to drop the restriction to adiabatic processes demanded by SKS. We thus have

$$\mathrm{SKS}\wedge \mathrm{Ax\ I}\wedge \mathrm{Ax\ II}\rightarrow \mathrm{CI}. \tag{39}$$

Beginning with (37), which is valid for arbitrary processes connecting equilibrium states, we obtain the SKS as the special case for adiabatic processes shown in Sec. V:

$$\mathrm{CI}\rightarrow \mathrm{SKS}. \tag{40}$$

Because the equality in the CI is exactly valid for reversible processes, by comparing (37) with (35), we get

$$\mathrm{CI}\rightarrow \mathrm{Ax\ I}. \tag{41}$$

Because, in addition to this, the integrals in (33) are nonnegative according to (28) and (37), we conclude that

$$\mathrm{CI}\rightarrow \mathrm{Ax\ II}. \tag{42}$$

The union of statements (40)–(42) results in the implication

$$\mathrm{CI}\rightarrow \mathrm{SKS}\wedge \mathrm{Ax\ I}\wedge \mathrm{Ax\ II}, \tag{43}$$

so that by use of (39) we get

$$\mathrm{SKS}\wedge \mathrm{Ax\ I}\wedge \mathrm{Ax\ II}\leftrightarrow \mathrm{CI}. \tag{44}$$

This statement shows that the SKS is fundamental for the derivation of the CI, although it is only formulated for adiabatic processes. The supplementary axioms relate to properties of reversible processes and of compound systems, and therefore they affect the second law only indirectly. The SKS and the two axioms are sufficient and necessary for the validity of the CI.

ACKNOWLEDGMENTS

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Do darker objects really cool faster?

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A common laboratory experiment in radiative heat transfer involves measuring the cooling rate of two water-filled metal cans, one can left unpainted and the other painted black. The shiny unpainted can cools more slowly, and the student can be led to the incorrect conclusion that bodies that are poor emitters (good reflectors) of light are also poor emitters of heat. A variation of this experiment was done with the shiny unpainted can replaced by one painted white. The experiment still compares a good and poor reflector of light, but in this case there is essentially no difference in cooling rates. These results are explained and are discussed in terms of other phenomena involving radiative heat transfer.

I. INTRODUCTION

The statement "a good absorber is a good emitter" is often heard or read, but its careless application often leads to incorrect conclusions. The more cautious statement would be "a substance that is a good absorber of certain electromagnetic wavelengths is a good emitter of those same wavelengths." A problem can arise from assuming that a body's reflecting properties at visible wavelengths will be the same at other wavelengths.

For example, a body on the Earth's surface that is exposed to solar radiation will absorb according to its reflecting properties at wavelengths near 0.5 μm. The body may absorb solar radiation and attain a temperature somewhere in the 300- to 400-K range where it will radiate according to its reflecting (or emitting) properties in the infrared at wavelengths near 10 μm. It is not true that a good reflector at visible wavelengths will necessarily be a good reflector (and thus a poor absorber and poor emitter) at infrared wavelengths; in fact, most nonmetals are good infrared emitters regardless of their visible reflectivity.

In other words, it does not follow that a warm (300- to 400-K) black object will cool any more quickly than a comparable warm white object. Their infrared emissivities may be nearly the same. These facts are well known to persons who work in the field of radiative heat transfer or in such areas as the development of surfaces for solar collectors, but they do not seem to be part of the general knowledge of many scientists.

Even some popular physics texts are wrong, or at least misleading, in this area. For example, Hewitt[1] in discussing absorption and radiation states: "This can be illustrated by placing thermometers in a pair of metal containers of the same size and shape, one having a brightly polished surface and the other with a blackened surface. If they are filled with hot water, we will find that the container with the blackened surface cools faster. The blackened surface is a better radiator." While that statement is true, it is correct because the "bright" container is metal and not simply because it is a better reflector of visible radiation. As will be shown, if the shiny container were painted white, it would cool at, or nearly at, the same rate as the blackened one.

Similarly, the following statement is found in White[2]: "If the outside surface of a hot coffee cup were painted a dull black, the rate of cooling would be more rapid than if it were chromium plated." Again the statement, though true, can be misleading. Replace the words "chromium plated" with "painted white," and it would be wrong.

I found the most misleading statement on this subject in Miller.[3] While discussing thermal radiation he states: "According to these ideas, a living room 'radiator' should be black for efficient radiation of heat. A light-colored paint job might be decorative, but it is hardly in accord with the laws of physics." While coating a metal radiator with (nonmetallic) paint will increase its efficiency of thermal radiation, the actual color used will be of little consequence since normal paints of all colors have very similar radiative properties[4] in the infrared at wavelengths near 10 μm.

Also misleading is a statement by Epstein[5] where, in discussing the advantages of a white painted house, he says "White radiates least and so keeps your house warm at night."

A commercial laboratory apparatus is available to illustrate the experiment described above in the quotation from Hewitt. Central Scientific Company[6] sells a "Two-Can Radiation Apparatus" to demonstrate the radiative cooling differences between a "dark" and a "light" surface. This simple apparatus consists of two metal cans, similar except that one is left with a shiny metallic surface and the other is painted flat black. This apparatus, with some modification, was used in the radiative cooling experiments described below.

II. EXPERIMENTAL WORK AND RESULTS

Two sets of the apparatus mentioned above were purchased, and one of the unpainted cans was coated with flat white spray paint.[7] Similar cans with three different surfaces (black, white, and unpainted) were then available for use in radiative cooling experiments. The experimental arrangement that was used is outlined in Fig. 1. A pair of cans was filled with water, heated to near the boiling point, placed side by side as shown, and then left to cool for about 3 h, while temperature in the can was carefully measured every 3 min.

To reduce conduction and convection losses, the cans were placed on Styrofoam blocks, and the entire apparatus was covered with a large metal container. The 500-cm^3 cans were filled with about 480 g of distilled water and were adjusted to have the same total masses to within 0.1 g. Heat capacities of the two cans used in a given run were then equal to within 0.2%. Before starting a particular run, the two cans were placed on an electric hot plate and heated while making adjustments to keep their temperatures within a degree of one another. When the temperatures reached

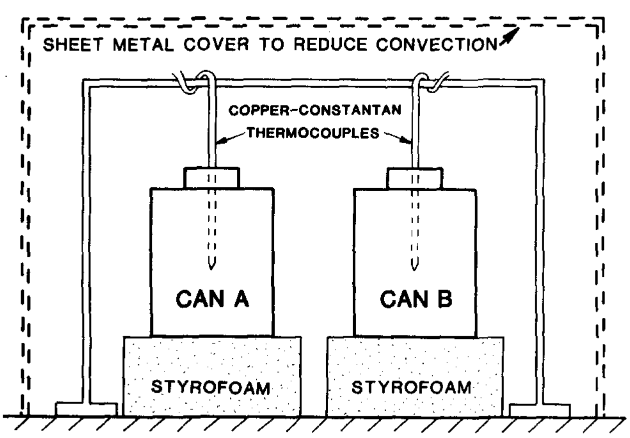

Fig. 1. The apparatus used to measure the relative cooling rates of two cans initially filled with hot water. In a given run, for example, can A may be black and can B may be white.

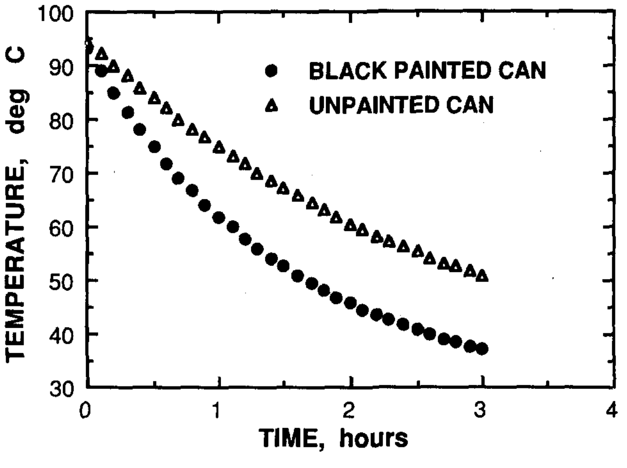

Fig. 2. Results of the experiment comparing the cooling rates of the can painted black with the one having the shiny unpainted surface. The unpainted can cools much more slowly, illustrating the fact that its surface is a poorer radiator of heat.

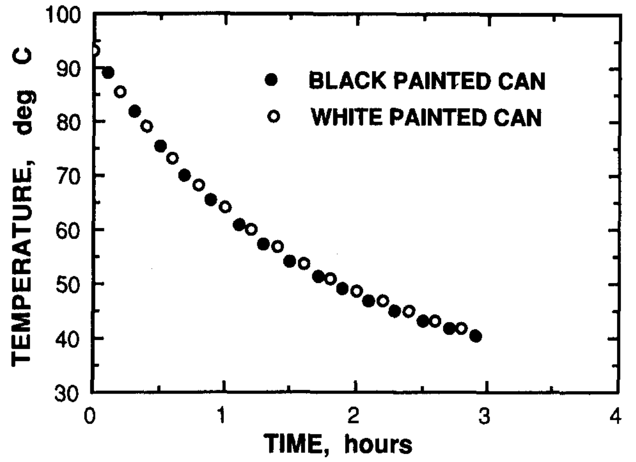

Fig. 3. Results of the experiment comparing the cooling rates of the can painted black with the one painted white. Here, the cooling rates are nearly identical, illustrating the fact that the white and black surfaces are essentially equivalent radiators of heat.

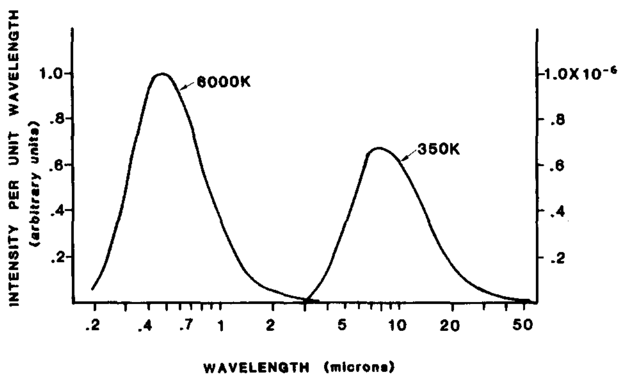

Fig. 4. Radiation curves for blackbodies at temperatures of 6000 and 350 K. The 6000-K curve peaks in the visible part of the electromagnetic spectrum at a wavelength of about 0.5 μm. The 350-K curve peaks in the infrared at about 8 μm. Note that practically all of the energy radiated by the 6000-K body is at wavelengths less than 3 μm, while for the 350-K body nearly all the energy is at wavelengths greater than 3 μm. The vertical scale, the intensity per unit wavelength range, is given in the same units for both bodies, the left-hand scale for the 6000-K body and the right-hand for the 350-K body.

about 97 °C, the cans were placed on the Styrofoam blocks, and measurements of temperature versus time were begun. Temperatures were measured to 0.2 °C using copper-Constantan thermocouples controlled and sampled by a Hewlett–Packard model HP85 microcomputer and model 3497A data-acquisition control unit. All runs comparing two particular radiating surfaces were done twice with the positions of the cans interchanged: No asymmetry was observed.

The experiment for which the commercial apparatus is designed is the cooling rate of the black versus the unpainted can, and the results of that experiment are shown in Fig. 2. We see that indeed the black can is a better radiator and cools considerably faster than the bare metal can. If the difference is due to the much "lighter color" (higher visible reflectivity) of the unpainted can, then a similar difference in cooling rate should be seen between the white and the black surfaced cans. However, Fig. 3 shows that there is very little difference in these cooling rates; that is, the white and black cans radiate heat at essentially the same rate.

III. DISCUSSION

Figures 4 and 5 are presented to explain the experimental results described above. Figure 4 compares the blackbody radiation curves for a 6000-K object and a 350-K object. A 6000-K body, like the Sun, has its peak radiation in the visible at about 0.5 μm, whereas a body at 350 K, like a steam radiator, has its peak radiation in the infrared at about 8 μm. The main thing to notice is that there is almost no overlap in the curves; that is, we are dealing with an entirely different set of wavelengths when comparing visible properties with those relevant to what we usually call "heat" radiation. There is no reason to believe a poor visible emitter (good visible reflector) must be a poor infrared emitter.

Figure 5 shows idealized curves for the wavelength dependence of the reflectivity of white paint, black paint, and bare metal. Real curves for these materials[8] are considerably more complicated than Fig. 5, but they do show the same overall behavior. The metal is a good reflector (poor emitter) at all wavelengths, black paint is a poor reflector (good emitter) at all wavelengths, and white paint, though a good reflector in the visible, is a poor reflector in the infrared. In fact, compilations of reflectivity data[4] show that ordinary paints of all colors are very "black" in the 8- to 10-μm region of the infrared. For example, Kreith and Bohn[9] give the following emissivity values: for white zinc oxide paint, 0.18 at 0.6 μm and 0.95 at 9.3 μm; for lampblack paint, 0.97 at 0.6 μm and 0.96 at 9.3 μm.

Figure 5 explains the results of the relative cooling experiments. The infrared radiation properties of the unpainted and black painted surfaces are very different, and

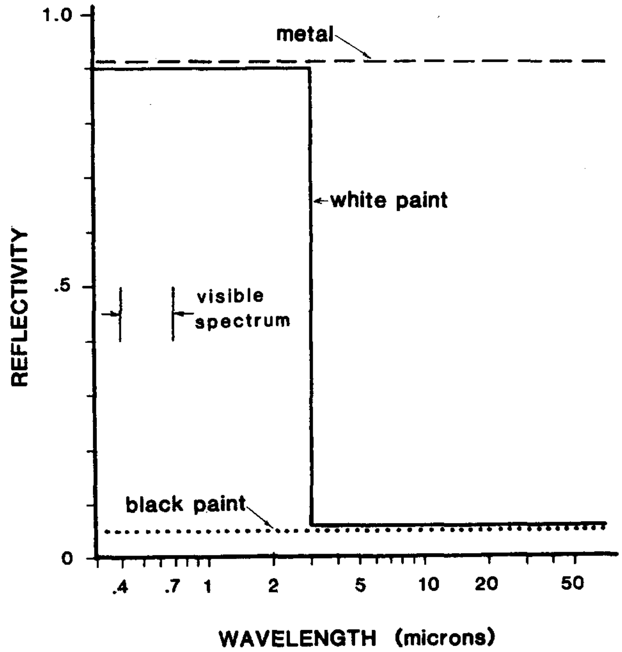

Fig. 5. A simplified representation of the wavelength dependence of the reflectivity of a metal, black paint, and white paint. The metal is a good reflector (poor emitter) at all wavelengths, and the black paint is a poor reflector (good emitter) at all wavelengths. The white paint, however, shows a wavelength dependence; it is a good reflector for visible wavelengths, but a poor reflector at infrared wavelengths greater than 3 μm.

the black can, being a much better infrared emitter, cooled off faster than the unpainted can. The black and white surfaces, however, had very similar properties in the relevant region of the infrared, and there was very little difference in their cooling rates.

Because the nonmetallic paint used here is a poor thermal conductor, one might expect, if only conduction were considered, that painting a can would reduce the rate of heat transfer from the can's interior. The fact that painting increases the rate of heat transfer means that the increase in surface emissivity more than compensates for the reduced conduction due to the paint layer. The mass of the cans was determined before and after painting, and the results were used to estimate the thickness of the paint layer to be about 0.03 mm. By making other reasonable assumptions appropriate to the experimental conditions employed, it can be shown[10] that the temperature drop across the paint should have been less than 0.1 K. The rate of heat transfer via radiation was not limited by the rate of conduction through the paint layer.

IV. A RELATED TOPIC: THE BACKWARD-RUNNING CROOKE'S RADIOMETER

The Crooke's radiometer has four vanes, black on one side and white on the other, mounted so they can rotate inside a partially evacuated transparent bulb. When exposed to sunlight, the black vanes reach a higher temperature than the white vanes, and this temperature difference causes the vanes to rotate in what we will call the "forward" direction; that is, with the black vanes moving away from and the white vanes moving toward the Sun.

In a note in this Journal, Crawford[11] pointed out that a room-temperature Crooke's radiometer will, for a while, run backward when placed in a refrigerator. A letter by Leff[12] confirmed Crawford's observation and also pointed out an excellent radiometer article by Woodruff[13] that also mentions the backward-running radiometer. The reason for the backward rotation is presumably because the black vane cools off faster in the refrigerator since it is a better infrared emitter.

According to the ideas discussed above, this behavior should not be obvious; that is, compared to the white vane, the black vane should not be the better infrared emitter just because it is the better visible absorber.

I purchased four of Crooke's radiometers and found that indeed they all initially exhibited backward rotation after being placed in a refrigerator. I opened one of the radiometers in order to inspect the construction of the vanes. To help maintain a temperature difference between the black and white surfaces, the vane material should be a thermal insulator such as paper or mica. A reasonable vane construction might then be a sheet of thermal insulator coated on one side with white paint and on the other with black paint.

However, that is not exactly what I found. The vanes apparently consisted of white paper sandwiched between transparent plastic sheets with one outside surface of a plastic sheet coated with a rather thick layer of loosely bound powder resembling lampblack. See Fig. 6. Though I am not sure I completely understand why the black side cools off faster, my conjecture would be that the phenomenon has more to do with surface texture than with surface color (though admittedly the two are not completely independent). The black powder has a considerably greater radiating (and absorbing) surface area than the other smooth side of the vane. The thermal insulating properties of the powder layer may also be advantageous to the (forward and backward) operation of the radiometer.

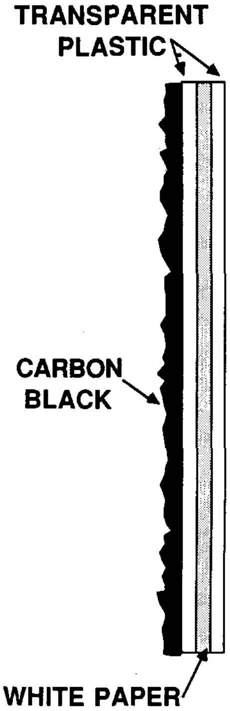

Fig. 6. The apparent structure of a vane on a Crooke's radiometer. What appears to be white paper is laminated between two transparent sheets, and then carbon black powder is loosely attached to one side of the vane. Not to scale.

If a Crooke's radiometer were constructed with something like paper or mica vanes coated with ordinary black and white paint layers of equal roughness and equal heat capacity, I would predict that, though it would run forward in strong sunlight, it would not run backward in a refrigerator. That is, compared to the white side, the black side would absorb more visible and near-infrared radiation from the sunlight, and thus reach a higher temperature, the condition necessary for the radiometer to run forward. However, if such a radiometer, initially at room temperature, were to be placed in a refrigerator, there would be practically equal radiation from the black and white sides of the vanes. No temperature difference would be established, and no rotation would occur.

A very interesting variation of the refrigerator experiment was described by Carter.[14] A radiometer with aluminum vanes, "shiny on one side, dull black on the other," was found to rotate "backward" when exposed to microwaves (11.5-cm wavelength). In this case, the aluminum vanes absorbed microwave energy and reached temperatures above that of their surroundings, thus duplicating the conditions of the refrigerator experiment. Interestingly, a similar radiometer with mica vanes rotated "forward" when exposed to the microwaves.

V. CONCLUSIONS

A body that is a good reflector of visible radiation will not necessarily be a good reflector (and thus a poor absorber and emitter) of infrared radiation. In fact, if the body is an electrical insulator and is "white" in the visible, it is probably "black" in the 8- to 10-μm region of the infrared. One should be careful in using the statement, "a good absorber is a good emitter," or misconceptions can occur. Even the results of a common laboratory excercise can be misleading if not properly interpreted. I would recommend that anyone who uses the cooling-rate experiment with the black and shiny cans also include a white painted can in the experiment.

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The fighter pilot's egg

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Of common interest to fighter pilots is what maneuver should be flown to accomplish a 360° heading change in the least time. Among the infinite possibilities, analytic expressions for a very commonly used maneuver, a roughly circular planar turn that accomplishes the required heading change, are derived and examined under four realistic assumptions. Contrary to common belief, the time to complete the stated heading change turns out to be independent of the inclination of the plane of the turn.

I. INTRODUCTION

Air-to-air weaponry and tactics aside, aerial "dogfights" between modern jet fighters often boil down to which aircraft can turn tighter and quicker. An aircraft turning tighter, that is, with a smaller instantaneous radius of turn, and quicker, that is, changing the direction of its velocity vector faster, will normally be able to gain a position of advantage behind its adversary. (Because the air mass through which an aircraft flies may itself be moving relative to the Earth, I assume such motion to be approximately uniform, and all vectors in this article are with respect to an inertial reference frame embedded in the air mass above a flat, nonrotating Earth.)

It is well known that the main factors that explicitly influence such optimum turns are velocity relative to the air, and lift. This is so because the definition of lift $\mathbf{L}$ is that component of force perpendicular to the velocity vector $\mathbf{v}$ resulting from the aircraft's motion through the air. (Thrust may also have a component perpendicular to $\mathbf{v}$, but for modern fighters this effect is small when compared to maximum lift. For the analysis to follow, thrust is assumed to be parallel to $\mathbf{v}$.) Clearly, to change the direction of $\mathbf{v}$ most rapidly, one should maximize L and minimize v. The same strategy results when one considers minimizing turn radius.

Since L depends quadratically on v, but is limited to a given maximum by the structural integrity of the aircraft, the basic idea is to fly at the slowest air speed at which safe maximum lift can be generated. For a given set of atmospheric conditions such as air density, compressibility, and viscosity, every aircraft has such a unique air speed known as its corner velocity. What is not so easily understood is how a pilot should specifically employ that maximum lift at corner velocity in turning 360°.

In all generality, the problem is extremely complex due to the four independent forces to be dealt with: lift, thrust, drag, and weight, with the first three dependent on velocity, air density, etc., and the last dependent on the changing mass of the fuel-burning aircraft. Because of this complexity, flight-path optimization almost always ends up as a numerical problem for computers.[1] I was surprised to discover that the problem becomes *completely* solvable when four realistic restrictions are imposed. I was even more surprised not to discover the solution in published literature containing analytic flight-path discussions.[2] The beginning