For $n = 0$, (9.1) simplifies since the only possible event is "no particles in t, no particles in Δt," and (9.4) becomes, for $n = 0$,

$$\frac{dP_0}{dt} = -\mu P_0. \tag{9.5}$$

Then, since $P_0(0) =$ "probability that no particle is emitted during a zero time interval" $= 1$, integration of (9.5) gives

$$P_0 = e^{-\mu t}. \tag{9.6}$$

Substituting (9.6) into (9.4) with $n = 1$ gives a differential equation for $P_1(t)$; its solution (Problem 1) is $P_1(t) = \mu t e^{-\mu t}$. Solving (9.4) successively (Problem 1) for P_2, $P_3, \cdots, P_n$, we obtain

$$P_n(t) = \frac{(\mu t)^n}{n!} e^{-\mu t}. \tag{9.7}$$

Putting $t = 1$, we get for the probability of exactly n counts per unit time

$$P_n = \frac{\mu^n}{n!} e^{-\mu}. \tag{9.8}$$

The meaning of μ is important for applications of this formula; a proof is outlined in Problem 2 that μ is just $\bar{n}$, the average number of counts per unit time. The probability function (9.8) is called the *Poisson distribution.*

The Poisson distribution is useful in a great variety of problems in which the probability of some occurrence is small and constant (see Problems 3 to 9, and Parratt, Chapter 5).

Example 1. The number of particles emitted each minute by a radioactive source is recorded for a period of 10 hours; a total of 1800 counts are registered. During how many 1-minute intervals should we expect to observe no particles; exactly one; etc.?

The average number of counts per minute is $1800/(10 \cdot 60) = 3$ counts per minute; this is the value of μ. Then by (9.8), the probability of n counts per minute is

$$P_n = \frac{3^n}{n!} e^{-3}.$$

A graph of this probability function is shown in Figure 9.1. For $n = 0$, we find $P_0 = e^{-3} = 0.05$; then we should expect to observe no particles in about 5% of the 600 1-minute intervals, that is, during 30 1-minute intervals. Similarly, we could compute the expected number of 1-minute intervals during which $1, 2, \cdots$, particles would be observed.

In Section 8, we discussed the fact that the binomial distribution can be approximated by the normal distribution for large n and large np. If p is very small so that np is very much less than n (say, for example, $p = 10^{-3}$, $n = 2000$, $np = 2$), the normal