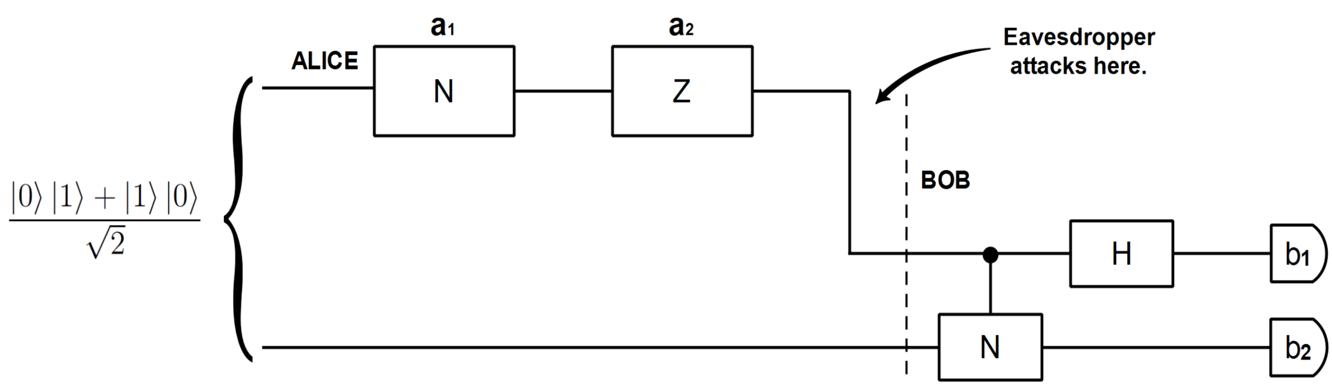
Assignment 3: Modern Physics
Due Date: 28th February 2018, 10:00 a.m.

Note: This is a collaborative assignment. Submit in up to groups of four. Write all four names and roll numbers clearly on the first sheet. Submit your assignments using A4 or similar sheets.

1. A quantum state is represented by $\frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$. Show this on the Bloch sphere. What kind of rotation (axis and amount of rotation) is needed to transform this state to $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$?

2. Consider two qubits prepared in the state

$$\frac{|0\rangle\,|1\rangle + |1\rangle\,|0\rangle}{\sqrt{2}}.$$



Alice wants to send two bits of information (a_1, a_2) to Bob. There are four possible messages:

$$(a_1, a_2) = (0, 0)$$
$$(a_1, a_2) = (0, 1)$$
$$(a_1, a_2) = (1, 0)$$
$$(a_1, a_2) = (1, 1)$$

Alice performs a sequence of operations on her qubit in the order shown.

$a_1 = 0$,	don't apply NOT gate (do nothing)
$a_1 = 1$,	apply NOT gate
$a_2 = 0$,	don't apply Z gate (do nothing)
$a_2 = 1$,	apply Z gate

Truth table for the two states is shown below.

NOT Gate	
Input	**Output**
$\|0\rangle$	$\|1\rangle$
$\|1\rangle$	$\|0\rangle$

Z Gate	
Input	**Output**
$\|0\rangle$	$\|0\rangle$
$\|1\rangle$	- $\|1\rangle$

(a) After these operations, Bob receives Alice's qubit. Bob then performs the sequence of measurement shown. What are Bob's binary outcomes b_1, and b_2? Show the full working.

(b) Comment on using this scheme for sending information (a_1, a_2) to Bob. How many bits are sent?

(c) If another attacker (eavesdropper) attacks Alice's qubit and tries to measure it, can he tap on the information (a_1, a_2)?

3. I have a single qubit in an unknown state which is either $\frac{|0\rangle + |1\rangle}{\sqrt{2}}$ or $\frac{|0\rangle - |1\rangle}{\sqrt{2}}$. I don't want to destroy this state by measuring it, but I want to distinguish between the '+' and '-' sign. I have access to another qubit. What kind of quantum circuit can I built that preserves the state of the given qubit but measures the '+' or '-' sign?

4. A Stern–Gerlach apparatus along x-axis creates two spin channels. We say that one channel is labeled $|x\rangle$ and the orthogonal channel is $|-x\rangle$. Likewise if the apparatus is physically oriented along y-axis, the output channels are labeled $|z\rangle$ and $|-z\rangle$. We know that

$$|z\rangle = \frac{1}{\sqrt{2}} \left(|x\rangle + |-x\rangle \right)$$

(a) Express $|-z\rangle$ as a superposition of $|x\rangle$ and $|-x\rangle$, given that $\langle z|-z\rangle = 0$.

(b) A state is prepared as

$$|\psi\rangle = \frac{2}{\sqrt{13}} |x\rangle + i\frac{3}{\sqrt{13}} |-x\rangle .$$

(c) Show this state on the Bloch sphere.

(d) If this state enters an Stern-Gerlach apparatus along x-axis, what are the probabilities of the outcomes?

(e) If this state rather enters an Stern-Gerlach apparatus along z-axis, what are the probabilities of the outcomes?