Principles and Applications of Superconducting Quantum Interference Devices

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Superconducting QUantum Interference Devices (SQUIDs) are sensitive devices that can detect small changes in the magnetic field. They take advantage of two important properties of superconductors, namely flux quantization and the Josephson effect. Furthermore, SQUIDs demonstrate quantum effects on the macroscopic scale, such as macroscopic wavefunctions, quantum interference and quantum mechanical tunneling. Mr. SQUID is a commercially available high temperature superconducting (HTS) DC SQUID magnetometer that will be used in our prototypical experiments.

KEYWORDS

Superconductivity · Josephson Junctions · Flux quantization · Meissner effect · Flux trapping · Phase order parameter · Superconducting ring · Critical current · Screening current

APPROXIMATE PERFORMANCE TIME: 2 weeks

Contents

1 Objectives

In this experiment, we will,

1. understand the phenomenon of superconductivity and macroscopic quantum behavior,
2. study the basic principles underlying SQUID applications,
3. observe zero resistance of superconductors and examine the superconducting phase transition,
4. observe the DC Josephson effect,
5. observe the periodically varying critical current in the resistive mode of the SQUID, and
6. learn about the detection of extremely small magnetic fields by converting them into voltages and using a feedback loop.

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2 Theoretical Introduction

2.1 Superconductors

A superconductor is an element, intermetallic alloy or a compound (may be organic) that loses its electrical resistance below a transition temperature T_C. Once setup, the super-currents in these materials have been observed to flow without measurable decrease for several years [1].

Superconductors can be categorized into type I and type II superconductors. The type I superconductors mainly comprising of metals and metalloids that require incredibly low temperatures to superconduct. On the other hand, type II superconductors comprise of metallic compounds, alloys, and cuprates. They achieve higher T_C's than type I superconductors and are more promising for practical applications. An important difference between the two types is that in that in type II, the transition from a normal to a superconducting state is gradual and involves a region of "mixed state" behavior.

In all superconductors, the current is carried not by single electrons but by pairs of electrons with opposite spins called Cooper pairs. These are quantum mechanical entities. For $T < T_C$, the binding energy of the Cooper pair is large as compared to the thermal energy scattering. As a result, Cooper pairs propagate through the material and current flows without any resistance. The typical transition from normal to superconducting behavior is shown in Figure 1.

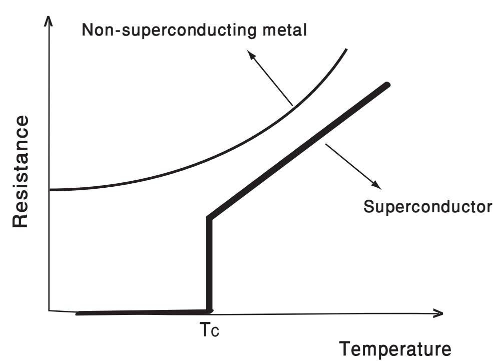

Figure 1: Resistance-temperature curve for Superconductors, $\delta R/\delta T|_{T=T_C} \longrightarrow \infty$.

2.2 Characteristics of Superconductors

2.2.1 Infinite conductivity

Superconductors exhibit the remarkable property of infinite conductivity. Within experimental error, their resistivities are found to be lower than 10^{-26} Ωm at $T < T_C$. For comparison the resistivity of copper, one of the finest conductors is at the most 10^{-8} Ωm at room temperature.

The well known relationship of current density is,

$$\mathbf{j} = \sigma \mathbf{E}.$$

Since the conductivity is infinite, to have finite values of **j** the electric field **E** should be zero inside a superconductor. But according to Faraday's law,

$$\begin{aligned} -\frac{\partial \mathbf{B}}{\partial t} &= \nabla x \mathbf{E} \\ &= 0 \\ \Longrightarrow B &= constant. \end{aligned} \tag{1}$$

Hence inside infinite conductivity materials including superconductors, the magnetic field **B** is constant in time. The property that differentiates a superconductor from a normal perfect conductor is that the magnetic field **B** is not only independent of time but is also zero inside a superconductor. This phenomenon is discussed in more detail in Section 2.2.4.

2.2.2 Cooper pairs

Cooper pair is the name given to the pair of electrons that are bound together at low temperatures. The American physicist Leon Cooper showed that an arbitrarily small attraction between electrons in a metal can cause a paired state of electrons to have a lower energy than the Fermi energy, which implies that the pair is bound. In superconductors, this attraction is due to the electron−phonon interaction. According to the BCS theory of superconductivity, these Cooper pairs are responsible for superconductivity.

The BCS theory is the first microscopic theory of superconductivity, proposed by Bardeen, Cooper, and Schrieffer in 1957, almost 46 years after the discovery of superconductivity in 1911. It describes superconductivity as a microscopic effect caused by the "condensation" of pairs of electrons into the bosonic Cooper state.

2.2.3 The Meissner Effect

The Meissner effect is the exclusion of magnetic flux from the superconductor. This is due to the electric currents known as the screening currents flowing on the surface of the superconductor. The screening currents flow in such a direction so as to generate a field equal and opposite to the applied field. This results in $\mathbf{B} = 0$ inside the superconductor. Superconductors expel the field even if they are cooled into the superconducting state in the presence of an applied field as shown in Figure 2. This behavior is in contrast with a normal, infinitely conducting sample.

Considering the superconductor from the point of view of a magnetic material in which screening currents produce an internal magnetic field $\mu_0\mathbf{M}$ expelling the applied field $\mu_0\mathbf{H}$, we require for the Meissner effect

$$\begin{aligned} \mathbf{B} &= \mu_0(\mathbf{H} + \mathbf{M}) \\ &= 0 \\ \Longrightarrow \mathbf{M} &= -\mathbf{H}. \end{aligned}$$

Hence the equation relating magnetization and applied magnetic field strength,

$$\mathbf{M} = \chi \mathbf{H}$$

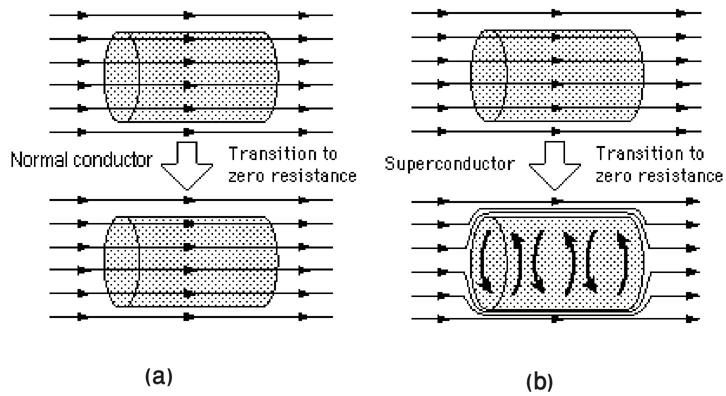

Figure 2: (a) A normal conductor cooled in the presence of magnetic field tends to sustain that field after transition to zero resistance. (b) Circular screening currents expels the magnetic field as the superconductor is cooled below its transition temperature.

shows that the superconductor behaves as though it has a magnetic susceptibility $\chi = -1$, a perfect diamagnet.

Meissner effect in type II superconductors A type II superconductor allows some of the external magnetic field to penetrate into its surface which creates some rather novel phenomena like superconducting "stripes" and "flux-lattice vortices".

2.2.4 London equation and description of the Meissner effect

To understand the London equation, we first need to understand the concept of electromagnetic momentum. Consider a stationary particle of charge q and mass m at a distance r from the axis of a long solenoid. For simplicity we consider the solenoid to be superconducting with its ends connected together to form a complete circuit as shown in Figure 3.

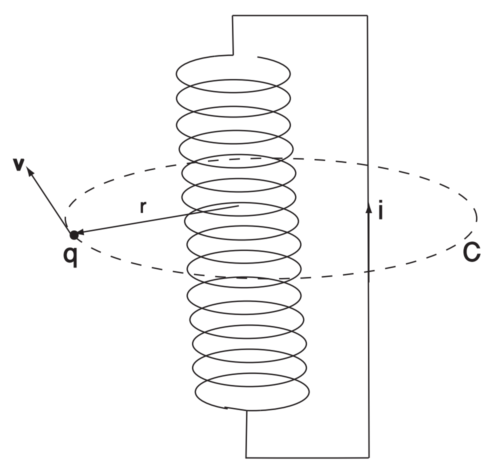

Figure 3: A charged particle accelerated by a decaying magnetic field.

Initially the solenoid is at $T < T_C$ and is carrying a supercurrent. The charge q is in a field-free region as all the field is concentrated inside the solenoid. Now If the solenoid is heated to $T > T_C$, the current and hence the magnetic field decays. By virtue of Faraday's law, a changing magnetic flux induces an emf around the loop C accelerating the charge particle, giving it a momentum m**v**. Where does this momentum come from? After all, we have not applied any force on the particle to boost its momentum from 0 to m**v**. Are we violating conservation of momentum here?

This paradox can be resolved by arguing that the particle possessed the momentum throughout! For the law of conservation of momentum to hold, we write the momentum of the particle as

$$\mathbf{p} = m\mathbf{v} + q\mathbf{A} \tag{2}$$

where **p** is the so-called canonical momentum, q is the charge and **A** is the magnetic vector potential ($\nabla x\mathbf{A} = \mathbf{B}$) [2]. The momentum $q\mathbf{A}$ possessed by a charged particle at rest is called the electromagnetic momentum. In the initial state, all the momentum of the charge particle is electromagnetic and in the final state, it is wholly kinetic.

Considering the current decay period to be very short, we can express the kinetic momentum as an impulse of force. Thus we can write the kinetic momentum as,

$$m\mathbf{v} = \oint_C \mathbf{F}_{elec} dt \tag{3}$$

$$= \oint_C q\mathbf{E} dt, \tag{4}$$

where F_{elec} is the electric force on charge q. The expression for the electric field at the position of the particle can be written using the Faraday's law,

$$\oint_C \mathbf{E}.d\mathbf{l} = -\frac{d\phi_B}{dt},$$

where ϕ_B is the magnetic flux through the circular loop C.

The magnetic field **B** at the position of particle is zero owing to the long size of the solenoid. The above equation can also be expressed in terms of **A** (which is non-zero at the position of the particle) using the expression for magnetic flux ϕ_B, and Stoke's theorem that relates a line integral to an area integral. Hence the equation is transformed into,

$$\begin{aligned}\oint_C \mathbf{E}.d\mathbf{l} &= -\oint_C \frac{d\mathbf{A}}{dt}.d\mathbf{l} \\ &= -\frac{d\mathbf{A}}{dt}.\end{aligned}$$

Equation 4 can then be written as,

$$\begin{aligned}m\mathbf{v} &= -q\int \frac{d\mathbf{A}}{dt} dt \\ &= -q\Delta\mathbf{A}\Big|_{ti}^{tf} \\ &= -q(\mathbf{A}(t_f) - \mathbf{A}(t_i)) \\ &= q\mathbf{A}(t_i).\end{aligned}$$

The above equation shows that even thought the particle is kinetically accelerated, its canonical momentum defined by Equation 2 is indeed conserved.

Comparing the initial ($T < T_C$) and final momenta ($T > T_C$),

$$\begin{aligned} \mathbf{p}_i &= \mathbf{p}_f \\ m\mathbf{v}(t_i) + q\mathbf{A}(t_i) &= m\mathbf{v}(t_f) + q\mathbf{A}(t_f) \\ q\mathbf{A}(t_i) &= m\mathbf{v}(t_f). \end{aligned}$$

Since $\mathbf{v}(t_i) = \mathbf{A}(t_f) = 0$, we get a general expression for the drift velocity of electron as,

$$\mathbf{v}_d = -\frac{e\mathbf{A}}{m_e} \tag{5}$$

and the current density for the electrons turns out to become,

$$\begin{aligned} \mathbf{j} &= n\mathbf{v}_d e & (6) \\ &= -\frac{ne^2}{m_e}\mathbf{A}. & (7) \end{aligned}$$

Taking the curl of the above equation and replacing the electron current density n by the superconducting current density n_s, we arrive at the London equation for superconductors,

$$\nabla x\mathbf{j} = -\frac{n_s e^2}{m_e}\mathbf{B}. \tag{8}$$

We now set to show that Equation 8 implies the screening of the magnetic field by the superconducting electrons. We take the curl of the Maxwell equation

$$\nabla x\mathbf{B} = \mu_0\mathbf{j}. \tag{9}$$

Using the vector identity,

$$\nabla x\nabla x\mathbf{B} = \nabla(\nabla.\mathbf{B}) - \nabla^2\mathbf{B} \tag{10}$$

and Equations 9 and 10, we arrive at,

$$\begin{aligned} -\nabla^2\mathbf{B} &= \mu_0\mathbf{j} & (11) \\ &= -\frac{\mu_0 n_s e^2}{m_e}\mathbf{B}. & (12) \end{aligned}$$

Q 1. Derive Equation 12 using Equations 9 and 10.

The above equation 12 can also be expressed as,

$$\begin{aligned} \nabla^2\mathbf{B} &= \frac{\mu_0 n_s e^2}{m_e}\mathbf{B} \\ &= \frac{1}{\lambda_L^2}\mathbf{B}, & (13) \end{aligned}$$

where

$$\lambda_L = \frac{m_e}{\mu_0 n_s e^2}$$

is a characteristic length called the London penetration depth.

The above solution shows that **B** decays exponentially as we go into the interior of the superconducting region. Let the external magnetic field be applied along the z-axis parallel to the surface of a thin slab of superconductor (thickness x $\approx$ 0), we have for this one dimensional setup,

$$\frac{d^2\mathbf{B}}{dx^2} - \frac{\mathbf{B}}{\lambda^2} = 0. \quad (14)$$

Applying the boundary conditions $\mathbf{B} = \mathbf{B}_{ext}$ at x=0 and $\mathbf{B}$=0 at x=∞, we can get the solution of above differential equation as

$$\mathbf{B}(x) = \mathbf{B}_{ext} \exp(\frac{-x}{\lambda}). \quad (15)$$

This is the London equation indicating that the magnetic field exponentially decays to zero inside a superconductor. The magnetic flux thus penetrates the sample only for a small distance from the surface and becomes zero at x $\gg \lambda$. The length scale of the penetration is determined by λ.

Q 2. Why do we take divergence of **B**, $\nabla.\mathbf{B}$ as zero?

Q 3. Solve the second order differential equation 14 and apply the appropriate boundary conditions to derive the solution 15.

Q 4. Formulate the London equation in terms of the current density **j** similar to Equation 13. What do you conclude from this equation? HINT: Use Equation 8 along with the Maxwell $\nabla x\mathbf{B} = \mu_0\mathbf{j}$ and the continuity equation $\nabla.\mathbf{j} = 0$.

Q 5. What is the physical significance of the continuity equation, $\nabla.\mathbf{j} = 0$?

2.2.5 Macroscopic quantum phenomena in superconductors

Cooper pairs in a superconductor share a common wavefunction $\psi(\mathbf{r})$ and the behavior of the superconducting electrons is completely specified by this function. This is in complete contrast to the situation in a normal metal where the behavior can only be determined by specifying all of the occupied single-particle states. This coherence in wavefunction associated with macroscopic occupation of the same state by Cooper pairs causes a superconductor to directly manifest quantum mechanics at a large scale! How pleasing!

The macroscopic wavefunction is specified by the order parameter,

$$\psi(\mathbf{r}) = \psi_0 \exp(i\mathbf{k}.\mathbf{r}). \quad (16)$$

Here **k** is the wave vector and **r** is the position vector. The Copper pair density is,

$$\begin{aligned} |\psi(\mathbf{r})|^2 &= \psi_0^2 \\ &= n_s/2. \end{aligned}$$

This means that if the electronic density is n_s, then the Cooper pair density is $n_s/2$. Several interesting quantum phenomena can now be motivated from the order parameter.

Q 6. A normal metal has N non-interacting conduction electrons. These electrons have wavefunctions $\psi_1(\mathbf{r}_1)$, $\psi_2(\mathbf{r}_2)$, ..., $\psi_N(\mathbf{r}_\mathrm{N})$. What is the combined wavefunction of all electrons?

Q 7. What is the physical significance of the wave vector $\mathbf{k}$? How is it related to the momentum?

2.2.6 Supercurrent density derivable from the wavefunction

The canonical momentum for the Cooper pairs is

$$\mathbf{p} = 2m_e\mathbf{v} - 2e\mathbf{A} \tag{17}$$

From equations 6 and 17 we obtain,

$$\begin{aligned}\mathbf{j}_s(\mathbf{r}) &= -\frac{n_s}{2}(2e)\frac{(\mathbf{p}+2e\mathbf{A})}{2m_e} \\ &= \frac{-e}{m_e}|\psi(\mathbf{r})|^2(\mathbf{p}+2e\mathbf{A}) \\ &= \frac{-e}{m_e}\psi^*(\mathbf{p}+2e\mathbf{A})\psi.\end{aligned}$$

Substituting the momentum operator by its differential operator expression $-i\hbar\nabla$ we get,

$$\mathbf{j}_s(\mathbf{r}) = \frac{e}{m_e}\Big[i\hbar\psi^*(\mathbf{r})\nabla\psi(\mathbf{r}) - 2e\mathbf{A}\psi^*(\mathbf{r})\psi(\mathbf{r})\Big].$$

This expression is, in general, complex.To get a real expression for the current density, we add the complex conjugate of first term on the right hand side of above equation and take its average,

$$\begin{aligned}\mathbf{j}_s(\mathbf{r}) &= \frac{e}{m_e}\left[\frac{(i\hbar\psi^*(\mathbf{r})\nabla\psi(\mathbf{r})) + (i\hbar\psi^*(\mathbf{r})\nabla\psi(\mathbf{r}))^\dagger}{2} - 2e\mathbf{A}\psi^*(\mathbf{r})\psi(\mathbf{r})\right] \\ &= \frac{ei\hbar}{2m_e}[(\psi^*(\mathbf{r})\nabla\psi(\mathbf{r}) - \psi(\mathbf{r})\nabla\psi^*(\mathbf{r})] - \frac{2e^2}{m_e}\mathbf{A}\psi^*(r)\psi(r)\end{aligned} \tag{18}$$

which is an expression for the superconducting current density based on the order parameter.

In its most general form we write the order parameter as

$$\psi(\mathbf{r}) = \big|\psi(\mathbf{r})\big|\exp(i\alpha(\mathbf{r}))$$

where $\alpha(\mathbf{r}) = \mathbf{k}.\mathbf{r}$ is a phase.

Q 8. Defining the gradient operator ∇ in spherical polar coordinate system (r, θ, ϕ) and using the above expression for $\psi(\mathbf{r})$ in Equation 16, derive the following alternative expression for $\mathbf{j}_s(\mathbf{r})$,

$$\mathbf{j}_s(\mathbf{r}) = \frac{-e}{m_e}\big|\psi(\mathbf{r})\big|^2(\hbar\nabla\alpha + 2e\mathbf{A}). \tag{19}$$

Q 9. Derive the London Equation 8 by taking curl of Equation 19 assuming that the Cooper pair density is independent of position, i.e., $\psi(\mathbf{r})^2 = n_s/2$.

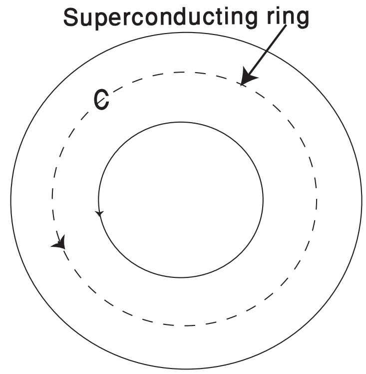

Figure 4: Closed super conducting ring containing a closed path C far from the surface.

2.2.7 Flux quantization

Let's consider a superconductor fashioned in the form of a ring as shown in Figure 4. The superconductor is in its Meissner state and allows a circular path along C, far from the surface, the current density $\mathbf{j} = 0$. Equation 19 thus becomes,

$$\hbar\nabla\alpha \;=\; -2e\mathbf{A}.$$

We integrate both sides of this equation around the closed curve C, apply Stoke's theorem and use the definition of the magnetic flux

$$\begin{aligned}
\hbar\oint_C \nabla\alpha.d\mathbf{l} &= \hbar\triangle\alpha \\
&= -2e\oint_C \mathbf{A}.d\mathbf{l} \\
&= -2e\int_S (\nabla x\mathbf{A}.d\mathbf{s}) \\
&= -2e\int_S \mathbf{B}.d\mathbf{s} \\
\Longrightarrow \hbar\triangle\alpha &= -2e\phi \\
\phi &= \frac{-\hbar}{2e}\triangle\alpha. \qquad (20)
\end{aligned}$$

Here $\triangle\alpha$ is the change in phase of the order parameter as one makes a complete round trip along C. Since the order parameter $\psi(\mathbf{r})$ is a legitimate wavefunction, it must be single valued and the phase change $\triangle\alpha$ around the closed loop must be $\pm 2\pi n$, where n is an integer (positive or zero). Thus,

$$\begin{aligned}
\phi &= \pm\frac{2\pi n\hbar}{2e} \\
&= \pm\frac{nh}{2e} \\
&= \pm n\phi_0,
\end{aligned}$$

which shows that the magnetic flux through any closed area within a superconductor, on whose perimeter $\mathbf{j} = 0$, is quantized in units of the flux quantum, $\phi_0 = h/2e = 2.07x10^{-15}$

Tm^2. The flux in quantized just like charge or spin quantization. The smallest unit ϕ_0 is, quite aptly called the "fluxon".

2.2.8 The DC Josephson effect

The Josephson effect is a manifestation of long-range quantum coherence of superconductors. Josephson was a Ph.D. student in Cambridge when he discovered this phenomenon. It occurs when two superconducting regions are weakly coupled. A Josephson junction (JJ) is formed by placing an insulating gap between two superconductors. If the gap is thin enough, electron pairs can tunnel from one superconductor across the gap to the other superconductor. By quantum tunneling, a resistanceless current can flow across the insulator. This is called the DC Josephson effect.

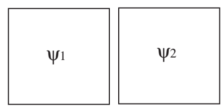

Figure 5: Schematic diagram of two super conducting regions separated by a thin gap.

To understand this effect, we consider two isolated samples of a superconductor with spatially constant order parameters, ψ_1 and ψ_2 as shown in Figure 5. In general, $\psi_1 \neq \psi_2$. Let the order parameter in the left region be $|\psi_1|exp(i\alpha_1)$ and that on the right be $|\psi_2|exp(i\alpha_2)$. In the absence of interaction between the two samples the phases α_1 and α_2 will in general be different. Strongly coupling the two samples by bringing them into contact over a large area causes the phases to equalize $\alpha_1 = \alpha_2$, so that all Cooper pairs will be in the same state; this equality is then very difficult to disturb. But if there is a weak coupling, the phases α_1 and α_2 will not equalize between the two regions. It is possible to maintain a phase difference between the two regions by passing a small current through the JJ. This is the DC Josephson effect.

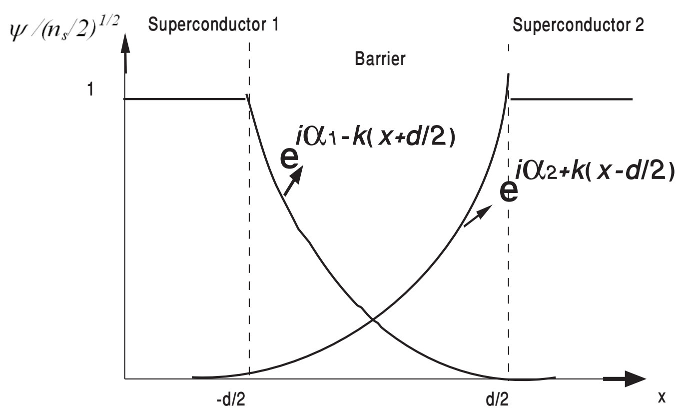

Figure 6: Contribution to the super conducting order parameter within the oxide barrier associated with the tunneling of Cooper pairs through the barrier.

Below T_C it is possible for the Cooper pairs to tunnel through the barrier, a net flow can take place even in the absence of an applied potential difference. This corresponds to a dissipationless supercurrent whose density is calculated below. Because of the tunneling of pairs, the superconducting order parameter extends throughout the barrier. Inside the barrier, we regard it as being the sum of the contributions shown in Figure 6. We thus write the order parameter inside the barrier as,

$$\begin{aligned}\psi &= |\psi_0|[\exp(i\alpha_1 - k(x+d/2)) + \exp(i\alpha_2 + k(x-d/2))] \\ &= (n_s/2)^{1/2}[\exp(i\alpha_1 - k(x+d/2)) + \exp(i\alpha_2 + k(x-d/2))] \qquad (21)\end{aligned}$$

where the barrier extends from $x = -d/2$ to $x = d/2$ and k^{-1} is the characteristic length for decay of the order parameter within the barrier. Furthermore, we assume $|\psi_1|^2 = |\psi_2|^2 = n_s/2$.

Q 10. Calculate the pair current density through the barrier by using the order parameter of Equation 21 and $\mathbf{A} = 0$. The phases α_1 and α_2 are assumed to be spatially constant within the superconducting region.

The expression you achieve should be similar to,

$$\begin{aligned}j &= \frac{ie\hbar n_s}{2m_e} k \exp(-kd)[-\exp i(\alpha_1 - \alpha_2) + \exp i(\alpha_2 - \alpha_1)] \\ &= j_0 \sin(\alpha_1 - \alpha_2) \qquad (22)\end{aligned}$$

where

$$j_0 = \frac{e\hbar k e^{-kd}}{m_e} n_s. \qquad (23)$$

The expression shows that the difference in the phase factor on either side, leads to a tunneling current, even when there is no applied potential difference. The tunneling current is a supercurrent. It is possible to directly observe the DC Josephson effect, and in the process, quantum mechanical tunneling in the present experiment. SQUID provides a highly inspirational way of practically observing tunneling!

2.2.9 The SQUID as a magnetometer

A superconducting quantum interference device (SQUID) uses the properties of flux quantization and the DC Josephson effect to detect very small magnetic fields. They are sensitive enough to measure fields down to the range of 10^{-15} T. For comparison, a typical refrigerator magnet produces 0.01 T, and some processes in animals produce very small magnetic fields between 10^{-9} T and 10^{-6} T.

The central element of a SQUID is a ring of superconducting material with one or more JJs. An example is shown in Figure 7. The critical current, I_C, of the junctions is much less than the critical current of the main ring. This produces a very low current density making the momentum of the electron-pairs small.

To understand the working of SQUID, let's consider the scenario when we bias it with a current well below its critical current value. Then, if we apply a tiny magnetic field to the SQUID, the applied magnetic field tends to change the superconducting wave function $\psi(\mathbf{r})$. But the superconducting wavefunction doesn't want to change. As discussed in Section 2.2.7, it must maintain an integral number of wavefunction cycles around the loop. So the superconducting loop does what you would expect; it opposes the applied magnetic field by generating a

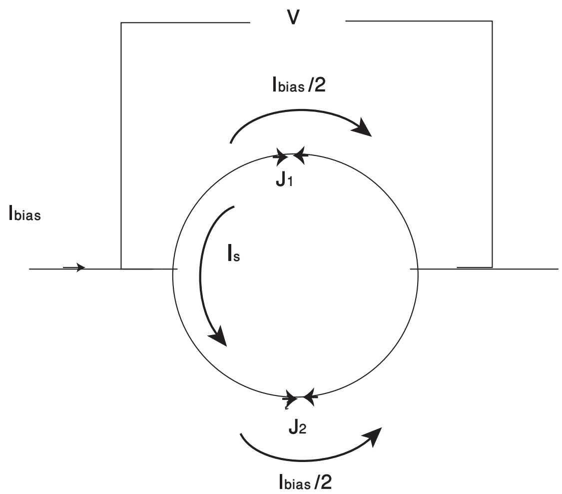

Figure 7: A DC SQUID in the presence of an applied magnetic field.

screening current I_s, that flows around the loop as shown in Figure 7. The screening current creates a magnetic field equal but opposite to the applied field, effectively canceling out the net flux in the ring.

In this way, the applied magnetic field has lowered the effective critical current of the SQUID. In other words, it has reduced the amount of bias current we can pass through the SQUID without generating a resistive voltage, since the screening current has superimposed itself on top of the bias current. The situation is depicted in Figure 7, where I_s flows parallel to $I_{bias}/2$ in one of the arms, increasing the total current, making it more likely to achieve the superconducting to normal transition.

Now, as we increase the applied magnetic flux, the screening current increases. But when the applied magnetic flux reaches half a flux quantum, something interesting happens: the superconducting junctions momentarily become resistive. If we try to observe this transition in terms of screening current, we can conclude that in order to have lower energy at $\phi \approx \phi_0/2$, it is little easier for the SQUID to keep 0.49 flux quanta in rather than keeping 0.51 flux quanta out. Of course, the screening current will have to change direction at this point. The variation of I_S with ϕ is shown in Figure 8.

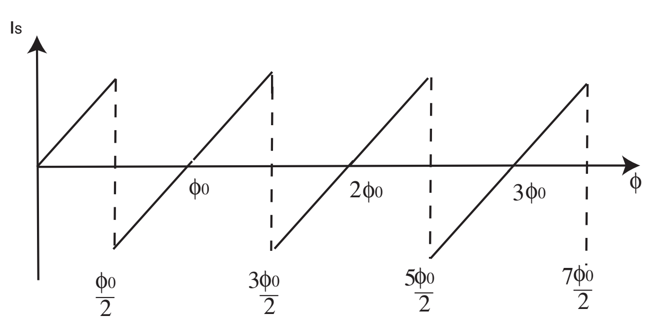

Figure 8: Relationship between screening current and applied magnetic flux. (This figure is taken from [3] page no. 37.)

Figure 8 clearly shows that the screening current changes sign when the applied flux reaches half of a flux quantum. Then, as the applied flux goes from half a flux quantum toward one flux quantum, the screening current decreases. When the applied flux reaches exactly one flux quantum, the screening current goes to zero. At this instant, the magnetic flux inside the loop and the magnetic flux applied to the loop are equal, so there is no need for a screening current. If we increase the applied magnetic flux a little more, a small screening current starts to flow in the positive direction, and the cycle begins again. The screening current is periodic in the applied flux, with a period equal to one flux quantum, ϕ_0.

Hence we conclude that,

- the screening current of a SQUID is periodic in the applied flux, and
- the critical current of a SQUID depends on the screening current.

Thus it makes sense that the SQUID critical current is also periodic in the applied magnetic flux. The critical current goes through maxima when the applied magnetic flux is an integral multiple of the flux quantum (I_s=0) and it goes through minima when the applied magnetic flux is an integral multiple of one plus half flux quantum.

The critical current is usually obtained by measuring the voltage drop across the junction as a function of the total current through the device. To make a magnetometer, the SQUID is operated at a biasing current slightly greater than I_C, so the SQUID always operates in the resistive mode. Under these conditions, there is a periodic relationship between the voltage across the SQUID and the applied magnetic flux with a period of one flux quantum ϕ_0. This is shown in Figure 9.

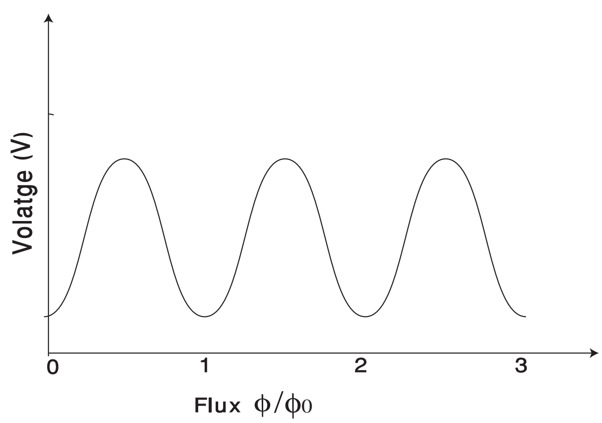

Figure 9: Voltage-Flux characteristics of DC SQUID showing the periodic dependence of the SQUID voltage on applied flux for a fixed bias current.

Q 11. The periodically varying critical current as a function of applied magnetic flux can be expressed mathematically as

$$I_C(\phi) \quad \propto \quad I_C(0)\Big|\cos\big(\frac{\pi\phi}{\phi_0}\big)\Big|. \tag{24}$$

Derive Equation 24 from [6], page no. 308-310. How will the equation's graphical depiction differ from Figure 9?

The sinusoidally varying critical current is, in fact, a direct demonstration of quantum interference. The variation in the current can be compared to the variation in the intensity of light detected on the screen of a Young's double-slit experiment. In the latter experiment, a phase difference is achieved by varying the optical path length, as we remember from our basic physics course. In the case of the SQUID, the phase difference is maintained by the currents flowing through the two arms of the SQUID. Refer to [6], page 310 for details. The phenomena of interference and flux quantization are exploited to create the world's most sensitive magnetic field detectors [3, 5]. These ideas will be directly verified in the present experiment.

Remember, when using the SQUID as a highly sensitive superconducting detector for magnetic flux, we bias it with a current slightly higher than the critical current I_C so that a voltage drop occurs across the JJ's with the SQUID operative in the resistive mode.

3 Overview of the Mr. SQUID apparatus

Our experiment uses "Mr. SQUID", a commercial DC Superconducting QUantum Interference Device (SQUID) magnetometer, incorporating the following components,

- a high-temperature superconductor (HTS) thin-film SQUID chip,
- two feedback coils to modulate the SQUID and to couple an external signal to the SQUID
- a cryogenic probe with a removable magnetic shield
- an electronic control box containing all the circuits needed to operate the SQUID, and
- a cable to connect the probe to the electronics box.

A schematic diagram of Mr. SQUID with the above mentioned components is shown in Figure 10.

At the heart of Mr. SQUID is a small integrated circuit chip made of yttrium barium copper oxide (YBCO) that is fashioned into a ring containing two Josephson junctions. For a full description and working of this device, refer to the relevant sections of the Mr. SQUID manual [3].

3.1 Brief description to the Electronic Box

POWER switch The power switch of the SQUID's control box is normally in the OFF (down) position.

$\mathbf{V-I}$, $\mathbf{V-\phi}$ mode switch The mode switch is normally in the $\mathbf{V-I}$ (up) position. This mode allows us to observe the voltage-current characteristics of the SQUID, i.e., we apply a biasing current I to the SQUID and observe the voltage drop V across the parallel JJs. In this mode we can directly observe the DC josephson effect. The $\mathbf{V-\phi}$ (down) position switch

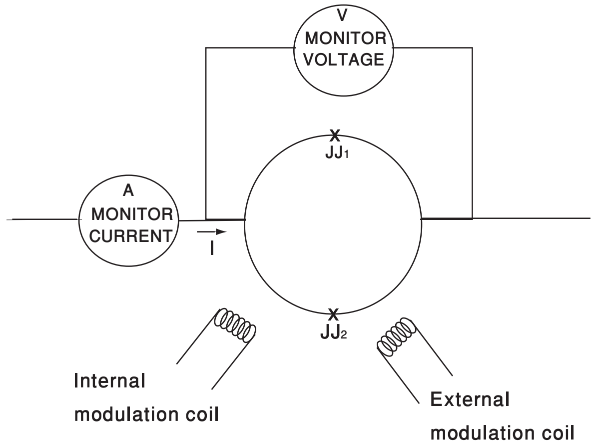

Figure 10: A simplified schematic diagram of the Mr. SQUID experiment. Current flowing into the superconducting ring is monitored by an ammeter 'A' in series and voltage developed across the JJs can be detected by a voltmeter 'V' connected in parallel. Current in the external and internal modulation coils links flux to the ring.

Figure 11: Photograph of the Mr. SQUID probe along with its electronic box.

allows us to observe the voltage-flux characteristics of the SQUID, where ϕ is the externally applied magnetic flux and V is the periodically changing voltage across the SQUID as the flux is varied. The period of the voltage modulation is determined by the quantum of the magnetic flux, the fluxon.

PROBE Nine-pin DB-9 socket for connections to the Mr. SQUID probe.

BIAS OFFSET knob This knob applies a fixed DC current through the SQUID. In the 12 o'clock position, this current is approximately zero. Turning the knob either clockwise or anticlockwise supplies a fixed current through the SQUID in either of the two directions.

EXT INPUT A BNC connector used to couple an external voltage signal to the "external" feedback coil on the Mr. SQUID chip. A MODE switch inside the Mr. SQUID electronics box (accessible by removing the top cover) selects whether this signal is routed directly through a

100 mA fuse at location $F1$ inside the box to the “external” feedback coil on the Mr. SQUID chip (switch position DIR) or is converted to true differential input using a buffer amplifier and then routed to the feedback coil (switch position BUF). The current output from the buffer amplifier (i.e., the current applied to the external coil) is 100 μA/V. The buffered configuration is the default factory setting.

SWEEP OUTPUT knob This control sets the amplitude of the triangle wave test signal in either the $\mathbf{V}-\mathbf{I}$ or the $\mathbf{V}-\phi$ mode. The control sweeps the current back and forth between the two extreme values. These values are set by rotating the position of this knob while the center position of the current is determined by the BIAS OFFSET knob. This arrangement is shown in Figure 12. In the $\mathbf{V}-\mathbf{I}$ mode, the triangular current wave is applied to the bias

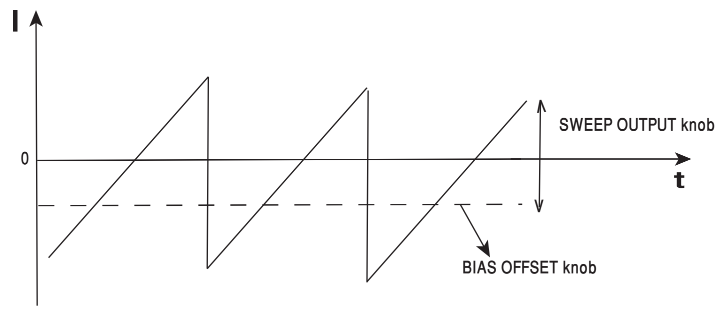

Figure 12: Function of the BIAS and SWEEP outputs of the SQUID electronic box.

terminals of the SQUID and in the $\mathbf{V}-\phi$ mode, the triangular wave is applied to the internal modulation coil.

FLUX OFFSET knob This knob applies a fixed DC current to the internal modulation coil and thus acts as a source of applied magnetic field for the SQUID. In the 12 o'clock position this current is approximately zero. This function controls the amount of magnetic flux through the central annular region in the SQUID loop.

CURRENT Output A BNC female connector provides the output current of the Mr. SQUID box. The current is converted to an output voltage by dropping across a 10kΩ resistor. Hence to determine the current flowing through the JJs, we divide the measured voltage by 10kΩ.

- In the $\mathbf{V}-\mathbf{I}$ mode this output represents the total current through the SQUID (sum of the fixed bias current and triangular wave provided by BIAS OFFSET and SWEEP OUTPUT respectively).

- In the $\mathbf{V}-\phi$ mode this output represents the current through the modulation coil (sum of fixed modulation current through the FLUX OFFSET control and the triangular wave).

VOLTAGE Output Another BNC female connector provides the voltage across the SQUID (in both the $\mathbf{V}-\mathbf{I}$ and $\mathbf{V}-\phi$ modes). This voltage is amplified by a factor of 10,000. So the actual voltage across the SQUID is the measured voltage divided by 10,000.

4 Experiments

4.1 The DC Josephson effect and $\mathbf{V} - \mathbf{I}$ characteristics of the SQUID

4.1.1 Objective

In this experiment we will observe the resistanceless current in the SQUID and also measure the critical current.

4.1.2 Apparatus

1. Mr. SQUID probe
2. Electronic control box
3. DB-9 M/M cable with 9-pin male connector
4. Liquid nitrogen
5. Magnetic shield

4.1.3 Procedure

1. Configure the oscilloscope for the dual X-Y mode and set the vertical sensitivity to 0.5 V/div and the horizontal sensitivity to 0.1 V/div. With the GND on the scope, use the horizontal and vertical positioning knobs to set the CRO dot to the middle of the screen.
2. Using BNC cables connect the VOLTAGE output of the SQUID's control box to the Y-channel of the CRO and the CURRENT output to the X-channel and set the source switch to CH I.
3. Confirm that the POWER switch of the electronic box is OFF and the mode switch is in the $\mathbf{V} - \mathbf{I}$ position, FLUX OFFSET and BIAS OFFSET are in the 12 o'clock position and the SWEEP OUTPUT knob is fully counterclockwise.
4. Plug the 5-pin power cable into the POWER connector at the rear panel of the electronic box.
5. Plug one end of the 9-pin DB-9 M/M cable into the PROBE connector and the other end into the Mr. SQUID probe.
6. Now turn the POWER switch ON and switch the CRO input coupling to DC mode. Use the BIAS OFFSET knob to position the dot in the middle of the CRO screen. In case you are unable to achieve this, check the connections and see Section 6 of the Mr. SQUID (page no. 49-55) manual.
7. Fill the dear about 3/4-full with LN_2. Fix the shielded probe into its styrofoam black cover and slowly lower the sensor end into the LN_2 dewar.

8. The critical temperature for the YBCO superconductor in SQUID is $T_C \approx 90$K and it will take some minutes for the SQUID sensor to reach the boiling point of LN_2, 77K.

9. It is important to cool the SQUID while minimizing the presence of magnetic fields in the surroundings. This will reduce the effect of flux trapping.

10. Minimize the BIAS OFFSET control and increase the SWEEP OUTPUT. The **V** − **I** curve will appear on the CRO screen. Use the BIAS OFFSET to symmetrize the trace and FLUX OFFSET to maximize the supercurrent (zero voltage drop) region. A good **V** − **I** curve should look like that shown in Figure 13.

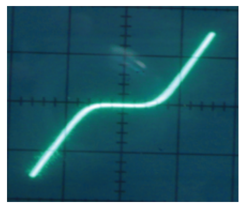

Figure 13: **V** − **I** characteristics of SQUID with CH I at 0.1 V/div and CH II at 0.5 V/div.

4.1.4 Determining the critical current

The **V** − **I** curve has a flat region at the center where the current is flowing with zero voltage drop. This region represents the supercurrent or zero-resistance current flowing through the JJ's, exhibiting DC Josephson effect. The current exists even though there is no voltage drop. You must have guessed that the current flows because of a nonzero phase difference $\triangle\alpha$ between the two superconducting regions across the JJ. But the superconductor ceases to be resistanceless as soon as the current exceeds the critical current, I_C. This critical current can be determined through the procedure given below.

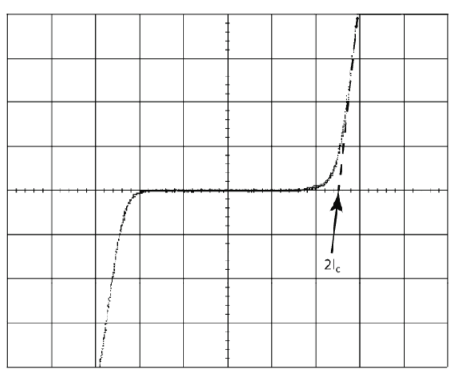

Figure 14: Method to determine the critical current $2I_C$.

1. Measure the voltage at the "knee" in the **V** − **I** curve from the CRO screen after maximizing the zero voltage current by using the FLUX OFFSET control.

2. Divide that voltage by 10,000 to convert the voltage to current.

3. The $\mathbf{V} - \mathbf{I}$ curve is typically rounded owing to the thermal noise. To measure the current more accurately , increase the vertical and horizontal sensitivities at the CRO channels.

4. As shown in Figure 14, extrapolate the straight part of the $\mathbf{V} - \mathbf{I}$ curve in the resistive region down to the horizontal axis. The point of intersection with the horizontal axis corresponds roughly to the critical current in the absence of thermal noise.

5. However this current is through both the junctions in the SQUID, so the critical current through one Josephson junction is half this value.

4.1.5 Calculation of the normal state resistance R_N

The $\mathbf{V} - \mathbf{I}$ mode also helps us measure another parameter of the JJ known as the normal state resistance. It can be measured by calculating the slope of the $\mathbf{V} - \mathbf{I}$ curve in the resistive region of the SQUID as described below.

1. Increase the horizontal and vertical sensitivities on the CRO channels

2. Increase the SWEEP OUTPUT control almost to the point where the horizontal and vertical outputs saturate, i.e., the ends of the $\mathbf{V} - \mathbf{I}$ curve get clipped and appear as flat lines as shown in Figure 15.

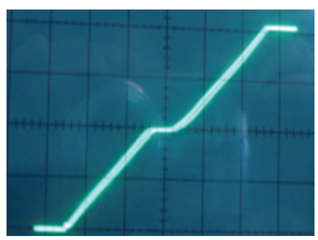

Figure 15: Saturation in the $\mathbf{V} - \mathbf{I}$ curve as the SWEEP OUTPUT is increased.

3. The slope of an imaginary line drawn between the endpoints of the $\mathbf{V} - \mathbf{I}$ curve passing through the origin, corresponds to the normal resistance of the SQUID. Refer to Figure 16.

4. Since the SQUID ring contains two JJs and hence two resistances in parallel, so the normal state resistance for a single JJ would be twice the slope of the dashed line drawn in Figure 16.

4.2 Flux quantization and V-ϕ characteristics of the SQUID

4.2.1 Objective

This experiment investigates the quantization of flux through SQUID ring. We will observe one of the most remarkable properties of the DC SQUID: the development of a periodic

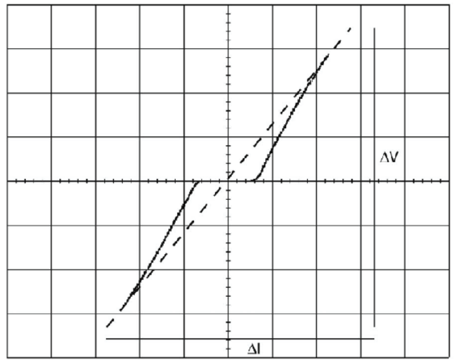

Figure 16: Method to determine the normal resistance of the JJ. R_N is twice the slope of the dashed line.

voltage across it in response to an applied magnetic flux, when biased slightly above the critical current.

Q 12. Equation 24 shows the sinusoidal variation of the critical current with flux. What is your prediction about the voltage developed across the SQUID as a function of the flux?

4.2.2 Apparatus

1. Mr. SQUID probe
2. Electronic control box
3. DB-9 M/M cable with 9-pin connector
4. Liquid nitrogen
5. Magnetic shield

4.2.3 Procedure

1. In the $\mathbf{V} - \mathbf{I}$ mode, rotate the SWEEP OUTPUT control completely counterclockwise (so as to see a dot on the CRO). Now rotate the BIAS OUTPUT knob, the dot will trace the usual $\mathbf{V} - \mathbf{I}$. Bias the SQUID just slightly above the critical current by adjusting the dot slightly above the knee of the $\mathbf{V} - \mathbf{I}$ curve. This is the most sensitive point on the $\mathbf{V} - \mathbf{I}$ curve.
2. Manually modulate the SQUID with the FLUX OFFSET control so that the point on the CRO screen will move up and down in response to the changing flux as an integral multiple of fluxons threading the superconducting ring. This periodic motion arises because the screening current in the SQUID body depends on the applied magnetic flux in a periodic manner where the period is determined by the magnetic flux quantum (ϕ_0).
3. Turn the MODE switch to $\mathbf{V}{-}\phi$, the down position. We will now attempt to automate the above step.

4. Rotate the SWEEP OUTPUT knob clockwise. This increases the sweep current through the internal modulation coil coupling a magnetic field to the SQUID and the periodic **V**$-\phi$ curve appears on the CRO screen.

5. The voltage change that occurs due to the influence of magnetic field now appears on the vertical axis of the CRO. Observe the modulation depth at increased vertical sensitivity.

6. The maximum peak-to-peak voltage swing of the SQUID modulation $\triangle V$ is measured which is called the modulation voltage for the SQUID. A typical curve is shown in Figure 17.

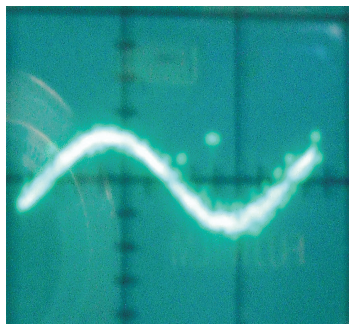

Figure 17: **V**$-\phi$ curve for the SQUID with CH I at 0.2 V/div and CH II at 20 mV/div.

7. The BIAS OFFSET can be adjusted to maximize the modulation depth.

8. Since this is an amplified signal so divide the $\triangle V$ value (obtained from the CRO) by 10,000 to get the actual magnitude of the voltage swing.

9. Adjusting the FLUX OFFSET control enables us to view a specific region of the **V**$-\phi$ curve. It actually allows us to apply a static magnetic field on the top of the oscillating field applied using the SWEEP OUTPUT. Turning this knob thus moves the **V**$-\phi$ curve left or right and enables us to explore points along the **V**$-\phi$ curve.

Q 13. Using the **V**$-\phi$ curve, find the mutual inductance of the SQUID chip with the internal modulation coil. HINT: Mutual inductance between the internal coil and the SQUID, M_{int} is expressed as the ratio of the magnetic flux threading coil 2 produced by current flowing through coil 1,

$$\begin{aligned} M_{int} &= \frac{\phi_2}{I_1} \\ &= \frac{\phi_0}{\triangle I_{period}} \end{aligned} \tag{25}$$

where $\triangle I_{period}$ is the current required by the flux ϕ to complete 1 waveform. Refer to the description found in [3], Section 4.4, page no. 27-28.

Q 14. Discuss the function of the FLUX OFFSET control knob in the **V** $-$ **I** and **V**$-\phi$ mode. Refer to page no. 17 and 23 of [3].

4.3 Resistance vs. temperature of the YBCO SQUID

4.3.1 Objective

This experiment helps us track the normally resistive to superconductive transition of the YBCO film that forms the SQUID in Mr. SQUID.

4.3.2 Apparatus

1. A commonly available silicon diode e.g., 1N914.
2. 200 cm long insulated copper wire (magnet wire).
3. A digital voltmeter (DVM) with sub millivolt resolution.
4. A soldering iron and electronics-grade solder.
5. A binder clip (stationery item).
6. Cotton wool.
7. Teflon tape.
8. DC constant current source of 10μA, constructed from the following equipment
   - An operational amplifier e.g., 741.
   - A zener diode 2.5-7 volts.
   - A selection of resistors in the range 1 kΩ through 100 kΩ.
   - A capacitor in the range 100 pF to 100 nF.
   - Two 9-volt batteries.
   - A solderless breadboard.

4.3.3 Constructing a constant current source (10μA)

In this part of the experiment, we will use a silicon diode as a thermometer. A constant current is supplied through the diode, operating in the forward bias region and the voltage drop across it is measured. The voltage is related to the temperature, as we show in the following discussion.

Q 15. Why does the resistance of a semiconductor decrease with increase in temperature? Consider the response of electron hole pairs to changes in temperature.

A simple constant current source can be build using a zener diode, two resistors, a capacitor, a general purpose operational amplifier and two 9 volt batteries. Build the circuit shown in Figure 18. The specific selection of the resistor $R_{current}$ is determined by the zener diode voltage. The circuit is designed in such a way so that the ratio of the zener diode voltage to the output resistance $R_{current}$ gives the value of the constant current generated by the circuit. Choose the components such that the output current is nearly 10μA.

Q 16. Describe the working of the constant current source.

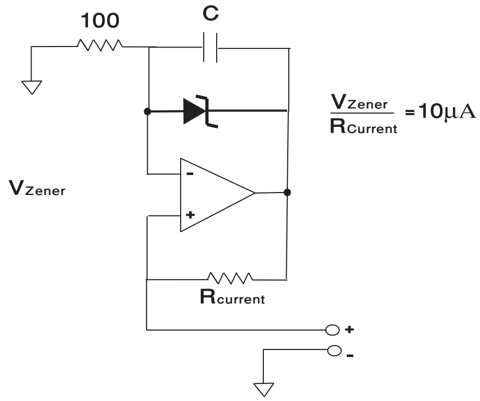

Figure 18: Circuit diagram for the constant current source.

4.3.4 Silicon Diode as a temperature sensor

1. We will use the commonly available silicon diode to measure the temperature of the SQUID inside the probe because the voltage across the diode varies almost linearly with temperature.

2. This diode voltage as a function of temperature is given by the following equation

$$V_f(T) = \frac{E_g}{2q} - \left[\ln(\alpha) + \frac{3}{2}\ln(T) - \ln(I_f)\right]\frac{k_B T}{q}, \tag{26}$$

   where

$$\alpha = \frac{1}{4}\left(\frac{2mk_B}{\pi\hbar^2}\right)^{3/2}\frac{Ak_B}{\tau E}.$$

   (The details are provided in [3] page no. 62, however there is a calculational error in [3].)

   Q 17. Derive Equation 26 by using the diode equation [4] for the forward bias current,

$$I_f = I_S \exp\left(\frac{qV_f}{k_B T}\right),$$

   where I_S is the saturation current through a *pn* junction semiconductor diode, q is the charge of carriers, T is the absolute temperature, k_B is Boltzmann's constant and V_f is the voltage across the diode.

   Q 18. Using Equation 26, plot V_f as a function of temperature T.

3. After trimming the leads of the diode, solder the copper wires to its ends.

4. Connect the diode leads to the 10 μA DC constant current source and the DVM as shown in Figure 19. Use at least 50 cm of copper wire between the diode and the DVM.

5. Now turn on the current source and the DVM. There should be a voltage drop of approximately 0.3-0.4 V across the forward biased diode.

4.3.5 Temperature calibration

1. We can now calibrate the temperature response of the diode since the voltage drop across the diode increases almost linearly with decreasing temperature.

2. Note the temperature of the laboratory with the help of a thermometer and record the voltage drop across the diode at his temperature. The room temperature can be the first calibration point for the diode sensor.

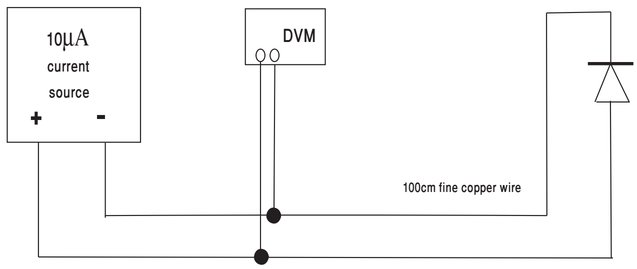

Figure 19: Setup for diode calibration.

3. Now slowly lower the diode right to the bottom of the dewar, three-fourth filled with LN_2. After the voltage reading has stabilized, again record the voltage across the diode. This voltage will lie somewhere between 0.9 and 1.1 volts and serves as the second calibration point, at 77 K.

4. A linear diode requires two calibration points, but the diode may be placed in an ice bath to get a third point also.

5. Now plot the diode voltage as a function of temperature and draw the line of best fit to the acquired data. In this way we have produced a calibration curve that interconverts diode voltage to temperature. Hence we have constructed a cryogenic temperature sensor using a Si-diode, a constant current source and voltmeter.

4.3.6 Procedure for detecting the superconducting phase transition

1. After performing the calibration we are required to plot the R-T curve for the SQUID. We will now use the diode (attached with long copper wires) to sense the temperature in the vicinity of the SQUID chip and calculate the corresponding resistance by measuring the slope of the SQUID's $\mathbf{V}-\mathbf{I}$ curve which keeps on changing while reducing temperature.

2. For this purpose first remove the magnetic shield of the probe carefully, keeping the screw in a safe place to be recovered later.

3. We will temporarily mount the diode with the probe using the teflon tape as shown in Figure 20.

4. **The diode is mounted to the back side of the chip.This process should be carried out very carefully so that the probe is not damaged in any case. Also avoid sticking teflon tape to the front side of the chip. DO NOT use ordinary tape to mount the diode.**

5. Now put the magnetic shield back on its place with the help of its screw. The copper wires should trail down the bottom of the shield.

6. Use small cotton stuffing to close the bottom of the probe in order to improve the temperature uniformity.

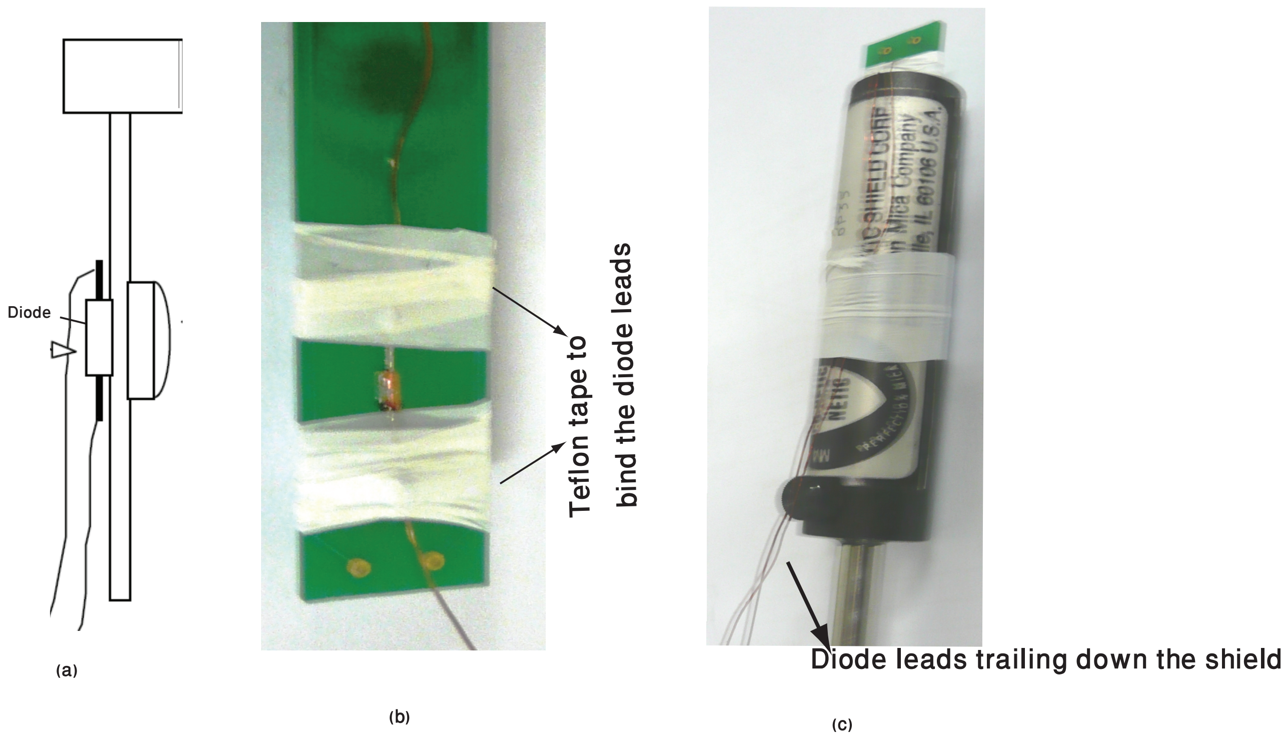

Figure 20: Setup for diode mounting. (a) Diode attached to Mr. SQUID. (b) Diode is attached with the teflon tape. (c) Shield the probe with the copper wires trailing downward.

7. Now empty the dewar until there is about 10-12 cm of LN_2 left at the bottom and lower the probe into the dewar.

8. The binder clip along with the foam cover is used to prevent the probe from sliding down right to the base of the dewar. Start with the SQUID probe at the very top of the dewar as shown in figure 21.

9. Now connect the SQUID probe to the electronic box and the current and voltage outputs of the electronic box to the CRO. Turn the mode switch to the $\mathbf{V} - \mathbf{I}$ position.

10. Connect the leads of the diode (coming from the bottom of the dewar) to the current source and DVM.

11. A straight line will be seen on the CRO screen, the slope of which gives the resistance of Mr. SQUID. At near room temperature (probe's chip end near the top end of dewar) it should be several hundred ohms.

12. **Measuring resistance as a function of T**

    - When the voltage across the mounted diode stabilizes, record its value. Wait patiently for the DVM to show you a stable value of voltage drop across the diode and then measure the corresponding slope.

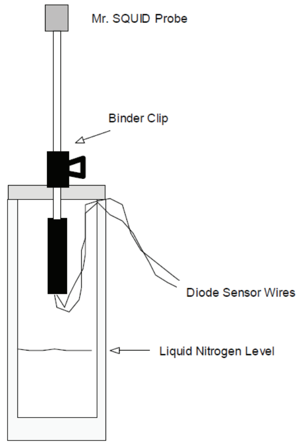

Figure 21: Position of Mr. SQUID's probe inside the LN_2 dewar.

- Calculate and record the slope of the **V** − **I** curve near the origin (V=0).
- Holding the probe, carefully loosen the binder clip and lower the probe about 1 cm further down into the dewar and record the voltage as well as slope of the **V** − **I** curve.
- Repeat this procedure until the probe's chip end is completely immersed in LN_2 and attained the temperature of 77 K. The diode's voltage should correspond to the LN_2's temperature at this point.

13. The expected transition of the slope while cooling the SQUID to LN_2's temperature is shown in Figure 22. The R-T graph should appear similar to Figure 23.

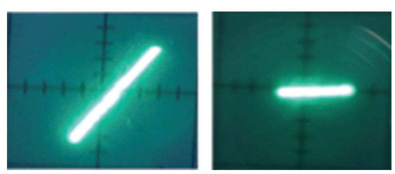

Figure 22: Transition of slope (resistance) from a finite value to zero during cooling to liquid nitrogen temperatures.

Q 19. Plot the R-T curve and note down the transition temperature of the YBCO film. Is the superconducting transition sudden? Is the resistance of the superconductor really zero?

Q 20. How does the resistance of a normal metal such as copper or silver change with temperature? What happens to the resistance at 0 K?

4.4 Analog flux-locked loop (FLL)

Mr. SQUID can be used as a sensitive magnetometer when employed in the so-called flux-locked loop FLL configuration. We know that in the resistive mode, the voltage across the SQUID is a sinusoidal function of the applied magnetic flux, with a period of one flux

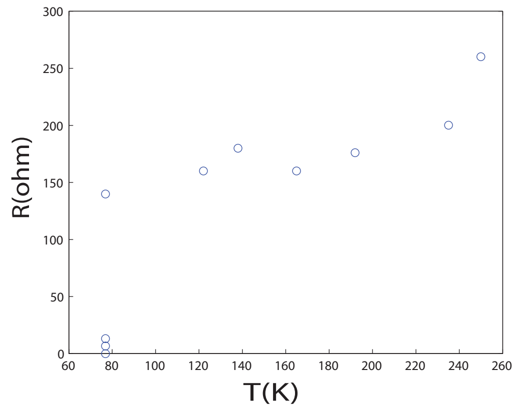

Figure 23: A typical $R - T$ graph.

quantum ϕ_0, but this is not the limiting resolution for the flux measurement. We can, quite accurately measure flux changes that are much smaller than a flux quantum. This extremely high sensitivity, makes the SQUID the most sensitive magnetometer!

4.4.1 Objective

In the present experiment, we will learn how the external coil of Mr. SQUID is used in a negative feedback loop for maintaining a constant magnetic flux through the SQUID ring. In the process, we will detect and measure ultra-small magnetic fields.

4.4.2 Principle of operation of the FLL

The FLL circuit is schematically represented in Figure 24. Here is the basic principle of operation.

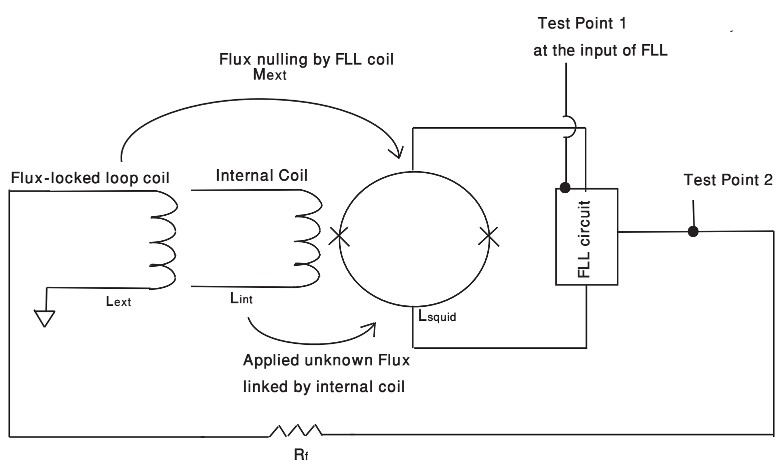

Figure 24: The flux-locked loop circuit, a schematic representation.

1. The SQUID (in the $\mathbf{V} - \mathbf{I}$ mode) is biased with $i > I_C$.

2. The $\mathbf{V}-\phi$ mode is switched on and a sweep current is applied across the inner modulation coil of the SQUID. This induces a magnetic flux in the SQUID loop which results in a periodic voltage across it.

3. This periodic voltage is then fed into the input of the FLL circuit, where it is amplified and inverted. The resultant negative voltage is used to drive the external coil of the SQUID.

4. By using the gain of the inverted amplifier and adjusting the variable resistance of the potentiometer, (shown in Figure 24), the FLL is set up in such a way that the current flowing through the external coil creates a magnetic flux of opposite polarity and equal magnitude to the applied flux produced by the inner coil.

5. Thus two fluxes cancel the effect of each other and the SQUID will be in a zero magnetic flux state: it will be locked in a zero-flux condition.

6. By measuring the current being used to generate the opposing flux through the external coil and using the mutual inductance of the external coil to the SQUID, M_{ext}, we can determine the magnitude of the applied unknown magnetic flux.

7. The FLL technique is based on the principle of the conversion of magnetic flux, which is hard to measure, into voltage, which is easier to measure.

Figure 25: Copper wire connection to the EXT. COIL terminals to find its the coil's resistance at liquid nitrogen temperature.

Q 21. The terminals of external coil at low temperature are provided at the bottom of the probe as shown in Figure 25. Bind an $\approx$ 50 cm long copper wire to each terminal of the coil and us find the coil's resistance R_{ext} at 77K.

4.4.3 Apparatus

1. Mr. SQUID and LN_2.
2. Oscilloscope.
3. Two dual-operational amplifiers (e.g., HA17458P).
4. Two 9-volt batteries.
5. One 10 kΩ potentiometer.
6. A selection of resistors in the range of 1 kΩ through 100 kΩ.
7. A selection of capacitors in the range of 0.001 μF through 1 μF.
8. A selection of BNC connectors, hook up wires and alligator clips.
9. Copper wire to connect to the external coil connections.

4.4.4 Procedure

1. Connect the FLL circuit on the bread board, as shown in Figure 26.

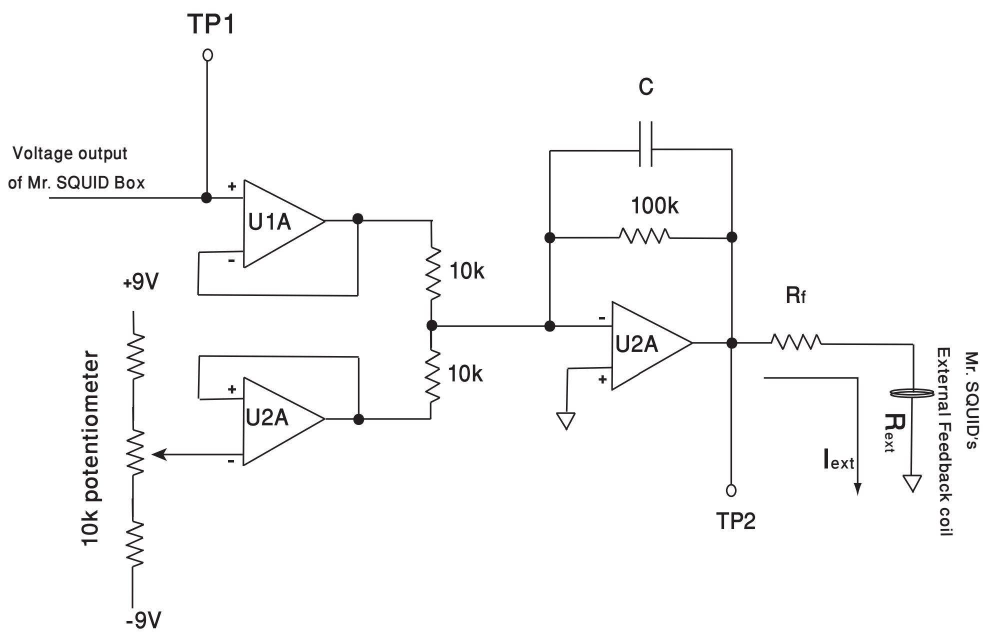

Figure 26: Circuit diagram for the flux-locked loop circuit.

2. Before setting up Mr. SQUID, remove the top cover of the MS-EB03 electronics box and move the MODE switch next to the BNC connector into the direct (DIR) position. In this position, the signal connected to the EXT INPUT BNC on the rear panel of the Mr. SQUID electronics box is directly coupled through a 100 mA fuse at location F1 to the external feedback coil on the Mr. SQUID chip. **Be sure to reset the MODE switch in the buffered (BUF) position after you are finished with the experiment.**

3. In the $\mathbf{V}-\mathbf{I}$ mode, turn the SWEEP OUTPUT to its minimum so that you can just see a point on the CRO screen. Using the BIAS OFFSET knob adjust this point at the knee of the $\mathbf{V}-\mathbf{I}$ curve and turn on the $\mathbf{V}-\phi$ mode.

4. Turn the SWEEP OUTPUT knob to set the amplitude of the flux (ϕ) to about $\pm$ 0.5 flux quantum by adjusting the single waveform symmetrically on the x-axis of the oscilloscope about the origin. This is shown in Figure 27, the top left corner.

5. Connect the voltage output of the electronic box to the input of the FLL circuit and CRO's channel II to either test point TP1 or test point TP2 as required.

6. The outputs at the TP1 and TP2 should look similar to Figure 27 (a) and (b). Compare the phase shift and amplitudes of TP1 and TP2.

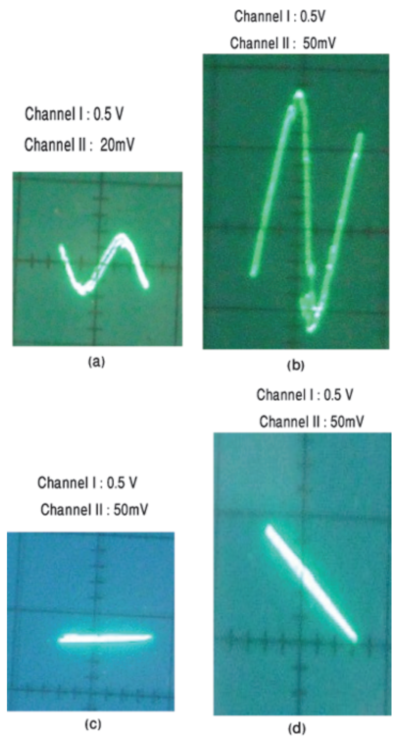

Figure 27: Scope photos of signals at TP1 (left) and TP2 (right) without external feedback. (a)TP1 showing flux being applied to the SQUID, (b) Amplified and inverted signal at TP2, (c) TP1 with the feedback flux superimposed, (d) Left over flux negatively amplified at TP2.

7. TP1 simply shows the $\mathbf{V}-\phi$ output of Mr. SQUID as a magnetic flux of $1\phi_0$ threads the SQUID loop. We can suppress the DC offset at TP1 by ac-coupling into the CRO.

8. TP2 also shows the $\mathbf{V}-\phi$ output of Mr.SQUID but magnified by a factor of -10 and with an extra DC offset from the FLL circuit's potentiometer. DC-couple the CRO at TP2 to see this offset and adjust the potentiometer so that the TP2 signal doesn't contain this DC offset anymore.

Q 22. Describe the working of the difference amplifier circuit shown in Figure 26.Calculate its gain?

9. Now the circuit is ready to lock the flux state of Mr.SQUID. This is done by carefully connecting the FLL circuit output to the BNC connector labeled EXT. COIL on the back of your Mr. SQUID electronic box. This BNC is a direct connection to the external coil.

10. Reconnect the power to your flux-locked loop circuit. The outputs at the TPs should now look like those in Figures 27 (c) and (d).

11. At this stage, TP1 should ideally would be a flat line if the FLL circuit perfectly canceled the flux applied by the Mr. SQUID box. But our output at TP1 Figure 27 (c) is slightly sloped indicating that the cancelation was not perfect due to the noise of the SQUID and the noise of the electronics.

12. The amount of flux threading the SQUID loop (before the feedback is superimposed) is,

$$\phi_{SQ} = N L_{SQ} I_{SQ} + M_{int} I_{int}, \tag{27}$$

where N is the number of turns of SQUID coil, i.e., 3/4 [3], L_{SQ} is the SQUID's self-inductance, I_{SQ} is the amount of biasing current flowing through the SQUID loop, M_{int} is the mutual inductance of the internal modulation coil and I_{int} is the current flowing through it. The mutual inductance M_{int} was calculated in Equation 25.

Q 23. What is the value of the current I_{SQ}, when the SQUID is biased approximately at the knee of the $\mathbf{V} - \mathbf{I}$ curve?

13. After providing feedback through the external coil, the net flux linking the SQUID is

$$\begin{aligned} \phi_{Net} &= L_{SQ} I_{SQ} + M_{int} I_{int} + M_{ext} I_{ext} \\ &= \phi_{SQ} + M_{ext} I_{ext} \\ &= \phi_{SQ} + \phi_{FB}, \end{aligned} \tag{28}$$

where ϕ_{FB} is the negative feedback flux provided through the external coil. Ideally $\phi_{Net} \approx 0$, i.e., $\phi_{FB} = -\phi_{SQ}$.

14. The values of the relevant inductances and resistances are given in Table 1.

Component	Values
M_{int}	Calculate from Q. 13, Section 4.2.3
M_{ext}	35 pH
L_{SQ}	73 pH
R_{fb}	2400 Ω
R_{ext}	Calculate from Q. 21, Section 4.4.2

Table 1: Values of Inductances and Resistances required for flux calculations

15. Calculate I_{ext} by measuring the voltage at TP2.

Q 24. Using the ideas developed above, estimate the efficacy of the FLL, i.e., how much flux is the circuit actually canceling out.

Q 25. Describe how the FLL is actually working as a magnetometer. What is the magnitude of the applied flux ϕ_{SQ} before the feedback is applied? Calculate the un-canceled flux as a fraction of ϕ_0.