HW 2: Maxwell's Equations: A Holistic View

You are welcome to directly use the vector identities given on the inside flap of Zangwill's book.

1. A static boundary exists in space at $z = 0$. Use the properties of the Heaviside function $\Theta(z)$ to derive the matching condition,

$$\hat{\mathbf{e}}_z \times (\mathbf{B}_R - \mathbf{B}_L) = \mu_o \mathbf{K}.$$

Here $\mathbf{B}_{R,L}$ are the magnetic fields in the region $z > 0$ and $z < 0$ respectively. The vector $\mathbf{K}$ represents the areal current density at the interface.

2. Discrete charges constitute a distribution,

$$\rho(\mathbf{r}, t) = \sum_k q_k(t)\delta(\mathbf{r} - \mathbf{r}_k). \tag{1}$$

   (a) Show that the current density,

$$\mathbf{j}(\mathbf{r}) = \frac{1}{4\pi} \sum_k \dot{q}_k \nabla \frac{1}{|\mathbf{r} - \mathbf{r}_k|}$$

   satisfies the continuity equation.

   (b) Write down an expression for the electric field $\mathbf{E}(\mathbf{r}, t)$ for the given charge distribution.

   (c) Find $\nabla \times \mathbf{B}$. What can be said about its magnitude as a function of time?

   (d) Show that given the expression of the charge density in Eq. (1), $Q = \sum_k q_k$.

3. A layer of polonium is deposited n the surface of a sphere of radius R. The metal emits alpha particles. Assume that these particles are emitted radially outward, thus forming a current. Will there be a magnetic field associated with this current?