in the identical manner in which the cross hair was formerly set on S_1. Any difference is to be added or subtracted from the rated scale distance $S_2 - S_1$. For instance, suppose that when the micrometer microscope was originally set on S_1 the average of the readings on the head was 87.5, the pitch of the micrometer screw being 1 mm. That when the corrective setting was finally made on S_2, the average of the readings on the head turned out to be 88.8. The difference in the settings is 0.013 mm. This amount is to be added to or subtracted from the rated distance between S_2 and S_1, depending on whether increased readings on the head correspond to an advance of the cross hair toward S_2 or a motion back toward S_1.

Another method which may be successful if it is practiced before the experiment is under way is as follows: Start counting fringes *a little before* the cross hair reaches S_1, and when the observer at the microscope decides the cross hair is just on S_1, note the extent of the count. Also, continue counting *a little after* the cross hair is seen to reach S_2. If n_1 is the number in the count as the cross hair passes S_1, and n_2 is the number in the count as it passes S_2, then $n_2 - n_1$ is the number of fringes corresponding to $S_2 - S_1$.

$$(S_2 - S_1) = (n_2 - n_1)\frac{\lambda}{2}.$$

Obviously, if the rated distance $S_2 - S_1$ is known accurately, the wave-length of the light may be found, instead of the scale being calibrated.

EXPERIMENT 10

MEASUREMENT OF INDEX OF REFRACTION WITH A MICHELSON INTERFEROMETER

Theory.—If a plane-parallel plate of index of refraction n is inserted normal to the path of one of the beams of light traversing the arms of a Michelson interferometer, the increase of optical path introduced will be $2(n - 1)t$, where t is the thickness of the plate. The factor 2 occurs because the light traverses the plate twice. For monochromatic light of wave-length λ, the difference of path introduced is $N\lambda$, where N is the number of fringes displacement introduced when the plate is inserted. Hence, if a

Michelson interferometer is adjusted for white-light fringes, a parallel plate of index n inserted in one of the paths, and a count made of the number of fringes which cross the field when equality of optical path is reestablished, it would be possible to measure n with a high degree of accuracy. This is not a satisfactory method of measuring the index of refraction, first, because N is too large a number to be conveniently counted unless the plate is very thin; second, because it is extremely difficult to determine the center of a white-light-fringe pattern when the two arms of the interferometer contain unequal thicknesses of glass. If, however, a parallel plate in one of the arms is rotated through a small measured angle, the path of the light will be changed, and the number of fringes N corresponding to this change may be counted. The exact method of performing this experiment will be described in a later paragraph.

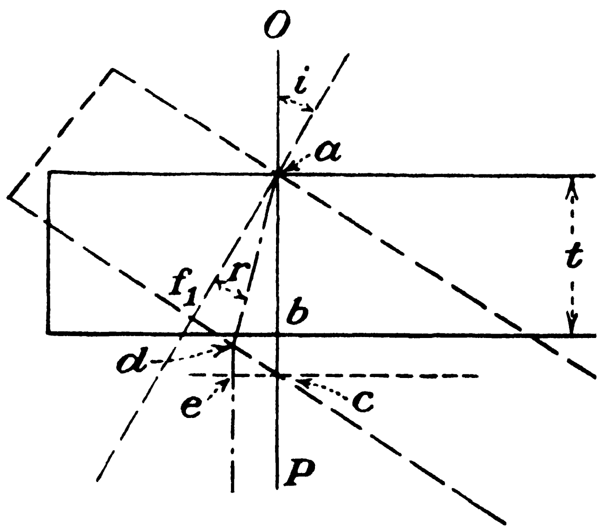

FIG. 1.

The change of path through the glass plate depends upon the thickness of the plate, the angle through which it is turned, and the index of refraction. The last of these three may be calculated if the other two are measured. Let OP (Fig. 1) be the original direction of the light normal to plate of thickness t. The total optical path between a and c for the light going in one direction is $nt + bc$. After the plate is rotated through an angle i, this optical path has been increased to $ad \cdot n + de$. Hence the total increase of optical path, since the light travels over the path twice, is

$$2(ad \cdot n + de - nt - bc) = N\lambda. \qquad (1)$$

But

$$ad = \frac{t}{\cos r},$$

$$de = dc \sin i = (fc - fd) \sin i = t \tan i \sin i - t \tan r \sin i,$$

$$bc = \frac{t}{\cos i} - t.$$

So,

$$\frac{nt}{\cos r} + t \tan i \sin i - t \tan r \sin i - nt - \frac{t}{\cos i} + t = \frac{N\lambda}{2}.$$

Using Snell's law, $n \sin r = \sin i$, this may be reduced to

$$n(1 - \cos i)2t - N\lambda = (2t - N\lambda)(1 - \cos i) + \frac{N^2\lambda^2}{4t}. \quad (2)$$

Since the last term is small compared to the others, it may be neglected, leaving for the index of refraction

$$n = \frac{(2t - N\lambda)(1 - \cos i)}{2t(1 - \cos i) - N\lambda}. \quad (3)$$

In the experiment, two such plates, P_1 and P_2, are used, one in either arm of the interferometer. These are made only half as high as the mirrors A and B so as to permit the observation in the field of view above them of fringes unaffected by the change of angle i. The use of two plates insures equal optical paths in the two arms, at all times when the angles of these plates with the direction of the light beams are the same, making possible the observation of white-light fringes through the plates when they are tilted at the same angle with the beam.

First, by the method outlined in Experiment 8, obtain in the upper part of the field vertical white-light fringes. This had better be done with the half plates P_1 and P_2 already in place, as inserting them afterward may be the cause of an accidental displacement of the other parts of the interferometer. With the white light fringes obtained, next set P_1 and P_2 normal to the light path as nearly as can be done while looking down on the instrument. Then, while observing the fringes, turn P_1 slowly until the fringes appear also in the lower part of the field. Now observe what happens if half plate P_2 is rotated a slight amount in one direction. If the lower fringes move completely out of the field and do not return, rotate P_2 in the other direction. What will usually happen is that either in turning one way or the other the fringe system will be displaced a number of fringes, say, to the right, and then move in the opposite direction. This indicates that the half plate P_1 was not, in the rough adjustment of the plate normal to the light path, set normal with sufficient precision. Hence it is to be rotated by such an amount that eventually the white-light fringes in the lower part of the field will move continuously out of the field in one direction upon a turn of P_2 in one sense, and out of the field in the same direction with a turn of P_2 in the opposite sense, without returning in either case. If the half plates P_1 and P_2 are cut from the same parallel plate, *i.e.*, are of exactly the same thickness, the white-light fringes should coincide in the upper and lower parts of the field.

Sometimes it is impossible to obtain the adjustment described in the preceding paragraph. This may be due to the fact that one of the half plates is "leaning" slightly in its frame, a condition which may be corrected by rocking the plate slightly. Another reason for lack of adjustment may be that the half plates are not cut from a parallel plate, but from one which has a slight wedge shape, the two sides being out of parallelism by a fringe or two. In this case the fringes in the upper and lower parts of the field of view of the interferometer may not be parallel, and one of the plates should be turned over in its frame.

After the white-light fringes extend across both the upper and lower portions of the field, and the half plates are precisely normal to the beams, turn P_1 through an angle of about 15 deg. This should be done in the direction in which the last adjustment of that plate was made, so that there is no lost motion to be taken up. (If no micrometer attachment is available for determining exactly the angle that P_1 is turned through, a small mirror fastened to the cell for P_1 and facing in the direction of a telescope and scale placed about 6 ft. away may be used. The angle will then be measured in the conventional manner with the telescope and scale.) Having turned P_1, and measured its angle, slowly turn Q_2 through the same angle, meanwhile counting fringes to the number (N) of monochromatic light which pass a selected point in the field, until the white-light fringes reappear in the lower part of the field and coincide with those in the upper. For this purpose, the green line of mercury may be used, and a source of white light be held or clamped in such a way that part of the field is illuminated by it. Thus the monochromatic fringes may be observed to pass, and at the same time the white-light fringes will be detected when they appear. An excellent check on the value of N is then to turn P_2 in the opposite direction, meanwhile counting fringes, until the white-light fringes once more appear in coincidence. P_2 will then have been turned through twice the angle i, and the number of fringes in this second count should be $2N$.

Remove P_2 and measure its thickness t with a micrometer caliper. Then calculate the value of n, using eq. 3.

Answer the following questions:

1. What percentage of error is introduced in the measurement of the index of refraction by an error of 10 min. of arc in the measurement of the angle through which P_2 is turned from the normal position?

2. What percentage of error is introduced in the measurement of the index by an error of 0.005 mm. in the thickness of P_2?

3. What percentage of error is introduced in the measurement of the index by an error in the count of N of five fringes?

4. Would any appreciable improvement in the result be obtained by retaining the last term in eq. 2?

EXPERIMENT 11

THE RATIO OF TWO WAVE-LENGTHS WITH THE MICHELSON INTERFEROMETER

Read Sec. **11-7** for the discussion of visibility fringes.

Apparatus.—A Michelson interferometer, a mercury arc, a filter of didymium glass about 5 mm. thick, aqueous solutions of copper nitrate, cobalt sulphate, and nickel acetate, an assortment of gelatin filters, a condensing lens. Uranine may be substituted for the solution of cobalt suphate.

Part A.—The solutions are to be prepared of sufficient density so that the combined filter will permit the transmission of only $\lambda4358$ and $\lambda5461$. It is essential that these be of about the same visual intensity. The transmission of the filter can be tested with a direct-vision spectroscope or a spectrometer and 60-deg. prism. Of the stronger mercury lines, copper nitrate transmits only those from $\lambda4046$ to $\lambda5790$, inclusive. Nickel chloride cuts out $\lambda4046$ and $\lambda4071$. Didymium glass cuts out $\lambda5770$ and $\lambda5790$, and cobalt sulphate or uranine cuts out the faint green line $\lambda4916$. These solutions may be mixed together in a filter cell about 1 cm. thick, or better still, in separate cells. If there is any precipitate present, the addition of a little hydrochloric acid will remove it.

Adjust the interferometer for white-light fringes. With the mercury arc and the combined filter the succession of maxima