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Exploring phase space using smartphone acceleration and rotation sensors simultaneously

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Abstract

A paradigmatic physical system as the physical pendulum is experimentally studied using the acceleration and rotation (gyroscope) sensors available on smartphones and other devices such as iPads and tablets. A smartphone is fixed to the outside of a bicycle wheel whose axis is kept horizontal and fixed. The compound system, wheel plus smartphone, defines a physical pendulum which can rotate, giving full turns in one direction, or oscillate about the equilibrium position (performing either small or large oscillations). Measurements of the radial and tangential acceleration and the angular velocity obtained with smartphone sensors allow a deep insight into the dynamics of the system to be gained. In addition, thanks to the simultaneous use of the acceleration and rotation sensors, trajectories in the *phase space* are directly obtained. The coherence of the measures obtained with the different sensors and by traditional methods is remarkable. Indeed, due to their low cost and increasing availability, smartphone sensors are valuable tools that can be used in most undergraduate laboratories.

Keywords: physical pendulum, smartphone, acceleration sensor, rotation sensor, gyroscope

(Some figures may appear in colour only in the online journal)

1. Introduction

According to press releases, in 2013 more than one billion smartphones were sold worldwide. These devices usually incorporate several sensors, including accelerometers, gyroscopes, and magnetometers. Although these devices are not supplied with educational purposes in mind, they can be employed in a wide range of physical experiments, especially in college and undergraduate laboratories. Thanks to their low cost, great adaptability and widespread availability among students, their usage has considerably expanded in laboratories.

Recently, in several articles [1–9], the use of smartphones has been proposed for use in several physics experiments. Several of these experiments are focused on classical mechanical studies; for example the analysis of free fall using acceleration sensors [1, 4], the kinematics of a pendulum [3], the impulse–momentum relation [9], coupled, forced and damped oscillations [5–7], or rotatory motion [8]. Acoustic and optical phenomena have also received considerable attention [10–12]. Several experiences related to electromagnetism were also proposed; for example, smartphones were employed as portable oscilloscopes in [13], or as a tool to measure magnetic fields [14].

Although most previous articles focused on mechanical experiments, an aspect that has received considerably less attention is the use of rotation sensors or gyroscopes. Indeed, the use of these sensors paves the way for new experiments, especially in classical mechanics, enabling the measurement of angular velocities. Very recently, the use of gyroscopes has been introduced to analyse the conservation of the angular momentum [8] or to measure the rotational energy in a simple system [15]. In the present work, rotation sensors are employed to measure the acceleration and the angular velocity of a physical pendulum. A step forward in the use of these devices is given by the possibility of measuring various components of the acceleration and the angular velocity *simultaneously*. In this way, a more complete picture of the dynamics of simple systems can be obtained. In the simplest case of systems with only one degree of freedom, two independent coordinates allow one to fully determine the temporal evolution; that is, to follow trajectories in the phase space. In this way, an abstract concept of classical mechanics becomes more tangible.

Nowadays, there is intense debate and research on the teaching and learning of Physics. Several authors have highlighted the importance of involving students in their own learning process (see [16] for a resource letter). A recent approach, known as *active-learning instruction*, insists on the involvement of the students in their own learning more deeply and more intensely than in traditional instruction schemes. One feature is the incorporation of classroom and laboratory activities that encourage students to express their thinking through several actions that go beyond the traditional, more *passive*, procedures. In this way, several techniques such as real-time computerized experiments, Socratic dialogues, interactive computer simulations, and structured problem-solving, among others, are included. A possible alternative is the analysis and discussion of everyday problems [17], or the proposal of simple demonstrative experiences using non-sophisticated equipment [18, 19]. Another option is non-classroom activities, such as those in amusement parks, either mechanical [20–22] or water based [23]. Indeed, as recent studies have pointed out [24], the rewiring approach to learning suggests that the knowledge that students bring to a classroom should be viewed as a productive resource upon which to build, rather than as an impediment. In the experience proposed here, the use of an ubiquitous and everyday device such as a smartphone is intended as a way to transform science into an activity firmly linked to the real world.

The focus of this work is the study of a paradigmatic system in classical mechanics, a physical pendulum, using smartphone acceleration and rotation sensors simultaneously. A physical pendulum is a rigid body, which can rotate around a fixed point, or *pivot*, located at non-zero distance from the centre of mass. In our experiment, a physical pendulum was built using a bike wheel and a smartphone as described in the next section. In section 3, the model and some fundamental concepts of classical mechanics are reviewed. The analysis of the motion is carried out in section 4. Finally, in section 5, concluding remarks are presented.

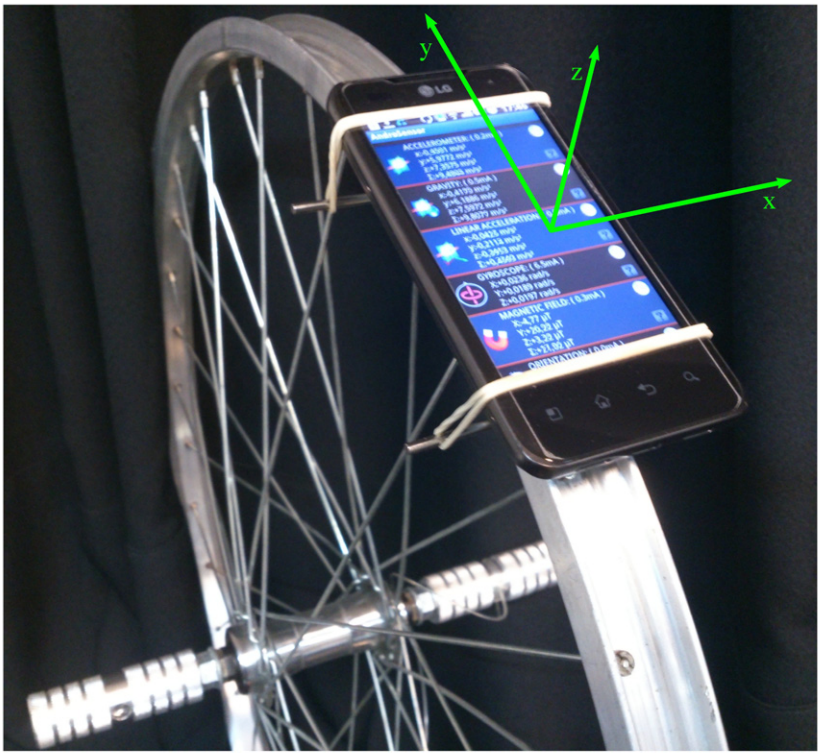

Figure 1. Smartphone mounted on a bike wheel moving in a vertical plane and a scheme indicating the axes orientation.

2. Experimental setup

The experimental setup consists of a smartphone mounted in the periphery of a bicycle wheel that can rotate freely in a vertical plane, as shown in figure 1. The distance between the centre of mass of the wheel and the centre of mass of the smartphone is $R = 0.300$ m and the mass of the smartphone is $m = 0.146$ kg.

The moment of inertia of the physical pendulum is determined as follows. First, the period of the small oscillations of the wheel (without the smartphone) suspended from a fixed point located at the inner part of the rim is obtained. Using the parallel axis theorem, the moment of the inertia of the wheel with respect to the geometrical centre is found to be $I_w = 0.039$ kg m^2. Next, thanks to the additivity, the moment of inertia of the compound system, wheel and smartphone, is easily calculated as $I = I_w + mR^2 = 0.052$ kg m^2.

The smartphone is an LG Optimus P990 2X (three-axis accelerometer KXTF9 Kionix, accuracy 0.001 m s^{-2}, three-axis gyroscope MPU3050 Invensense, accuracy 0.0001 rad s^{-1}) was used. The application AndroSensor [25] running under the Android operating system was employed to record the values measured by the sensors. Usually, the different sensors register vector magnitudes along three axes oriented, as indicated in figure 1. In the present experiment, the relevant magnitudes are the component of the angular velocity reported by the rotation sensor according to the x-axis and two components of the acceleration according to the y- and z-axis, corresponding to the tangential and radial accelerations respectively. Once registered, data can be exported and analysed using appropriate software.

3. Theoretical overview

The equation of motion of the physical pendulum can be readily obtained by using Newton equations applied to a rigid body. Defining the variable θ as the angle from the lowest position of the smartphone, the equation of motion reads as

$$I\dot{\omega} = -Rmg \sin\theta + M_r \tag{1}$$

where $\omega = \dot{\theta}$ is the angular velocity. The right-hand side of equation (1) accounts for the torque originated by the mass of the smartphone and for the rolling resistance, or friction, M_r originated mainly in the bearings of the wheel. In general, as bike wheels are designed to reduce friction, this term contributes little. In our setup, as shown in the next section, energy is slowly dissipated and friction can be neglected when considering a short time interval, i.e. of the order of a few periods. However, when analysing a larger time interval dissipation must be taken into account.

Under this assumption of weak dissipation, $M_r \sim 0$, the equation of motion can be integrated to obtain

$$E = \tfrac{1}{2}I\omega^2 + mgR(1 - \cos\theta) \tag{2}$$

representing the energy conservation. Depending on the value of the energy the pendulum would be either rotating (performing full revolutions in the one direction) or oscillating about the stable equilibrium position. In addition, if the pendulum is oscillating with small amplitudes, the equation of motion, equation (1), can be linearized

$$I\dot{\omega} = -Rmg \sin\theta \sim -Rmg\theta. \tag{3}$$

The energy conservation is expressed in this case as a function of the angle θ

$$E = \frac{1}{2}I\dot{\theta}^2 + \frac{1}{2}\frac{I^2}{mgR}\ddot{\theta}^2. \tag{4}$$

A central concept in classical mechanics is that of the *phase space*, see for example [26]. The state of a mechanical system can be represented by a unique point in this geometrical representation which consists of all possible values of the generalized coordinates and their conjugate variables (or generalized momenta). For systems with only one degree of freedom, the dimension of the phase space is equal to two. This is the case, for instance, of the physical pendulum in which the generalized coordinate usually chosen is the angle θ and the conjugate variable is the corresponding component of the angular momentum $I\omega$.

As the system evolves, it traces a path or trajectory in the phase space. In the present case of a system with one degree of freedom, the energy conservation, equation (2), determines the shape of phase space trajectories. In particular, in the limiting case of small oscillations, using the linearized equation of motion, equation (3), and the energy conservation, equation (4), we notice that trajectories in phase space are given by ellipses.

4. Analysis of the motion

We start analysing the dynamics of the system by plotting in figure 2 both the radial acceleration a_r and the angular velocity ω as a function of time. At the beginning, the wheel is rotating in one direction; however, at t = 30 s, (indicated by the green circle), due to the effect of the weak dissipation, the angular velocity vanishes for the first time, the wheel reverses the direction of spinning and starts to oscillate around the stable equilibrium point. During the rotational motion, the displayed maxima and minimums of both magnitudes correspond to the smartphone at the lowest or the highest point of the trajectory, respectively. However,

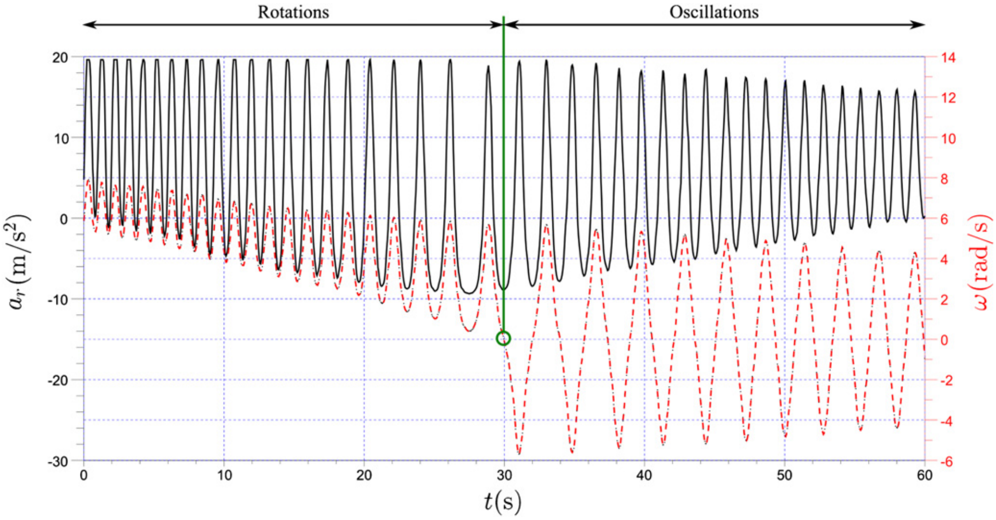

Figure 2. Radial acceleration (left axis, black lines) and angular velocity (right axis, red lines). At the initial stage when there is enough energy, the wheel is rotating. At the point indicated with the green circle, the angular velocity changes its sign and the wheel starts oscillating about the stable equilibrium point.

when the smartphone is oscillating around the equilibrium point, the interpretation of the extrema is a bit different. In this case, the minimums of the radial acceleration and the zeros of the angular velocity coincide with the smartphone at the turning points. The maxima of the angular velocity occur when the smartphone is passing through the lowest point and coincide either with a maximum or a minimum of the angular velocity according to the direction of the motion.

Two additional technical aspects in figure 2 can be remarked upon. The minimums of the radial acceleration, which occur when the smartphone is at the highest point of the wheel, do not vanish as would be expected when the wheel is almost stopped near the green circle, but approaches to -10 m s^{-2}. This is a consequence of the mode of operation of acceleration sensors that are, actually, force sensors [1]. The same fact can be noted at the last stage of the oscillations when the acceleration fluctuates around 10 m s^{-2} instead of 0 m s^{-2}. Another detail on the limitations of the sensors is that in the first 20 s the maximum acceleration saturates the sensor range (20 m s^{-2}).

To extend the analysis and further evidence the coherence of the measures of acceleration and rotation sensors in figure 3, the radial acceleration and the angle are plotted. The angle θ is obtained by numerical integration of the angular velocity measured by the gyroscope

$$\theta(t) = \int_{t_0}^{t} \omega(t')\,\mathrm{d}t'$$

where the integral is numerically integrated using a trapezoidal approximation [27]. The radial acceleration exhibits its relative maxima each time the angle is zero, which is when the smartphone crosses the lowest point of the trajectory. Moreover, the minimums of the radial acceleration occur at the turning points when the absolute value of the angular variable reaches its maxima.

To go in depth, in figure 4, the temporal evolution of the potential and kinetic energy is depicted. When the wheel is rotating, relative maxima and minimums of the kinetic energy are achieved when the smartphone is passing through the lowest or highest points respectively.

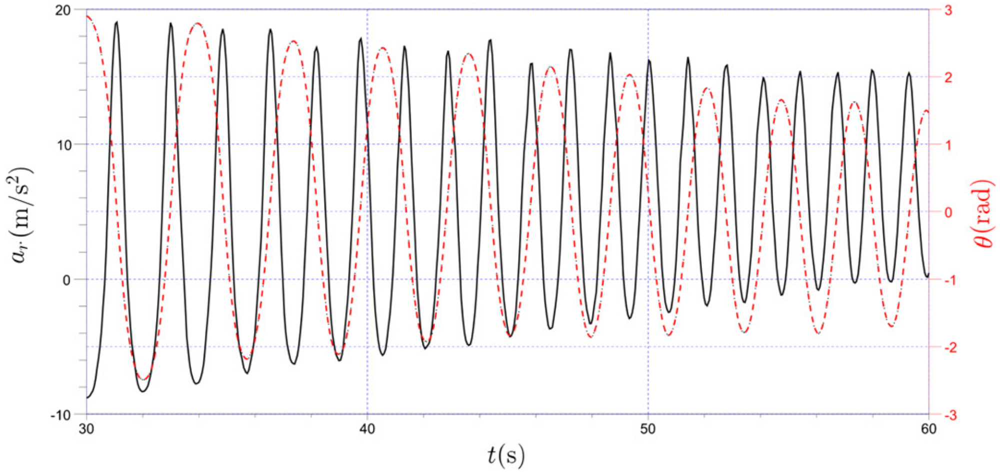

Figure 3. Radial acceleration and angular variable obtained from numerical integration of the angular velocity during the oscillation stage.

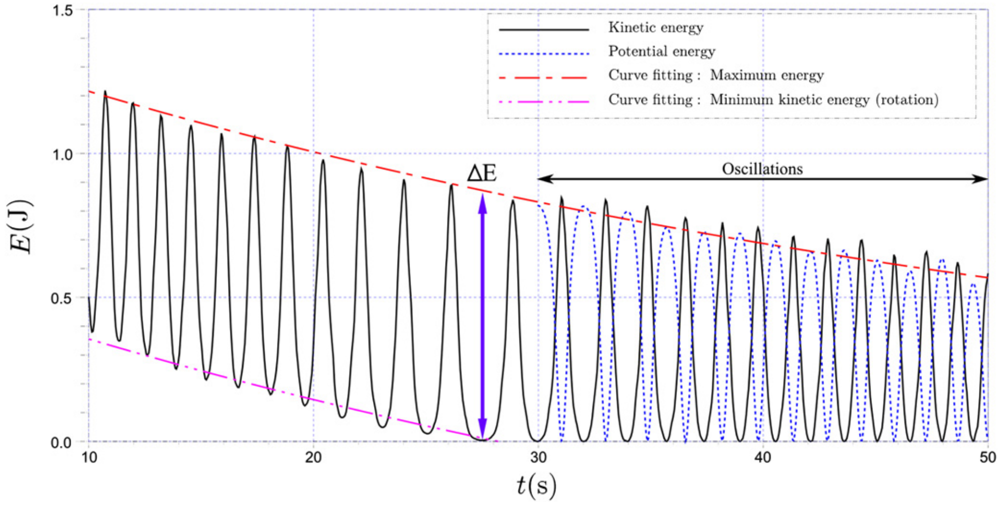

Figure 4. Temporal evolution of the kinetic (black) and potential (blue) energy. The potential energy is plotted only during the oscillation stage indicated with the horizontal arrow. The dashed curves are exponential fittings to the relative maxima (red) and minimums (pink). The numerical value of the vertical difference between these curves (see the violet arrow) is $\Delta E = 0.86$ J, which is in excellent agreement with the value obtained by measuring the physical parameters of the pendulum (see text).

After that, when the wheel is oscillating, relative minimums coincide with turning points or zeros of the angular velocity, while relative maxima correspond to the smartphone passing through the lowest point in one direction or the other according to the sign of the angular velocity.

Two exponential curves (fitting the relative maxima and minimums) included in figure 4 reveal that the energy decreases roughly exponentially. In addition, the vertical difference of these curves can be used to compare with measures obtained from the physical parameters of

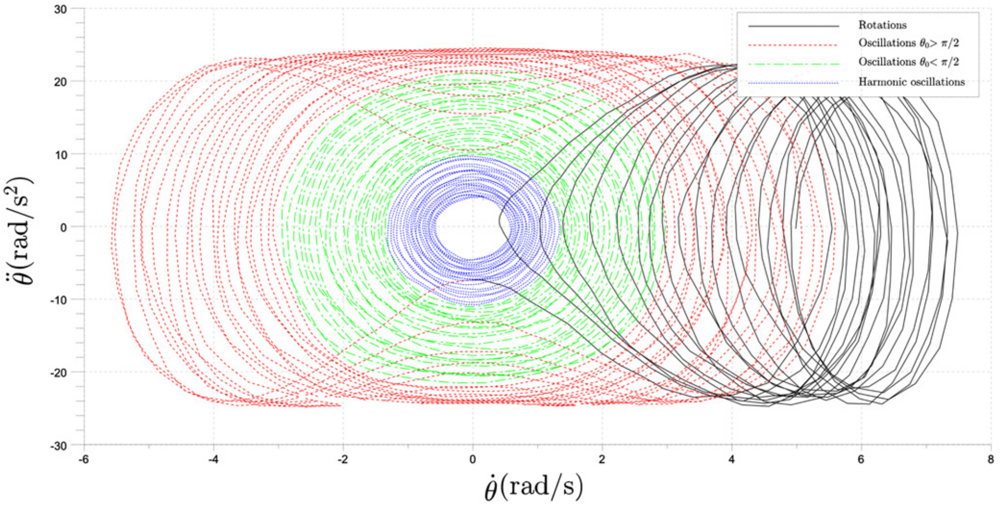

Figure 5. Typical trajectory in phase space using $\dot{\theta}$ (obtained from the gyroscope) and $\ddot{\theta}$ (obtained from the tangential acceleration $a_t = R\ddot{\theta}$), as generalized coordinates. The full evolution of the system, from the rotations until the wheel is almost stopped is plotted. At the beginning of the experiment the wheel was set rotating, i.e. performing full revolutions (black lines). Due to the presence of a weak dissipation, when $\dot{\theta}$ vanishes for the first time, the wheel starts oscillating about the equilibrium point. The oscillations are plotted in different colours, large amplitudes, $\theta_0 > \pi/2$ (red); intermediate, $\theta_0 < \pi/2$ (green) and harmonic oscillations (blue). When the amplitude of the oscillations is small enough the motion is harmonic and the trajectories can be approximated by ellipses

the system. The value obtained from the parameters of the fittings, $\Delta E = 0.86$ J, accounts for the difference of kinetic energy of the smartphone in the top respect to the bottom position. During a single rotation, as discussed in the previous section, the energy is very nearly conserved, and using the bottom of the bicycle wheel as the zero of potential energy, we can write the total energy of the system during the rotation as

$$E = K + U = \tfrac{1}{2}\omega_b^2 = \tfrac{1}{2}\omega_t^2 + 2mgR. \tag{5}$$

where w_b and w_t are the angular velocities when the smartphone is at the bottom or top position respectively. It is worth noticing the clear agreement between the value obtained using the sensors (0.86 J) and the value obtained from the physical parameters (0.86 J).

The same two qualitatively different motions discussed in figure 2, rotations and oscillations, can be distinguished in the phase space depicted in figure 5 where the trajectory in the phase space is plotted. To take advantage of the magnitudes measured by the sensors, the phase space is represented as a function of $\dot{\theta} = \omega$ (obtained from the gyroscope) and the angular acceleration $\ddot{\theta} \sim a_t$ (obtained from the accelerometer). At the first stage of the experiment, the wheel is rotating performing full revolutions in the same direction (black lines). As the energy is dissipated, when the angular velocity changes its sign for the first time, the wheel starts oscillating about the stable equilibrium position. According to the amplitude of the oscillations, the curve is plotted in different colours: large amplitudes (red), intermediate amplitudes (green) and, at the end of the realization, the trajectory can be approximated by ellipses, given by equation (4), indicating harmonic oscillations (blue lines).

5. Concluding remarks and perspectives

In this paper, the use of a smartphone is proposed for making direct measurements of the acceleration and the angular velocity of a physical pendulum. According to the initial conditions, the pendulum can be rotating or oscillating. The temporal series of the measured magnitudes allow these different motions to be distinguished. In addition, derived magnitudes as the kinetic and potential energies are also calculated and their relative extrema interpreted in terms of the characteristics of the motion. The comparison between the measures obtained by the different sensors (acceleration and rotation) and using traditional methods exhibits great coherence. The temporal evolution of these quantities is compared with the trajectories in the phase space. In this way a rather abstract concept becomes more substantial.

The use of smartphones in general presents a clear advantage in comparison with the use of other methods, such as interfaces or video analysis, which require costly devices and/or special training. We remark that the use of the smartphone sensors allows a broad spectrum of measures applicable in different mechanical experiments. The potential is more significant if we consider the simultaneous use of acceleration and rotation sensors that allows, among other things, us to access the phase space and get a physical representation of a rather abstract concept. This methodology can be extended to other experiments in classical mechanics, such as a simple pendulum, a torsion pendulum, a spring pendulum, and coupled or forced oscillators. The use of devices that are becoming increasingly available for secondary or university students, such as smartphones, helps to reduce the gap between science and everyday experience.

In spite of the several advantages of the smartphones, it is worth mentioning several drawbacks that need be addressed in the coming years. When using smartphones owned by students in a classroom, one concern could be the transfer of cost from the institution to the parents. Another could be the responsibility for possible damage incurred to the devices during an experiment. Possible discrimination against students not having or having older models of a device is another problem. Privacy issues are also especially relevant.

In recent years, mobile technologies have vertiginously developed and become available to a large number of users in the world and especially among young people. Because of their widespread use, these devices have begun to be used in education. Smartphones, as used almost permanently by students, can become a much more powerful, ubiquitous educational tool and display greater penetration than laptops. Moreover, although less exploited or investigated, these devices contain a large number of sensors, which allows students to perform multiple measurements. The learning of science, usually restricted to classrooms and laboratories, materializes in this way into the hands of all students, allowing them to carry out specific activities beyond traditional ones, such as measurements of multiple variables in non-classroom locations, for example by measuring the intensity of sound in a discotheque or the acceleration of a vehicle. These possibilities generate interest in measuring and meeting the physical world around them, besides the traditional classroom formats. It is therefore considered that the incentive of using smartphones in education encourages an interest in science in general. We expect, as cell phones change our way of life, they will also change our way of teaching and learning.

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References

[1] Vogt P and Kuhn J 2012 Analyzing free fall with a smartphone acceleration sensor *Phys. Teach.* **50** 182

[2] Chevrier J, Madani L, Ledenmat S and Bsiesy A 2013 Teaching classical mechanics using smartphones *Phys. Teach.* **51** 376

[3] Kuhn J and Vogt P 2012 Analyzing spring pendulum phenomena with a smart-phone acceleration sensor *Phys. Teach.* **50** 504

[4] Kuhn J and Vogt P 2013 Smartphones as experimental tools: different methods to determine the gravitational acceleration in classroom physics by using everyday devices *Eur. J. Phys. Educ.* **4** 16

[5] Castro-Palacio J C, Velázquez-Abad L, Giménez M H and Monsoriu J A 2013 Using a mobile phone acceleration sensor in physics experiments on free and damped harmonic oscillations *Am. J. Phys.* **81** 472

[6] Castro-Palacio J C, Velázquez-Abad L, Giménez F and Monsoriu J A 2013 A quantitative analysis of coupled oscillations using mobile accelerometer sensors *Eur. J. Phys.* **34** 737–44

[7] Sans J A, Manjón F J, Pereira A L J, Gomez-Tejedor J A and Monsoriu J A 2013 Oscillations studied with the smartphone ambient light sensor *Eur. J. Phys.* **34** 1349

[8] Shakur A and Sinatra T 2013 Angular momentum *Phys. Teach.* **51** 564

[9] Streepey J W 2013 Using ipads to illustrate the impulse-momentum relationship *Phys. Teach.* **51** 54

[10] Parolin S O and Pezzi G 2013 Smartphone-aided measurements of the speed of sound in different gaseous mixtures *Phys. Teach.* **51** 508

[11] Thoms L-J, Colicchia G and Girwidz R 2013 Color reproduction with a smartphone *Phys. Teach.* **51** 440

[12] Kuhn J and Vogt P 2013 Analyzing acoustic phenomena with a smartphone microphone *Phys. Teach.* **51** 118

[13] Silva N 2012 Magnetic field sensor *Phys. Teach.* **50** 372

[14] Forinash K and Wisman R F 2012 Smartphones as portable oscilloscopes for physics labs *Phys. Teach.* **50** 242

[15] Monteiro M, Cabeza C and Martí A C 2014 Rotational energy in a physical pendulum *Phys. Teach.* **52** 561

[16] Meltzer D E and Thornton R K 2012 Resource letter alip–1: active-learning instruction in physics *Am. J. Phys.* **80** 478–96

[17] Walker J 2008 *The Flying Circus of Physics: With Answers* (New York: Wiley)

[18] Ehrlich R 1997 *Why Toast Lands Jelly-Side Down: Zen and the Art of Physics Demonstrations* (Princeton, NJ: Princeton University Press)

[19] Ehrlich R 1988 *Turning the world inside out (and 174 other simple physics demonstrations)* (Princeton, NJ: Princeton University Press)

[20] Unterman N A 2001 *Amusement Park Physics: A Teacher's Guide* 2nd edn (Portland, ME: Walch Publishing)

[21] Bagge S and Pendrill A-M 2002 Classical physics experiments in the amusement park *Phys. Educ.* **37** 507

[22] Moll R F 2010 An amusement park physics competition *Phys. Educ.* **45** 362

[23] Cabeza C, Rubido N and Martí A C 2014 Learning physics at the water park *Phys. Educ.* **49** 187

[24] Etkina E and Planinšič G 2014 Thinking like a scientist *Physics World* **27** (3) 48

[25] Google Play 2014 (last visited, February) http://play.google.com

[26] Thornton S T and Marion J B 2004 *Classical Dynamics of Particles and Systems* (Belmont: Thomson Learning)

[27] Nakamura S 1995 *Numerical Analysis and Graphic Visualization With MATLAB* (Englewood Cliffs, NJ: Prentice-Hall)