Precise measurement of velocity dependent friction in rotational motion



2011 Eur. J. Phys. 32 1367

(http://iopscience.iop.org/0143-0807/32/5/024)

IOP PUBLISHING EUROPEAN JOURNAL OF PHYSICS

Eur. J. Phys. **32** (2011) 1367–1375 doi:10.1088/0143-0807/32/5/024

# Precise measurement of velocity dependent friction in rotational motion

**Junaid Alam, Hafsa Hassan, Sohaib Shamim, Waqas Mahmood and Muhammad Sabieh Anwar**[1]

School of Science and Engineering, Lahore University of Management Sciences (LUMS), Opposite Sector U, D.H.A, Lahore 54792, Pakistan

E-mail: sabieh@lums.edu.pk

Received 2 March 2011, in final form 5 July 2011
Published 12 August 2011
Online at stacks.iop.org/EJP/32/1367

**Abstract**

Frictional losses are experimentally determined for a uniform circular disc exhibiting rotational motion. The clockwise and anticlockwise rotations of the disc, that result when a hanger tied to a thread is released from a certain height, give rise to vertical oscillations of the hanger as the thread winds and unwinds over a pulley attached to the disc. It is thus observed how the maximum height is achieved by the hanger decrements in every bounce. From the decrements, the rotational frictional losses are measured. The precision is enhanced by correlating vertical motion with the angular motion. This method leads to a substantial improvement in precision. Furthermore, the frictional torque is shown to be proportional to the angular speed. The experiment has been successfully employed in the undergraduate lab setting.

(Some figures in this article are in colour only in the electronic version)

## 1. Introduction

The moment of inertia is the resistance to change in rotational momentum. It is the rotational analogue of mass and is a central concept in rotational mechanics. Measurement of rotational inertia is a popular experiment at the undergraduate and especially the freshman level. Concepts such as conservation of energy, torque, angular acceleration, moment of inertia and dynamic friction along with the quantitative relationships between linear and rotary motion can be verified with the help of suitably designed experiments. Furthermore, a practical understanding can be developed about ubiquitous mechanical devices, such as flywheels, pulleys and related concepts. Some experiments regarding the mathematical basis of rotational inertia [1], friction [2] and speed-dependent drag [3] as well as interesting procedures for measuring the moment of inertia [4] have already been presented. Computer-aided experiments [5] are enjoying popularity and have become common.

[1] Author to whom any correspondence should be addressed.

This paper describes a simpler, easier and more precise method to determine the frictional losses that take place while experimentally measuring the moment of inertia. The loss in the number of rotations of a disc is, in a calculative way, translated into the loss in vertical height of a mass fastened to a thread that winds and unwinds over a pulley fixed on to the disc. This *rotational* measurement is used to determine energy losses, overcoming the possibility of errors in *translational* measurement with a metre rod.

Although for a *stationary* mass-hanger, position can be measured accurately up to the least count of the rule, considering errors due to parallax, delayed human response and judgemental variations in measuring effective height for a *moving* mass-hanger, the uncertainty is no better than $\pm 1$ cm. Our method effectively brings down the uncertainty in height measurements of the *moving* mass up to $\pm 2$ mm. This is achieved by using basic relationships between rotational and linear motion. Furthermore, the measurement of the number of rotations is made easy and considerably accurate with an improvisation involving a high-resolution photogate and a digital counter, providing an accuracy of $1/50$ of a rotation. Observations carried out with different applied torques and resulting angular speeds are compiled and they show a linear relationship between frictional torque and angular velocity.

## 2. Theoretical motivation

The moment of inertia of a continuous and uniform object is given by

$$I = \int_{\mathbf{r}} \rho(\mathbf{r})\mathbf{r}^2\, \mathrm{d}^3\mathbf{r}, \tag{1}$$

where $\rho(\mathbf{r})$ is the density at a point designated by $\mathbf{r}$, and the integral spans over the whole volume of the object. It is well known that the moment of inertia for a uniform circular disc is given by [6]

$$I = \tfrac{1}{2}MR^2, \tag{2}$$

where $M$ is the mass and $R$ is the radius. Furthermore, the rotational kinetic energy of a rigid body is given by [6]

$$K = \tfrac{1}{2}I\omega^2. \tag{3}$$

Here, $\omega = 2\pi/T$ is the angular speed in rad s$^{-1}$, and $T$ is the time period.

When a mass $m$, attached with a thread wound to the axle of the disc, is released from a height $h$, it provides a torque to the disc. Suppose that the angular speed of the disc is measured just *after* the mass hits the ground and is given by $\omega$. The energy transferred after the complete vertical descent of the mass is captured by the approximate equation

$$mgh \approx \tfrac{1}{2}I\omega^2, \tag{4}$$

but an immediate inquiry of the real-life scenario suggests that the above equation is incomplete as it ignores the energy lost in the process. The correct equation, instead, is

$$mgh = \tfrac{1}{2}I\omega^2 + \Delta E = \tfrac{1}{2}I\omega^2 + \Delta E_f + \Delta E_m, \tag{5}$$

where $\Delta E$ is the energy loss which can be further divided into two parts: the frictional losses $\Delta E_f$ and the untransferred energy, that is, the kinetic energy retained by the mass $\Delta E_m = \frac{1}{2}mv^2$, where $v$ is the velocity at the instant the mass hits the ground. Although frictional losses are mainly due to friction in the axle and bearing assembly of the apparatus, it is worth mentioning that air friction acting on the surface of the rotating disc as well as the moving weights may also result in losses that are actually a part of the frictional losses accounted in equation (5). Quantifying the sources of friction is not possible in the given

**Table 1.** PASCO's rotational apparatus specifications for various components [7].

| Component | Parameters |
|---|---|
| Main platter | $M = 991$ g; $R = 12.7$ cm |
| | Moment of inertia = 0.0075 kg m$^2$ |
| Step pulleys | Radii = 1.50 cm, 2.00 cm, 2.50 cm |
| Super pulley | Inside groove radius = 2.38 cm |
| | Outside groove radius = 2.54 cm |

scenario; therefore $\Delta E_f$ represents the lumped-up sum of all frictional losses. Calculating $\Delta E_m$ is straightforward once we have calculated $v$ using the equation

$$v = r_p \omega, \tag{6}$$

where $r_p$ is the radius of the step pulley over which the thread is wound and $\omega$ is measured just after the mass hits the ground.

## 3. Experiment

### 3.1. Experimental setup

The experimental setup (figure 1) comprises PASCO's introductory rotational apparatus (Model: ME-9341) and other components from their ME series. The assembly comprises two major parts: a base with levelling supports and a solid disc referred to as the main platter. The base has two holes at the edges to support any peripheral components like holders, and a bearing and spindle assembly at the centre to serve as the axle of the disc. The main platter is fitted with three concentric pulleys of different radii that are called the step pulleys. A thread with a mass attached to the distal end can be wrapped around any one of the pulleys enabling the application of a torque to the disc. A super pulley (9448B) (number 1), that supports the thread tied to the mass is clamped to the base. For timing measurements, another super pulley (number 2) is fixed to a rod and with the help of a holder; it is fitted in such a way that its rim gently brushes that of the main platter. A photogate (9498A), connected to the Smart-timer (8930), is placed in such a way that the path of its beam is interrupted by the spooks of the second super pulley. A photogate employs an infrared light beam passing from one arm to the other, detecting when an obstacle interrupts the beam. Masses of different sizes are available along with the hangers and are used to apply different driving torques to the main platter. The physical attributes of some important components are summarized in table 1. The manufacturer did not provide any uncertainties for the tabulated values.

The accuracy and precision of the measurements is partially determined by the instruments used. Time measurements using a photogate and Smart-timer have an uncertainty equal to that of the resolution of the timer, i.e. $\pm 0.1$ ms. As such a metre rod is used for height measurement. For a static mass, the uncertainty is simply the least count ($\pm 0.5$ mm). However, for a moving mass, parallax, human response and arbitrariness of the position of the centre of the gravity of the mass results in considerable imprecision, which could be of the order of $\pm 1$ cm. The uncertainty in mass is taken to be $\pm 1$ g, which is the resolution of our mass measuring device. When the smart timer is used as a counter, measurements are uncertain by $\pm 1$ count. These uncertainties will propagate [8] and determine the precision of our results, as we describe later.

A bubble level is used to level the base to avoid any wobbling of the main platter. One end of the thread is tied to the hanger loaded with weights while the other end is wrapped around one of the step pulleys and passed over the super pulley in such a way that it is horizontal and in line with the edge of the step pulley being used. The thread should be of a reasonable length, neither too short that it slips well before the mass-hanger reaches the floor, nor too long that it hinders the rotation of the main platter when the torque is removed. Students try out different kinds of threads and wires: inflexible, elastic, cotton, nylon, metallic, etc.

### 3.2. Angular deceleration

In order to investigate the frictional losses, the first step is to observe the angular deceleration of the freely rotating disc, providing insight to the nature of the relationship between the angular speed and frictional loss. If the energy loss that takes place in one rotation is denoted as $\Delta E_R$, the value of frictional torque, $\tau_f$, is determined as

$$\Delta E_R = \tau_f \theta = 2\pi \tau_f, \tag{7}$$

where $\theta$ is the angular displacement over which the torque is applied, which in the case of $\Delta E_R$ is $2\pi$. If frictional torque were linearly dependent on the angular speed, we could also write

$$\tau_f = -I\frac{\mathrm{d}\omega}{\mathrm{d}t} = b\omega, \tag{8}$$

where $b$ represents a constant of proportionality, the coefficient of rotational friction. Solving equation (8) for angular velocity, we obtain

$$\omega = \omega_o\,\mathrm{e}^{-(b/I)t}, \tag{9}$$

showing that if the decay of the angular speed is found to be exponential, frictional torque is proportional to the angular speed.

In the first experiment, the main platter is driven manually and then allowed to rotate freely. After every 10 s, the angular speed is measured and subsequently plotted against time, and curve-fit as shown in figure 2 which clearly highlights the linear dependence of frictional torque on the angular speed. The value of $b$ estimated from the fit is $6.6 \times 10^{-5}$ kg m$^2$ s$^{-1}$. It should however, be remembered that this holds only for as long as the disc rotates freely. Section 3.3 deals with more quantitative and rigorous verification of this linear relationship.

### 3.3. Measuring rotational friction

The next experiment is determining the frictional losses. For this purpose, the free end of the thread is tied to the screw on the small step pulley and the smart timer is used as a counter, set in the manual count mode. In this mode, every interruption of the photogate beam causes the device to increment its reading by one until it is stopped manually. The thread should be of such a length that at its minimum height position, the mass-hanger does not touch the floor.

After wrapping the thread around the step pulley, the counter is started to record counts and the mass-hanger is released from a certain initial height (figure 3). During the clockwise and anticlockwise rotation of the main platter, the mass-hanger will oscillate vertically, the span of its movement decreasing with every bounce (figure 4). Our objective is to measure the height that is lost in successive oscillations, what we call the $\Delta h_n$, in order to measure the energy loss. To do this, counts from the counter are read every time the mass-hanger reaches its maximum height (position 1, 2, 3 and so on). Counts for individual oscillations can be determined from the difference of two consecutive readings. We know that, for one complete revolution of the platter, 50 counts are recorded, so the fractional number of rotations of the

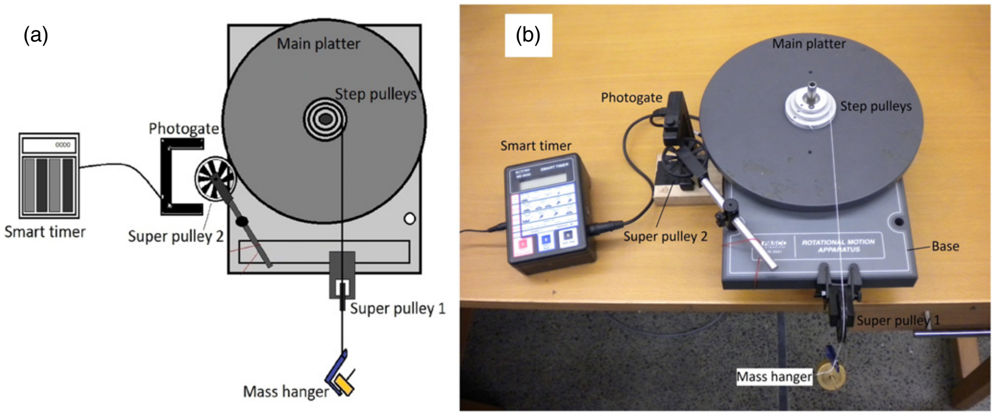


**Figure 1.** Moment of inertia apparatus: (a) schematic illustration and (b) photograph showing the various components.

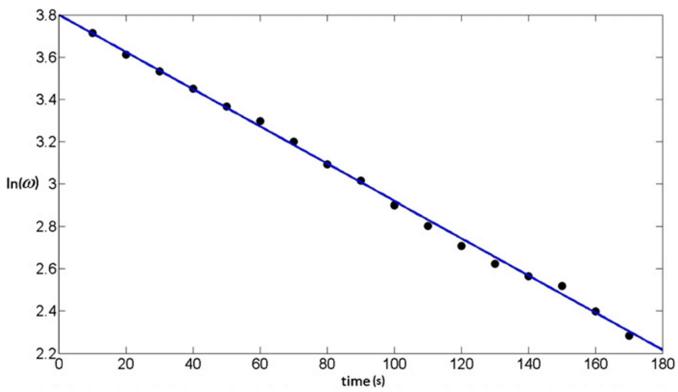


**Figure 2.** Decay of the angular speed of the freely revolving main platter shown as the linear fit of $\ln(\omega)$.

main platter, $N_n$, in a certain cycle (or bounce) of the mass-hanger can be calculated using the relationship

$$N_n = \frac{C_{n-1}}{50}, \tag{10}$$

where $C_n$ is the number of counts for the $n$th cycle. Using equation (10), the $\Delta h_n$ as indicated in figure 3, can be calculated using the following expression:

$$\Delta h_n = \Delta C_n \times \frac{2\pi r_p}{50}, \tag{11}$$

where $\Delta C_n = C_n - C_{n-1}$ represents the difference in counts of two consecutive bounces of the mass-hanger.

Once we have calculated the height loss in a certain bounce, we can determine the corresponding energy loss per bounce ($\Delta E_{B_n} = mg\Delta h_n$). The energy loss per rotation ($\Delta E_R$) can then be found:

$$\Delta E_{R_n} = \frac{\Delta E_{B_n}}{N_n}. \tag{12}$$

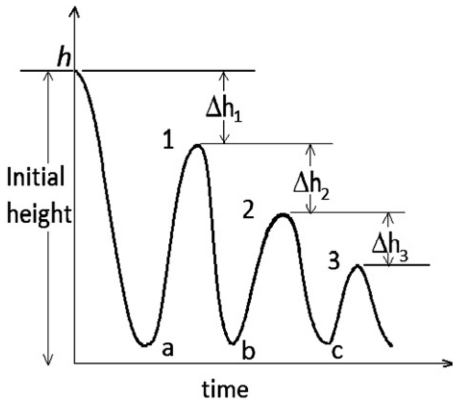


**Figure 3.** Frictional losses: scheme of observations for determining frictional losses. Counts are recorded at positions 1, 2, 3 and so on.

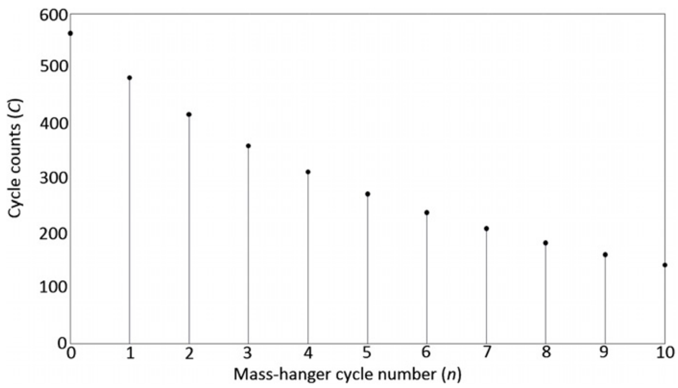


**Figure 4.** Decay in maximum height achieved by the mass-hanger: a plot of counts for every bounce.

### *3.4. Typical measurements and findings*

Following the procedure described in the previous subsection, observations are recorded for various weights, different pulleys and three types of thread. Table 2 summarizes a typical set of readings for a particular mass, pulley and thread. The only measured quantity is the number of counts, $C$, and the rest of the table can be constructed using equations (10)–(12). The deduced uncertainties in $\Delta h$, $\Delta E_B$ and $\Delta E_R$ are calculated to be $\pm 2$ mm, $\pm 0.003$ J and $\pm 0.0005$ J, respectively.

The values in the last column of table 2 clearly indicate that as the maximum velocity achieved by the mass-hanger or the disc decreases, energy loss per rotation also decreases for successive bounces resulting in gradually decreasing energy loss per rotation. This qualitative statement is now substantiated.

**Table 2.** Sample observations for frictional energy losses where the zeroth cycle is assumed to be the starting reference. Mass = 105 g, pulley: small, thread: thin polyester.

| Cycle '$n$' | $C$ $\pm 1$ | $\Delta C$ $\pm 2$ | $\Delta h$ $\pm 0.002$ (m) | $\Delta E_B$ $\pm 0.003$ (J) | $N$ $\pm 0.02$ | $\Delta E_R$ $\pm 0.0005$ (J) |
|---|---|---|---|---|---|---|
| 0 | 564 | | | | | |
| 1 | 483 | 81 | 0.153 | 0.157 | 11.28 | 0.0139 |
| 2 | 416 | 67 | 0.126 | 0.129 | 9.66 | 0.0133 |
| 3 | 359 | 57 | 0.107 | 0.110 | 8.32 | 0.0132 |
| 4 | 312 | 47 | 0.089 | 0.092 | 7.18 | 0.0128 |
| 5 | 271 | 41 | 0.077 | 0.079 | 6.24 | 0.0126 |
| 6 | 238 | 33 | 0.062 | 0.064 | 5.42 | 0.0118 |
| 7 | 209 | 29 | 0.055 | 0.056 | 4.76 | 0.0117 |
| 8 | 183 | 26 | 0.049 | 0.050 | 4.18 | 0.0119 |
| 9 | 162 | 21 | 0.039 | 0.040 | 3.66 | 0.0109 |

**Table 3.** Average angular speed ($\omega_{av}$) and corresponding frictional loss per rotation ($\Delta E_R$) for small and medium pulleys and three kinds of threads.

| Weight | *Thin thread* | | *Thick thread* | | *Nylon thread* | |
|---|---|---|---|---|---|---|
| (g) | $\omega_{av}$ (rad s$^{-1}$) | $\Delta E_R$ (J) | $\omega_{av}$ (rad s$^{-1}$) | $\Delta E_R$ (J) | $\omega_{av}$ (rad s$^{-1}$) | $\Delta E_R$ (J) |
| | | | Small pulley | | | |
| 55 | 4.11 | 0.0133 | 4.19 | 0.0117 | 4.27 | 0.0128 |
| 105 | 6.04 | 0.0144 | 6.10 | 0.0148 | 6.28 | 0.0162 |
| 155 | 7.54 | 0.0194 | 7.66 | 0.0187 | 7.66 | 0.0189 |
| 205 | 8.78 | 0.0219 | 8.73 | 0.0211 | 8.98 | 0.0213 |
| | | | Medium pulley | | | |
| 55 | 4.13 | 0.0139 | 4.16 | 0.0138 | 4.22 | 0.0134 |
| 105 | 6.10 | 0.0183 | 6.04 | 0.0182 | 6.16 | 0.0184 |
| 155 | 7.54 | 0.0218 | 7.66 | 0.0211 | 7.57 | 0.0219 |
| 205 | 8.83 | 0.0251 | 8.85 | 0.0242 | 8.73 | 0.0245 |

### *3.5. Dependence of frictional torque on angular speed*

In order to discover the quantitative relationship between the frictional torque and angular speed, we need to record the average speeds for the various cycles of the mass-hanger. This can be done by putting different weights in the hanger and observing the angular speed of the main platter when the mass-hanger reaches the first minimum (position 'a' in figure 3). The initial velocity for every cycle is zero, and because acceleration is constant for a particular weight, average angular velocity can be calculated in a straightforward manner:

$$\omega_{av} = \frac{0 + \omega_{\max}}{2} = \frac{\omega_{\max}}{2}, \tag{13}$$

where $\omega_{\max}$ is the angular velocity of the main platter when the mass-hanger is at the first minimum. The average energy loss per rotation measured for various weights and the resulting average angular velocities are listed in table 3.

The data recorded in table 3 are plotted in figure 5. It can be observed that the average energy loss per rotation increases in proportion to the average angular speed of the main

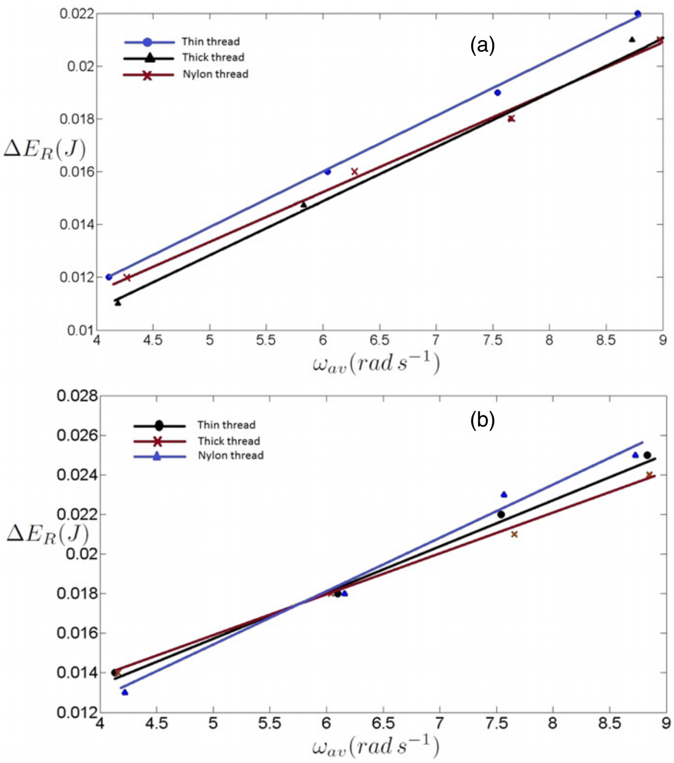


**Figure 5.** Average angular speed versus energy loss per rotation for (a) small pulley and (b) medium pulley.

platter. This clearly indicates a direct relationship between speed and frictional torque. Slight differences for different pulleys are due to unquantifiable errors that take place due to the difference of angle at which the thread meets the super pulley.

### *3.6. Calculating the moment of inertia*

As frictional losses can be calculated for a range of angular speeds, we are now in a position to measure the moment of inertia of the main platter. The mass-hanger is loaded with a weight of 105 g, the thread is wound on the small pulley and released from a height of 62 cm, and in this particular experiment, it is allowed to fall to the ground. As it hits the ground, the smart timer is used to measure the angular speed, $\omega$, of the main platter.

In one experiment, the average smart-timer reading was 10.4 ms. This corresponds to a time period of 0.52 s and an angular speed of 12.08 rad $s^{-1}$. Using equation (13), the average angular speed turns out to be 6.04 rad $s^{-1}$. From figure 5, we can see that this average speed corresponds to an energy loss of $1.44 \times 10^{-2}$ J. Using equation (6), the kinetic energy retained by the mass hanger, $\Delta E_m$, was calculated to be $1.72 \times 10^{-3}$ J. This results in a total frictional loss of $9.647 \times 10^{-2}$ J when the number of rotations of the main platter ($N = h/(2\pi r_p)$) was 6.58. Using equation (5), the moment of inertia was found to be 0.0074 kg $m^2$. Using the precision of the instruments and the formulations for error propagation [8], the uncertainty in the value determined for moment of inertia is calculated to be $\pm 0.0002$ kg $m^2$.

## 4. Conclusions

A simple, yet insightful, method of measuring the frictional losses is presented in order to find a precise value for the moment of inertia of a circular disc. The experiment goes hand in hand with the basic concepts of energy conservation, rotary motion and real life equations. Basic relationships between translational and rotational motion are employed to improve the precision of the measurements. The dependence of the frictional torque on the speed of rotation is also observed deductively. The beauty of the experiment is that the same apparatus is used to accomplish a range of tasks. Moreover, students also get to observe how improvements in the apparatus can help make simpler but accurate and precise experiments.

## References


[1] Pintao C A F, de Souza Filho M P, Grandini C R and Hessel R 2004 Experimental study of the conventional equation to determine a plate's moment of inertia *Eur. J. Phys.* **25** 409–17

[2] Shefinski J 2001 A rotational dynamics problem with friction and calculus *Phys. Teach.* **39** 150–1

[3] Xu Y, Yung K L and Ko S M 2007 A classroom experiment to measure the speed-dependent coefficient of rolling friction *Am. J. Phys.* **75** 6

[4] Griffiths I W, Watkins J and Sharpe D 2005 Measuring the moment of inertia of the human body by a rotating platform method *Am. J. Phys.* **73** 1

[5] Amrani D 2006 Computerized rotational system to study the moment of inertia of different objects *Eur. J. Phys.* **27** 1063–9

[6] Resnick R, Halliday D and Krane K S 1992 *Physics* vol 1, 4th edn (Toronto: Wiley)

[7] PASCO Scientific 1987 Introductory Rotational Apparatus: Instruction Manual and Experiment Guide for the PASCO Scientific Model ME-9341, ftp://ftp.pasco.com/Support/Documents/

[8] Kirkup L and Frenkel B 2006 *An Introduction to Uncertainty in Measurement* (Cambridge: Cambridge University Press)