fix our observation to a central reference point on the detector, we have that

$$\Delta d = \frac{(\Delta n)\lambda}{2} \quad (2)$$

where  $\Delta d$  is the change in displacement of  $M_1$  and where  $\Delta n$  is the number of fringes passing the reference point.

Now, we consider the case for when there are two wavelengths,  $\lambda_1$  and  $\lambda_2$  present in a dichromatic light source. The two interference patterns are dictated by equation 1 and are superimposed at the detector. The maxima in the combined interference patterns then, occur at displacements when each separate interference pattern is maximized, that is, when the optical path difference is an integer multiple of both  $\lambda_1$  and  $\lambda_2$ . The minima of the combined interference patterns occur directly between the maxima for symmetry reasons. Suppose we have found a displacement  $d_1$  which gives maximal fringe visibility in the field of view. Then, the next displacement which gives maximal fringe visibility occurs when

$$2(d_2 - d_1) = n\lambda_1 = (n + 1)\lambda_2 \quad (3)$$

for some integer  $n$ . In words, we require that the shorter wavelength wave shift one fringe more than the more slowly varying long wavelength in the course of a full period of beats. We can solve equation 3 for  $n$  as

$$n = \frac{\lambda_2}{\lambda_1 - \lambda_2} \quad (4)$$

and subsequent substitution of equation 4 back into equation 3 gives

$$\lambda_1 - \lambda_2 = \frac{\lambda_1 \lambda_2}{2(d_2 - d_1)} = \frac{\lambda_1 \lambda_2}{2\Delta d} \quad (5)$$

Let us denote  $\bar{\lambda}$  as the average wavelength. Then,  $2\bar{\lambda} - \lambda_1 = \lambda_2$  and  $2\bar{\lambda} - \lambda_2 = \lambda_1$ . If the wavelength separation is small, we define the small quantities  $\delta \equiv \bar{\lambda} - \lambda_1$  and  $\epsilon \equiv \bar{\lambda} - \lambda_2$ . Assuming the intensities of the two wavelengths are equal, we have  $\delta = -\epsilon$ . Then

$$\begin{aligned} \lambda_1 \lambda_2 &= (\bar{\lambda} + \delta)(\bar{\lambda} + \epsilon) \\ &= \bar{\lambda}^2 + \delta\bar{\lambda} + \epsilon\bar{\lambda} + \delta\epsilon \\ &= \bar{\lambda}^2 - \delta^2 \\ &\approx \lambda^2 \end{aligned}$$

to good approximation so that

$$\lambda_1 - \lambda_2 = \frac{\lambda^2}{2\Delta d} \quad (6)$$

This gives a way of determining the wavelength separation given the average of the wavelength. If it is assumed that the intensities are approximately the same, then the average is centered between  $\lambda_1$  and  $\lambda_2$ .

Finally, we discuss the possibility of the observation of white light fringes with the Michelson Interferometer. We first observe that equation 1 is wavelength independent only when  $d = 0$ , that is, when  $M_1$  coincides with  $M'_2$ . It is at this zero optical path difference that fringe effects for white light are observed, since white light contains a range of wavelengths from 400 nm to 750 nm.[1, 2] As  $M_1$  is displaced from  $M'_2$ , however, the fringes of different colors begin to separate until after roughly 8 to 10 fringes, many colors persist at any particular point, washing out the fringe pattern. Note that the presence of the compensating plate is crucial in this analysis so that the light travels through glass three times regardless of which path it takes.[2]

### III. DESCRIPTION OF THE APPARATUS

Figure 2 is picture of the apparatus used in this experiment taken onsite. The moveable mirror  $M_1$  is controlled by a lever arm attached to an adjustable micrometer. The light sources used were a standard mercury lamp and white light source. To isolate specific bandwidths of the mercury spectrum, a green or yellow filter was placed between the source and the beam splitter.  $M_2$  is tilt-adjustable with two screws at opposite corners and spring tension at a third corner. The center of the eyepiece is marked by a reference dot denoting the center of the field of view.

The experiment was divided into three parts as outlined by the Physics 410 experiment reference packet.[3] First, fringes passing the reference point were counted as

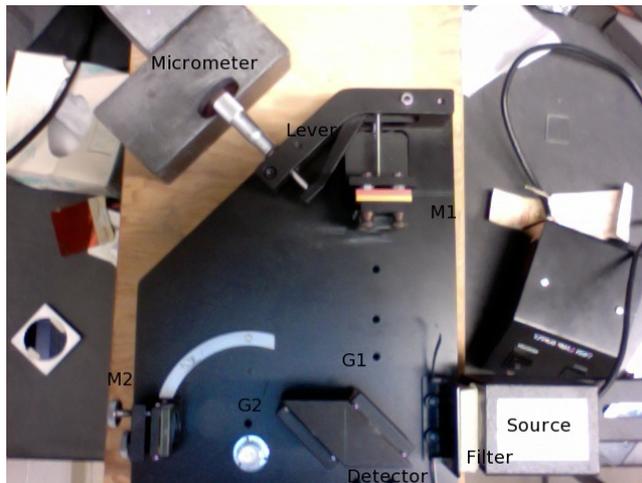


FIG. 2. Overview of Michelson Interferometer Setup (manufactured by Ealing). M1 and M2 are translatable and tiltable mirrors respectively. The micrometer is coupled to the lever arm and is used to finely adjust the position of M1. The source can produce mercury light or white light. A slot between the source and glass panels G1 and G2 can be used to filter out the green and yellow lines of the mercury spectrum.

a function of micrometer displacement for mercury green light. A picture of these fringes and the reference point is given in figure 3. A course ruler measurement was used to ensure that the readings corresponded to theory. Then, the known mercury green line wavelength was used to calibrate the screw and determine the screw error.

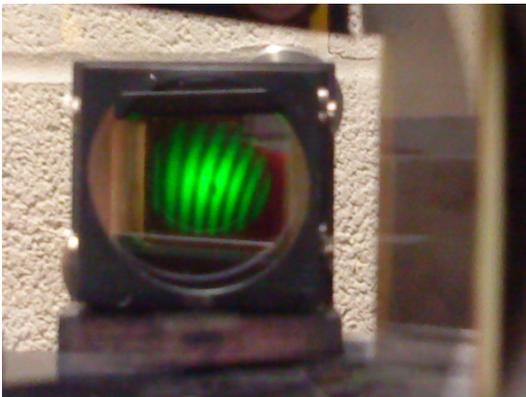


FIG. 3. A view of mercury green fringes. The central dot is a reference point from which fringes passing may be counted.

Next, the green filter was replaced by the yellow filter to observe the interference pattern produced by mercury's two yellow lines. The calibrated micrometer could then be used to measure the displacement required to move from one region of fringe non-visibility to the next.

Finally, the mercury source was swapped with a white light source with all filters removed and white light fringes were observed at the displacement which satisfied the zero optical path length difference condition.

#### IV. DATA TABLES

The data in table I was collected by first adjusting the tilt and  $M_1$  displacement to produce visible, distinct, and nearly straight fringes. Adjusting the fringes to appear this way made counting the fringes easier and less error prone. The micrometer was turned in one direction throughout the measurement to prevent backlash (indeed the micrometer was turned in one direction throughout this experiment). This data represents the third attempt at procuring accurate data. The first two sets were deemed invalid because of human error, either due to dry eyes or attempting to count too many fringes at once.

Fringes passing reference	Micrometer degrees
0	10' 0.00"
50	10' 6.75"
100	10' 13.50"
150	10' 20.00"
200	10' 27.00"
250	10' 35.75"
300	10' 40.25"
350	10' 47.00"
400	10' 53.50"
450	10' 60.25"
500	10' 67.50"
550	10' 74.00"
600	10' 80.75"
650	10' 87.75"
700	10' 94.25"
750	11' 0.50"
800	11' 6.75"
850	11' 13.75"
900	11' 19.75"
950	11' 26.25"
1000	11' 33.50"

TABLE I. Number of mercury green line fringes passing reference point counted as a function of micrometer rotation

Next, the green filter was exchanged with a yellow filter to observe the interference pattern produced by mercury's two yellow lines. The displacements recorded correspond to displacements of least fringe visibility.

Figure 4 shows a view of the white light fringes from the detector side of the interferometer..

#### V. ANALYSIS

To determine the wavelength of mercury, we first need to determine how a degree change in the micrometer corresponds to displacement of the moveable error. Rotation

Adjacent displacements corresponding to least fringe visibility
10' 82.0"
11' 20.0"
11' 62.0"
12' 1.0"
12' 43.5"

TABLE II. In micrometer degrees, this table records the readings for which the fringes from the two yellow mercury lines wash out. Note that due to the sinusoidal nature of the beat frequency, the displacements between disappearances is half the beat wavelength.

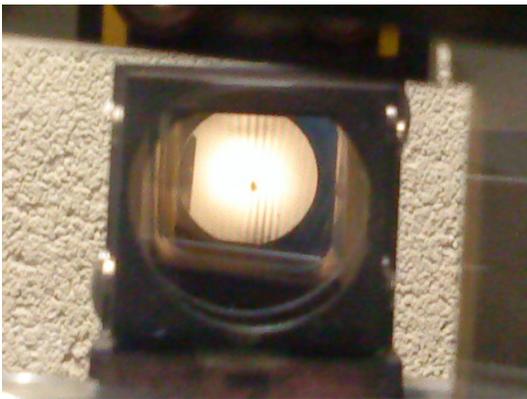


FIG. 4. White light fringes are visible here with a mirror displacement ranging from 10.5' 30.00" to 10.5' 31.50" on the micrometer scale. Roughly 16 clear to semi-clear fringes are visible. The white glare in the center is due to overexposure of the camera lens and is not actually seen when viewed with the naked eye.

of the micrometer by a full twenty degrees translates the mirror by  $5/32 \pm 1/128$ ths of an inch. This coarse measurement gives the correspondance of  $200 \pm 10 \mu\text{m}$  per degree. The average number of subdegrees required for fifty fringes to pass is  $6.7 \pm .3$  using data from table I, assuming a Gaussian distribution. In microns, this is approximately  $13.4 \pm 0.9 \mu\text{m}$  and from equation 2, we have

$$\lambda_{\text{green}} = \frac{2(\Delta d)}{\Delta n} \approx 540 \pm 36 \text{ nm} \quad (7)$$

This corresponds to within .02% of the accepted wavelength of mercury's green line, 546.0735(5) nm.

Usually, it is prudent to use the accepted value for the mercury green spectral line to calibrate the screw ratio, since a ruler measurement can't be expected to produce accurate data for displacements on the order of microns. The wavelength of the mercury green line is accepted to be 546.073(5) nm.[4] Using an average of  $6.7 \pm .3$ " per 50 fringes passing the reference point, we use equation 2 again to compute a ratio of  $204 \pm 9 \mu\text{m}$  per degree. The uncertainty margin here is similar to that produced by

the ruler measurement indicating that the uncertainty in counting is similar to the uncertainty introduced in measuring distances with a precision ruler.

Next, we compute the average of the separations of table II as  $40 \pm 2$ ", again assuming a Gaussian distribution. In microns, the separation distance is given by  $81.6 \pm 5.4 \mu\text{m}$ . Given that the average wavelength of the mercury yellow lines is 578.013(5) nm[4], we can use equation 6 to compute the separation.

$$\lambda_1 - \lambda_2 \approx \frac{578.013^2}{2 \cdot 81600} = 2.05 \pm .14 \text{ nm} \quad (8)$$

so that one line has a wavelength of approximately  $579.04 \pm 0.07 \text{ nm}$  and the other line has wavelength approximately  $576.99 \pm 0.07 \text{ nm}$ . All uncertainty estimates in this section were determined via standard error propagation procedures and we relegate assessments of uncertainty and possible systematic uncertainty to the next section.

The white fringes can be reproduced consistently after a few tries. The approximate range over which they are visible is 1.5" so the micrometer needs to be adjusted very slowly when searching for them. The number of white fringes visible in this interferometer agrees with Jenkins & White; in general, they report that 8 to 10 fringes are visible on each side of the central fringe.[2]

## VI. QUANTITATIVE UNCERTAINTY ASSESSMENT

Uncertainties reported in section V were computed using the method of quadrature. A sample calculation is shown below. Here, we compute the uncertainty in the wavelength separation of the mercury yellow lines given the average wavelength and the half-beat-wavelength using equation 9.

$$\sigma_f^2 = \sum_i^n \left( \frac{\partial f}{\partial x_i} \right)^2 \sigma_{x_i}^2 \text{ where } f = f(x_1, \dots, x_n) \quad (9)$$

$$f = \lambda_1 - \lambda_2 = \frac{\bar{\lambda}^2}{2d}$$

$$\sigma_f^2 = \left( \frac{\bar{\lambda}}{d} \right)^2 \sigma_{\bar{\lambda}}^2 + \left( \frac{\bar{\lambda}^2}{2d^2} \right)^2 \sigma_d^2$$

so that  $\sigma_f = .14 \text{ nm}$  as reported. Other uncertainties are computed in like manner.

As the device itself is fairly simple, there are few problems that might cause global systematic uncertainties. The most significant of such effects is the backlash present in the micrometer. Care was taken throughout the experiment to continue to rotate the micrometer in one direction. However, due to the nature of the record-taking, the micrometer motion needed to be stopped at each measurement. Thus, we can expect distance measurements to vary an additional quarter of a subdegree or about half a micron.

## VII. CONCLUSION

In conclusion, the experiments both determined values for the mercury green and yellow line wavelengths to great accuracy and demonstrated the existence of white light fringes. The mercury green line is reported to be  $540 \pm 36_{\text{theo}} \pm 10_{\text{sys}}$  nm in agreement with the accepted value, 546.0735(5) nm.[4] The mercury yellow lines from experiment are  $576.99 \pm 0.07_{\text{theo}} \pm 0.04_{\text{sys}}$  nm and  $579.04 \pm 0.07_{\text{theo}} \pm 0.04_{\text{sys}}$  nm in agreement with their accepted values, 576.9598(5) nm and 579.0663(5) nm, as

well.[4] Finally, the number of visible white light fringes corresponds roughly to the number of fringes expected in Jenkins & White and our experiment is in agreement with standard optics theory.

## ACKNOWLEDGMENTS

Thanks to Nick Szabo and Don Hartill at Cornell University for their guidance and assistance.

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 [2] Jenkins, F. A. and White, H. E., *Fundamentals of Optics*, 4th ed. (Mcgraw-Hill College, 1976) Chap. 13.  
 [3] Physics-410/510-staff, *Experiment O2 - Michelson Interferometer* (Cornell University, 1992).

- [4] Sansonetti, J. E. and Martin, W. C., “Mercury table,” (2010), <http://physics.nist.gov/PhysRefData/Handbook/Tables/mercurytable2.htm>.



FIG. 5. “Gah my finger twitched again!”

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