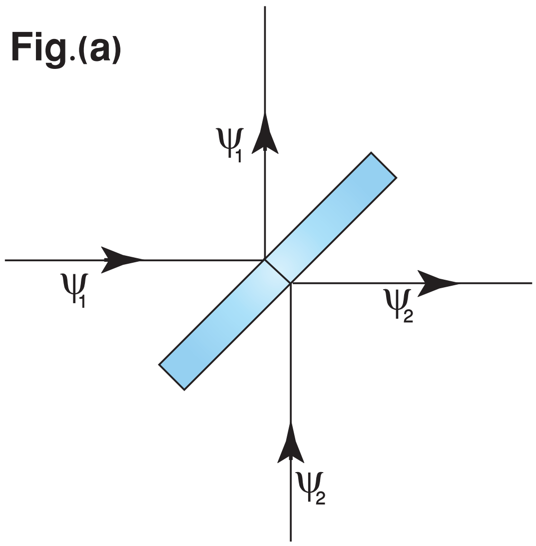
Tutorial 3: Modern Physics

1. Fig.(a) shows a 50:50 beamsplitter (BS) with two input ports and two output ports. The input rays have arrows coming into the BS and output rays have arrows going away.

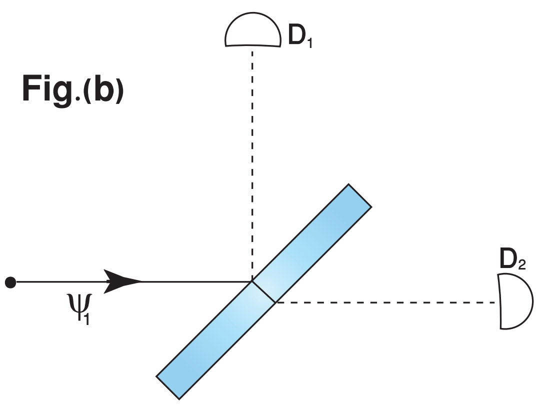


The input photon has two possible paths, labeled by ψ_1 and ψ_2 (as shown in the fig.(a)). The output photon also has two possible paths, also labeled as ψ_1 and ψ_2 as shown. The BS creates a superposition i.e., an input field goes into an output field according to the rule given below.

$$\begin{aligned} \psi_1 &\xrightarrow{\text{BS}} \frac{1}{\sqrt{2}}(\psi_1 + \psi_2), \quad \text{and} \\ \psi_2 &\xrightarrow{\text{BS}} \frac{1}{\sqrt{2}}(\psi_1 - \psi_2) \end{aligned} \tag{1}$$

Now answer these questions.

(i)

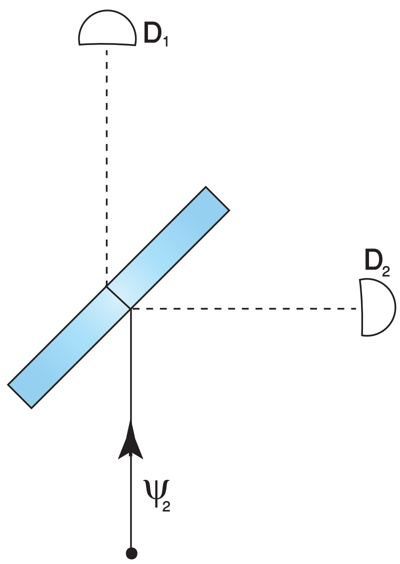


A single photon comes in as ψ_1 as shown in Fig.(b). Which of the detectors D_1 and D_2 clicks?

(ii) Many photons come in, all in the state ψ_1. How will the detectors click? Both at once, both but one after the other, simultaneously...?

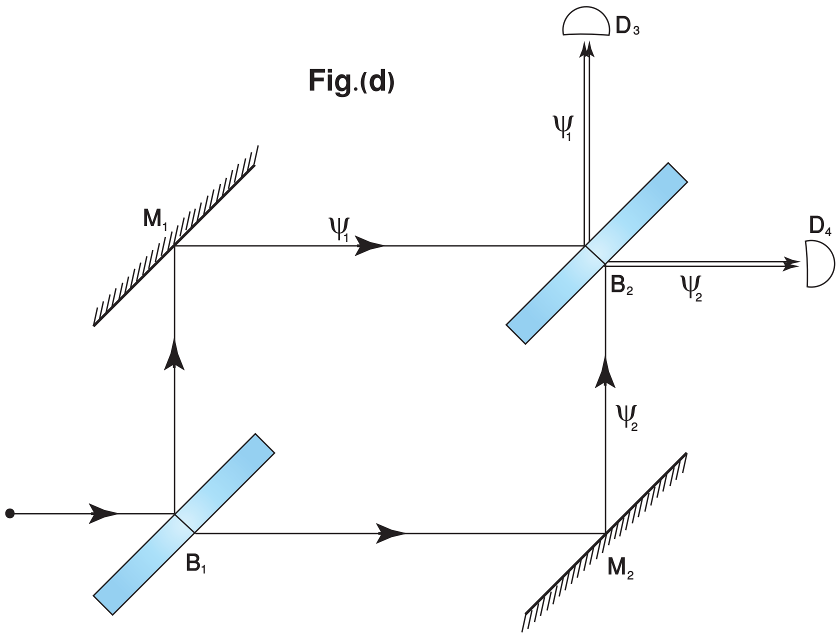
(iii) Answer questions (i) and (ii) when the photons come in as a field ψ_2 instead of ψ_1 as shown in Fig.(c).

Fig.(c)



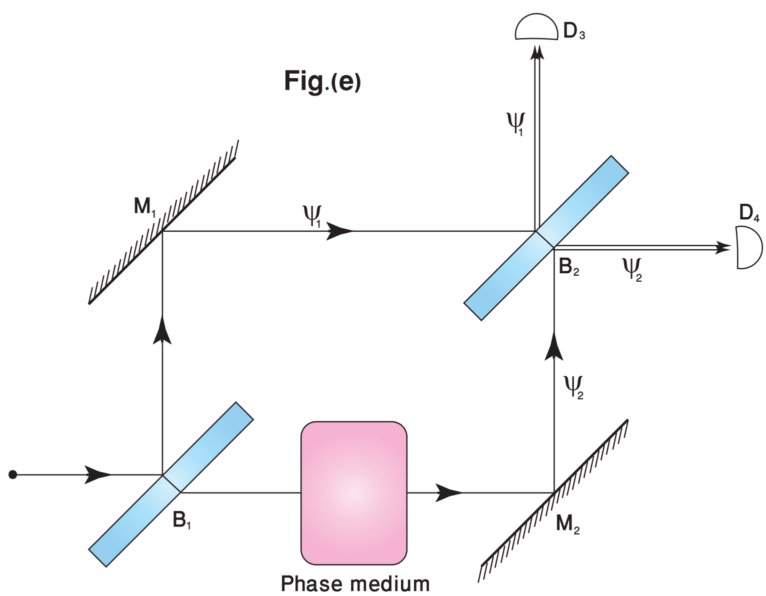
(iv) If a quantum object is in superposition field $(a\psi_1 + b\psi_2)$, the probability that upon collapse, ψ_1 is observed is $|a|^2 = a^*a$ and the probability that ψ_2 is observed is $|b|^2 = b^*b$. Clearly $|a|^2 + |b|^2 = 1$. (Remember that a^* represents the complex conjugate of a). Refer again to Fig.(b), but now suppose that our beamsplitter is not an ideal 50:50 device, rather a 70:30 device meaning that D_1 clicks 70% of the times and D_2 clicks 30% of the times. What is then the photon's superposition state after the beamsplitter?

(v)



Let's turn back to 50:50 beamsplitters. Consider Fig.(d). B_1 and B_2 are identical beamsplitters acting in the fashion presented in equation (1), while M_1 and M_2 are perfect mirrors. Show that D_3 will click 100% of the time. The labeling is as shown. How will you explain this strange outcome?

(vi) Now turn to Fig.(e).

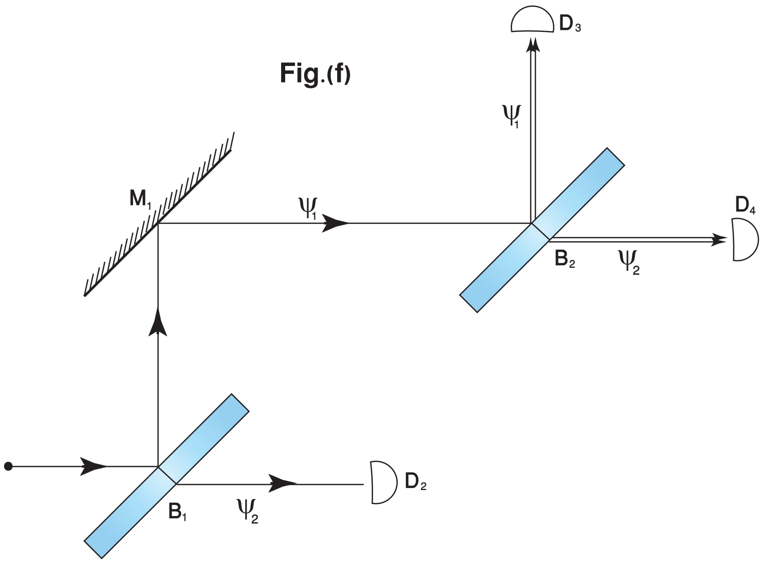


A nonlinear medium is placed in one of the "arms" of the apparatus. Our apparatus can be called an interferometer. The nonlinear medium has the property that it takes any input field ψ and adds a phase δ to it, i.e.,

$$\psi \xrightarrow{\text{phase medium}} \psi e^{i\delta}.$$

What is the probability that D_3 clicks and the probability that D_4 clicks? Can we adjust δ and make D_3 and D_4 click with equal probabilities?

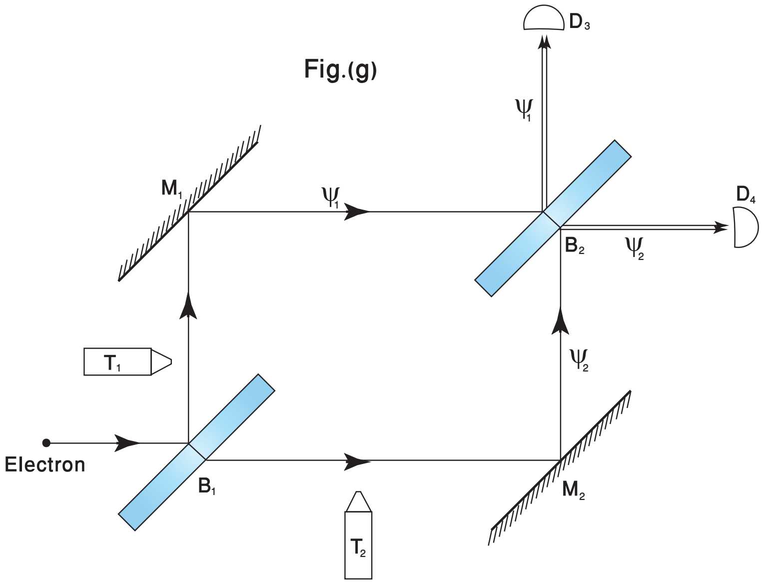
(vii) We now make yet another modification to the apparatus, and arrive at the assembly shown in Fig.(f).



What is the probability of D_2, D_3 and D_4 clicking? Can all of these detectors click together? Can any two click simultaneously? Can some detector never click?

(viii) Draw a sketch of a double slit interference experiment analogous to Fig.(f). Can you mention any analogies between experimenting with beamsplitters and experimenting with the double slit apparatus?

(ix) Finally refer to Fig.(g).



Instead of a photon, we now input an electron. In the two arms of the interferometer, we place two tightly focused telescopes T_1 and T_2 which can "illuminate" and show the presence of the electron. It is also assumed that the act of observing the electron through the telescopes is non-destructive, meaning that the electron is **not** destroyed upon observation by either T_1 or T_2. What are the relative probabilities of the detectors D_3 and D_4 clicking? Explain your reasoning in light of the uncertainty principle.

(x) Can you think **how** the arrangement in Fig.(d) needs to be modified if we would like to see a delayed choice of the photons "choosing" between interfering or not interfering. I expect you to provide a sketch as an answer to this question.

2. Much evidence indicates that the hydrogen atom is about 0.1 nm in radius. That is, the electron's orbit (regardless of whether it is circular) extends to about this far from the proton. Accordingly, the uncertainty in the electron's position is no longer than about 0.1 nm.

   (a) Find the uncertainty in momentum. Heisenberg's uncertainty relationship is $\Delta p \, \Delta r \geq \hbar/2$.

   (b) What is the minimum energy the electron must have if it is confined within a radius of 0.1 nm? Since we know nothing about the electron's velocity, be careful and use the relativistic expression for the energy. Reason how you choose the momentum of the electron.

(c) What is the theoretical minimum kinetic energy the electron will possess if it is confined this close to the nucleus?

(d) If an electron has this much K.E., can it remain bound with the nucleus? I am expecting a simple calculation to validate your answer.

(Note: An electron in an atom moves in three dimensions; the values in the example apply to the component of its motion along the radial direction.)