LAHORE UNIVERSITY OF MANAGEMENT SCIENCES- SCHOOL OF SCIENCE AND ENGINEERING- PHYSICS DEPARTMENT

SURFACE PLASMON RESONANCE

An Introduction

MUHAMMAD ALI VALLIANI under supervision of Dr. SABIEH ANWAR and Mr. Hassaan Majeed

1/18/2012

The document contains introductory theory needed to understand the topic and an experiment entailing a simplified process to observe the phenomenon.

INTRODUCTION:

"Surface Plasmon resonance" refers to the coherent oscillation of the electrons at the interface of a metal and a dielectric when the metal subjected to an incident light." The incident radiation gives rise to a surface wave with specific optical properties.

CONDITIONS FOR SPR:

This phenomenon is observed in when light from a medium of positive dielectric constant strikes a surface of large but negative dielectric constant.

As the dielectric constant $\varepsilon = (n+ik)^2$, where n is the refractive index and k is the absorption constant, the second surface should have $k>>n$. This is true for the case of metals. The condition of total internal reflection in the incident medium should also be satisfied.

Introductory Concepts:

The introductory understanding of the phenomenon requires knowledge of polarization and other properties of light, Maxwell's equations and plane waves and Fresnel's Equations.

Plane Waves:

Plane waves are ideal waves which have a constant frequency and infinite planes of constant peak to peak amplitude normal to the phase velocity vector.

$A(r,t) = A_o\, Cos(\mathbf{k}.\mathbf{r} - \omega t + \phi)$

Where:

k is the wave vector with magnitude $2\pi/\lambda$ and has the direction of propagation of the wave.

In the complex plane we may also represent it as:

$$U(r,t) = Uo\, e^{i(\mathbf{k}.\mathbf{r} - \omega t)}$$

A simplified wave equation can be represented as:

$$\frac{\partial^2 y}{\partial x^2} - \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2} = 0 \Leftarrow$$

Wave equation

Polarization:

"Polarization is the direction of the electric field vector."

Linearly polarized light with respect to y axis at a certain instant, propagating in the z direction can be represented as can be represented as:

$$E(y) = Eo\, Cos(kz - \omega t)$$

An elliptically polarized light can be represented as:

$$E(x,y) = Eox\, Cos(kz - \omega t) + Eoy\, Cos(kz - \omega t + 90)$$

And if $\underline{Eox = Eoy}$, we have a circularly polarized light.

A plane of incidence is a plane defined by the incident ray and the normal to the surface. A p-polarized light is polarized light parallel to the plane of incidence. An s-polarized light is light polarized perpendicular to the plane of incidence.

Maxwell's Equation:

There are four Maxwell's equations:

Name	Differential form	Integral form
Gauss's law	$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$	$\oiint_{\partial V} \mathbf{E} \cdot \mathrm{d}\mathbf{A} = \frac{Q(V)}{\varepsilon_0}$
Gauss's law for magnetism	$\nabla \cdot \mathbf{B} = 0$	$\oiint_{\partial V} \mathbf{B} \cdot \mathrm{d}\mathbf{A} = 0$
Maxwell–Faraday equation (Faraday's law of induction)	$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$	$\oint_{\partial S} \mathbf{E} \cdot \mathrm{d}\mathbf{l} = -\frac{\partial \Phi_S(\mathbf{B})}{\partial t}$
Ampère's circuital law (with Maxwell's correction)	$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}$	$\oint_{\partial S} \mathbf{B} \cdot \mathrm{d}\mathbf{l} = \mu_0 I_S + \mu_0 \varepsilon_0 \frac{\partial \Phi_S(\mathbf{E})}{\partial t}$

For the simplified case when an electromagnetic wave propagates in the y direction (E in the z axis and b in the x axis) when we solve the four equations they five us the following two conditions:

$$\frac{\partial E_z}{\partial y} = -\frac{\partial B_x}{\partial t}$$
$$\frac{\partial B_x}{\partial y} = -\mu_o \epsilon_o \frac{\partial E_z}{\partial t}$$

} Conditions

Taking the derivatives will give:

$$\frac{\partial^2 y}{\partial x^2} - \mu_o \epsilon_o \frac{\partial^2 y}{\partial t^2} = 0$$ ⟵ Electromagnetic wave equation

Comparing it with the wave equation will give the velocity of electromagnetic waves. This also points to the fact that light is an electromagnetic wave.

Representing E and B as functions of 'y' and 't' will be:

$E = Eo\ Cos\ (ky - \omega t)$

$B = Bo\ Cos\ (ky - \omega t)$

Using the above two conditions will give:

$$Eo = -\frac{\omega}{k} BoEo = -\frac{c^2}{\omega} Bo$$

This gives:

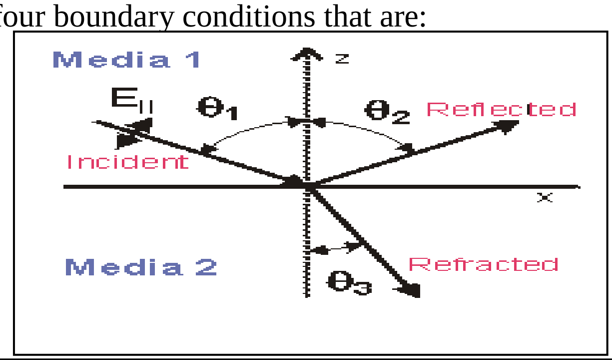
$\boldsymbol{\omega = kc}$ ⟵

$\boldsymbol{E_o = cB_o}$ ⟵

For Dielectrics, when light enters from n_1 to n_2, we have four boundary conditions that are:

$$k_{e1}E_{1\perp} - k_{e2}E_{2\perp} = \frac{\rho}{\epsilon_o}$$
$$\frac{B_{1\parallel}}{k_{1m}} = \frac{B_{2\parallel}}{k_{2m}}$$
$$B1 \perp = B2 \perp$$
$$E1 \parallel = E1 \parallel$$

} Boundary Conditions



Fresnel's Equations:

When we use the boundary conditions of the dielectric material for the ratio of components (s and p) of the electric field transmitted and reflected we get four equations. These are called Fresnel's Equations:

$$r_s = \frac{n_1 \cos\theta_\mathrm{i} - n_2 \cos\theta_\mathrm{t}}{n_1 \cos\theta_\mathrm{i} + n_2 \cos\theta_\mathrm{t}}$$

$$t_s = \frac{2 n_1 \cos\theta_\mathrm{i}}{n_1 \cos\theta_\mathrm{i} + n_2 \cos\theta_\mathrm{t}}$$

$$r_p = \frac{n_2 \cos\theta_\mathrm{i} - n_1 \cos\theta_\mathrm{t}}{n_1 \cos\theta_\mathrm{t} + n_2 \cos\theta_\mathrm{i}}$$

$$t_p = \frac{2 n_1 \cos\theta_\mathrm{i}}{n_1 \cos\theta_\mathrm{t} + n_2 \cos\theta_\mathrm{i}}$$

r stands for reflected and t stands for transmitted or refracted.

THEORETICAL DETAILS OF SPR:

1. ELECTRON OSCILLATION:

In a metal electron oscillates with plasma frequency $\omega_p = \frac{Ne^2}{m\epsilon_o}$. At the surface the oscillation occurs with low energy and gives rise to a surface charge density wave (Plasmon Polariton). The coupling of this wave with Electromagnetic Wave results in SP Wave, which can serve as a method to detect Plasmon Polariton.

2. EVANESCENT WAVES:

Since the result is a surface wave (evanescent), let the solution be of the form:

$U(r,\omega) = Uo\ e^{i(\boldsymbol{k}.\boldsymbol{r}-\omega t)}$ ⬅

Where the equation is:

$$\nabla^2 \vec{E}_1(\vec{r},\omega) + \varepsilon_1(\omega)\frac{\omega^2}{c^2}\vec{E}_1(\vec{r},\omega) = 0$$

Dispersion equations rule the behavior of the waves and the result comes out to be:

$$E = E_o e^{-k''_{1z} z} e^{i(k'_{1x} x - \omega t)}$$

3. SP WAVE:

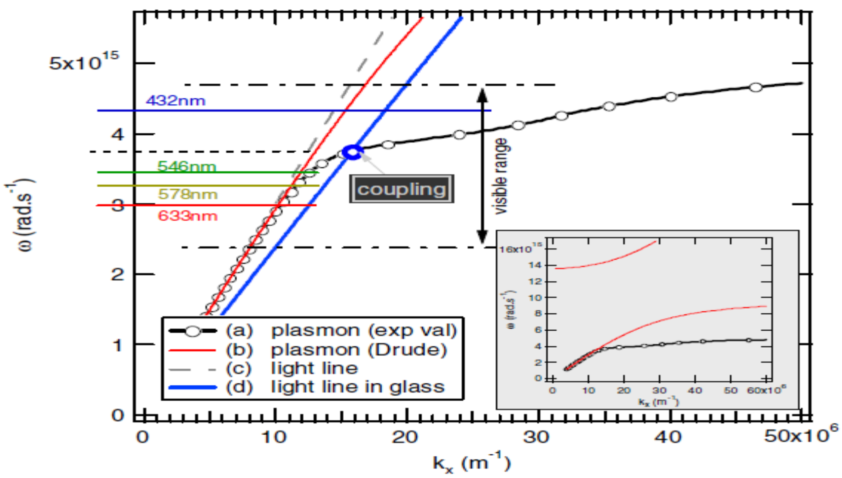
This evanescent wave is intrinsically coupled to a second wave on the dielectric side of the medium. This coupling generates a Surface Plasmon Wave.

With the Boundary conditions and Fresnel's Equations we can have for components of wave vector for the both media and the interface:

$$k_x^2 = \frac{\omega^2}{c^2} \cdot \frac{\varepsilon_1(\omega) \cdot \varepsilon_2(\omega)}{\varepsilon_1(\omega) + \varepsilon_2(\omega)}$$

$$k_{1z}^2 = \frac{\omega^2}{c^2} \cdot \frac{\varepsilon_1^2(\omega)}{\varepsilon_1(\omega) + \varepsilon_2(\omega)} \quad \text{and} \quad k_{2z}^2 = \frac{\omega^2}{c^2} \cdot \frac{\varepsilon_2^2(\omega)}{\varepsilon_1(\omega) + \varepsilon_2(\omega)}$$

We can demonstrate that the k vs ω curve for gold and air does not interact signifying no generation of SP wave, hence we generally use glass prisms.



The MATLAB code for the graph is (does not demonstrate experimental curve):

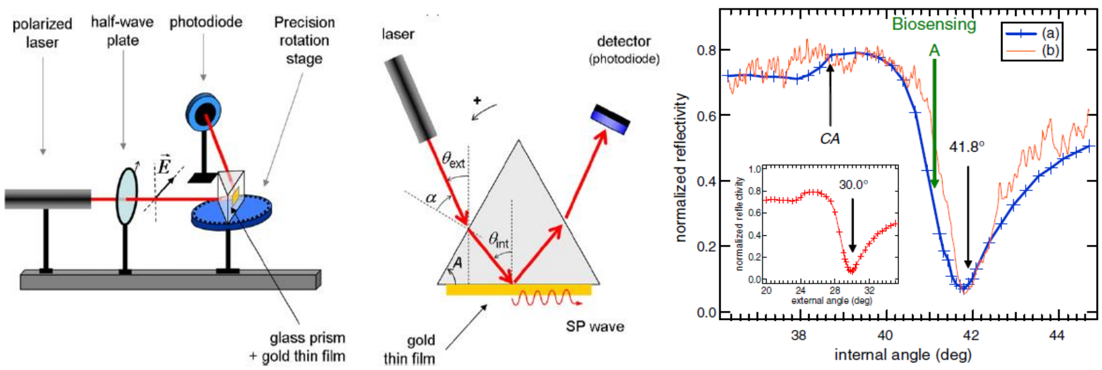
```
nprism=1.5;
T=10^14;
wp=1.36*10^16;
lambda=(189:1940);
c=3e8/10^-9;
w=2*pi*c./lambda;
Eg=1-((wp^2)./(w.^2+1i*T*w));
Kx=w/c.*((Eg)./(Eg+1)).^0.5;
Ka=w./c;
plot(Kx,w);
hold on
plot(Ka,w,':r');
plot(Ka*nprism,w,':g');
xlabel ('wavevector 1/m');
ylabel ('angular frequency rad/sec');
```

4. **<u>EXCITATION OF SP WAVE</u>**:

The coupling of two electromagnetic waves requires the conservation of both the energy $\hbar\omega$ and the momentum $\hbar k$ of the excitation wave. The component of the wave vector parallel to the interface kx obeys the dispersion relation $\omega = \frac{k_x \cdot c}{\sin\theta}$ which is represented by the light line in the case $\theta = \pi/2$. For an incidence angle smaller than $\pi/2$, the slope of the light line is even greater and the intersection of any of the light lines with the SP dispersion relation never occurs.

A Simplified Experiment:

Gold is used as the metal, as the above graph shows, the experiment can be conducted and analyzed by visible light. Monochromatic *p*-polarized light impinges on a prism in a total reflection configuration. An ultrathin gold film (thickness 50 nm) is placed on the prism so that it collects the evanescent waves produced by the total reflection and the SP wave is launched at the gold/air interface if the angle is set at a given value. The light intensity reflected by the prism is measured with the photodiode. The intensity will go through a minimum when the coupling with the plasmon wave occurs.



We will control the θ_{int} through θ_{ext} by the relation:

$$\theta_{int} = \sin^{-1}\frac{\sin(\theta_{ext} - A)}{n} + A$$

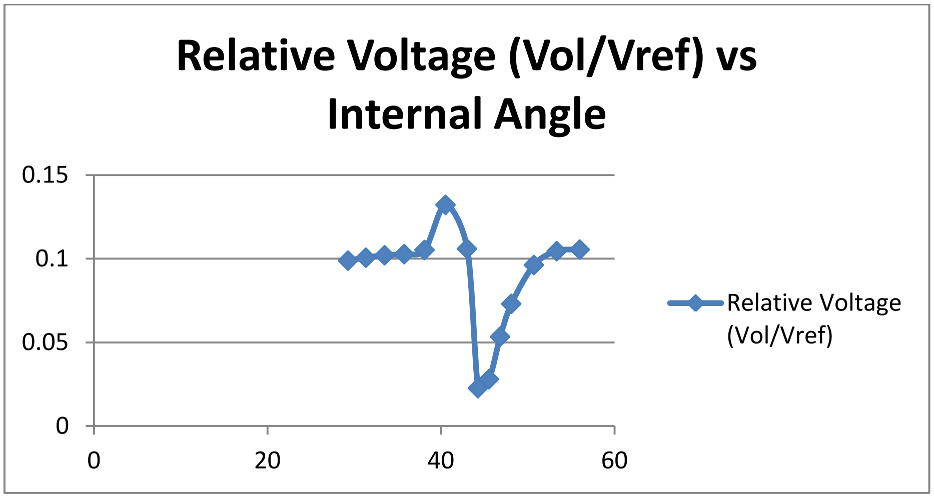
We use Lock in amplifier and an optical chopper for better accuracy. Normalization (using relative value of voltage) can also be done to reduce error.

Experimental Observations:

1. Incident Light- P-Polarized:

Internal Angle	External Angle	Reference Voltage Vref (V)	Voltage Vol (V)	Relative Voltage (Vol/Vref)
29.28977923	10	1.759	0.17398	0.098908471
31.34333983	14	1.754	0.17653	0.100644242
33.50708975	18	1.763	0.18002	0.10211004
35.76746029	22	1.761	0.181	0.10278251
38.11188761	26	1.768	0.1861	0.105260181
40.52877937	30	1.778	0.2351	0.132227222
43.00744554	34	1.775	0.18811	0.105977465
44.26682795	36	1.783	0.04044	0.022680875
45.53800712	38	1.78	0.04989	0.02802809
46.81985784	40	1.775	0.09479	0.053402817
48.11129228	42	1.783	0.13025	0.073051038
50.71872683	46	1.77	0.17038	0.096259887
53.35222291	50	1.783	0.18637	0.10452608
56.00406825	54	1.787	0.18854	0.105506435

CARACTERISTIC CURVE:

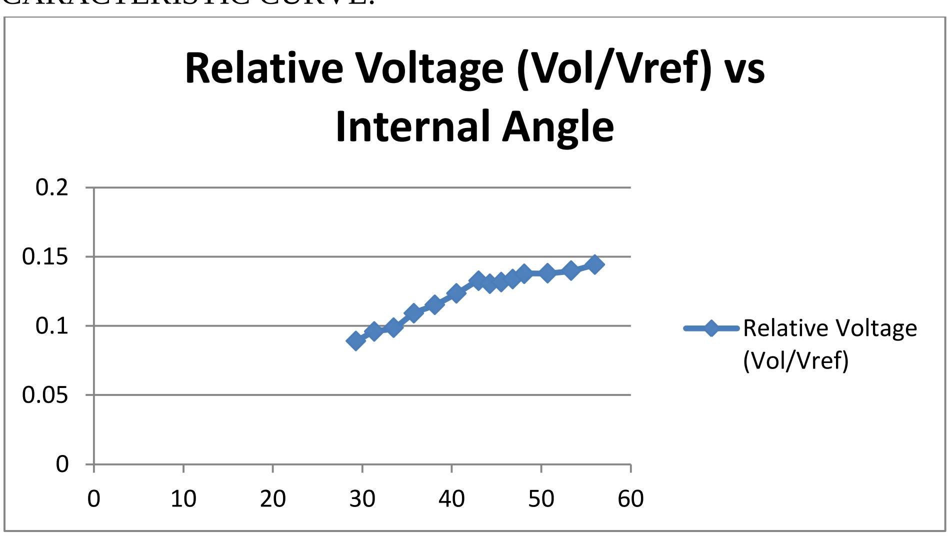


SPR angle= 44.2668 degrees

2. Incident Light-S Polarized:

Internal angle	External Angle	Reference Voltage Vref (V)	Voltage (V)	Relative Voltage (Vol/Vref)
29.28977923	10	1.681	0.14972	0.089066032
31.34333983	14	1.69	0.16214	0.095940828
33.50708975	18	1.701	0.16788	0.098694885
35.76746029	22	1.705	0.18606	0.1091261
38.11188761	26	1.706	0.19656	0.115216882
40.52877937	30	1.72	0.2124	0.123488372
43.00744554	34	1.708	0.2265	0.132611241
44.26682795	36	1.728	0.2254	0.130439815
45.53800712	38	1.735	0.2284	0.131642651
46.81985784	40	1.741	0.233	0.133831132
48.11129228	42	1.73	0.2381	0.137630058
50.71872683	46	1.732	0.2391	0.138048499
53.35222291	50	1.773	0.2481	0.139932318
56.00406825	54	1.775	0.2561	0.14428169

CARACTERISTIC CURVE:



SPR angle: not observed.

Experimental Results:

A sharp bent is observed just after the critical angle, indicating the excitation of the electrons in the gold film for the case of p-polarized light (SPR conditions). S-polarized light shows no SPR curve.

CONCLUSION:

There are many practical situations where we can use the sensitive nature of the SPR condition. Technology has already been created for Bio sensing. Researches can continue on the effects of magnetic field etc. on the nature of the SP wave.

References:

1. O. Pluchery, R. Vayron and K.-M. Van. "*Laboratory experiments for exploring the surface plasmon resonance"*. (2011)
2. M. Mansuripur. *"Classical Optics and its applications".* Cambridge University Press, Cambridge, 2002.