(*b*) From Eq. 11

$$R = Nm = (9600)(5) = 4.80 \times 10^4.$$

Thus, near $\lambda = 546$ nm and in fifth order, a wavelength difference given by (see Eq. 10)

$$\Delta\lambda = \frac{\lambda}{R} = \frac{546 \text{ nm}}{4.80 \times 10^4} = 0.011 \text{ nm}$$

can be resolved.

Sample Problem 4 A diffraction grating has 1.20×10^4 rulings uniformly spaced over a width $W = 2.50$ cm. It is illuminated at normal incidence by yellow light from a sodium vapor lamp. This light contains two closely spaced lines of wavelengths 589.00 and 589.59 nm. (*a*) At what angle does the first-order maximum occur for the first of these wavelengths? (*b*) What is the angular separation between these two lines (in first order)? (*c*) How close in wavelength can two lines be (in first order) and still be resolved by this grating? (*d*) How many rulings can a grating have and just resolve the sodium doublet lines?

Solution (*a*) The grating spacing d is given by

$$d = \frac{W}{N} = \frac{2.50 \times 10^{-2} \text{ m}}{1.20 \times 10^4} = 2083 \text{ nm}.$$

The first-order maximum corresponds to $m = 1$ in Eq. 1. We thus have

$$\theta = \sin^{-1}\left(\frac{m\lambda}{d}\right) = \sin^{-1}\left(\frac{(1)(589.00 \text{ nm})}{2083 \text{ nm}}\right) = 16.4°.$$

(*b*) Here the *dispersion* of the grating comes into play. From Eq. 9, the dispersion is

$$D = \frac{m}{d \cos\theta} = \frac{1}{(2083 \text{ nm})(\cos 16.4°)} = 5.01 \times 10^{-4} \text{ rad/nm}.$$

From Eq. 7, the defining equation for dispersion, we have

$$\begin{aligned}\Delta\theta &= D\,\Delta\lambda \\ &= (5.01 \times 10^{-4} \text{ rad/nm})(589.59 \text{ nm} - 589.00 \text{ nm}) \\ &= 2.95 \times 10^{-4} \text{ rad} = 0.0169° = 1.02 \text{ arc min}.\end{aligned}$$

As long as the grating spacing d remains fixed, this result holds no matter how many lines there are in the grating.

(*c*) Here the *resolving power* of the grating comes into play. From Eq. 11, the resolving power is

$$R = Nm = (1.20 \times 10^4)(1) = 1.20 \times 10^4.$$

From Eq. 10, the defining equation for resolving power, we have

$$\Delta\lambda = \frac{\lambda}{R} = \frac{589 \text{ nm}}{1.20 \times 10^4} = 0.049 \text{ nm}.$$

This grating can easily resolve the two sodium lines, which have a wavelength separation of 0.59 nm. Note that this result depends only on the number of grating rulings and is independent of d, the spacing between adjacent rulings.

(*d*) From Eq. 10, the defining equation for R, the grating must have a resolving power of

$$R = \frac{\lambda}{\Delta\lambda} = \frac{589 \text{ nm}}{0.59 \text{ nm}} = 998.$$

From Eq. 11, the number of rulings needed to achieve this resolving power (in first order) is

$$N = \frac{R}{m} = \frac{998}{1} = 998 \text{ rulings}.$$

Since the grating has about 12 times as many rulings as this, it can easily resolve the sodium doublet lines, as we have already shown in part (*c*).

47-4 X-RAY DIFFRACTION

X rays are electromagnetic radiation with wavelengths of the order of 0.1 nm (compared with 500 nm for a typical wavelength of visible light). Figure 12 shows how x rays are produced when electrons from a heated filament F are accelerated by a potential difference V and strike a metal target.

For such small wavelengths a standard optical diffraction grating, as normally employed, cannot be used. For $\lambda = 0.10$ nm and $d = 3000$ nm, for example, Eq. 1 shows that the first-order maximum occurs at

$$\theta = \sin^{-1}\left(\frac{m\lambda}{d}\right) = \sin^{-1}\left(\frac{(1)(0.10 \text{ nm})}{3 \times 10^3 \text{ nm}}\right) = 0.0019°.$$

This is too close to the central maximum to be practical. A grating with $d \approx \lambda$ is desirable, but, because x-ray wavelengths are about equal to atomic diameters, such gratings cannot be constructed mechanically.

In 1912 it occurred to physicist Max von Laue that a crystalline solid, consisting as it does of a regular array of atoms, might form a natural three-dimensional "diffraction grating" for x rays. Figure 13 shows that if a collimated beam of x rays, continuously distributed in wavelength, is allowed to fall on a crystal, such as sodium chloride, intense beams (corresponding to constructive interference from the many diffracting centers of which

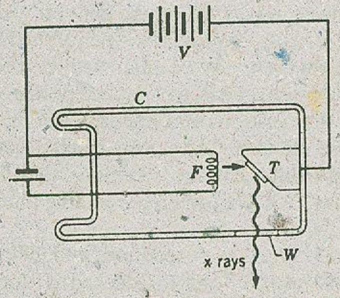

Figure 12 X rays are generated when electrons from heated filament F, accelerated through a potential difference V, strike a metal target T in the evacuated chamber C. Window W is transparent to x rays.

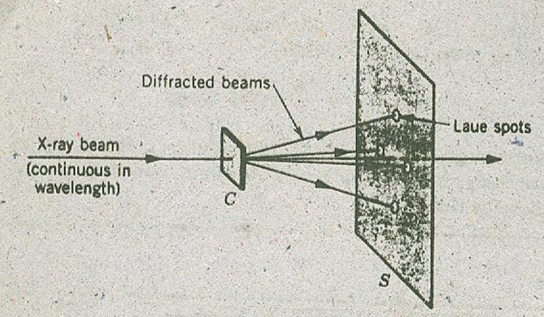

Figure 13 A beam of x rays strikes a crystal C. Strong diffracted beams appear in certain directions, forming a Laue pattern on the photographic film S.

Figure 14 A Laue x-ray diffraction pattern from a crystal of sodium chloride.

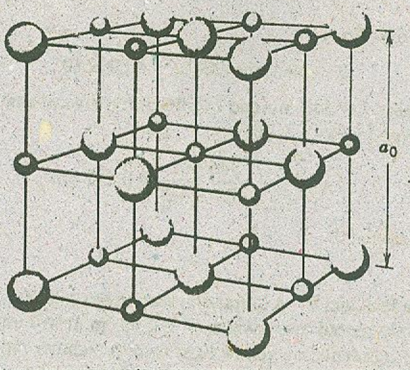

Figure 15 A model of a sodium chloride crystal, showing how the sodium ions Na^+ (small spheres) and chloride ions Cl^- (large spheres) are stacked in the unit cell, whose edge a_0 has the length 0.563 nm.

the crystal is made up) appear in certain sharply defined directions. If these beams fall on a photographic film, they form an assembly of "Laue spots." Figure 14, which shows an actual example of these spots, demonstrates that the hypothesis of Laue is indeed correct. The atomic arrangements in the crystal can be deduced from a careful study of the positions and intensities of the Laue spots in much the same way that we might deduce the structure of an optical grating (that is, the detailed profile of its slits) by a study of the positions and intensities of the lines in the interference pattern. Other experimental arrangements have supplanted the Laue technique to a considerable extent today, but the principle remains unchanged (see Question 25).

Figure 15 shows how sodium and chlorine atoms (strictly, Na^+ and Cl^- ions) are stacked to form a crystal of sodium chloride. This pattern, which has *cubic* symmetry, is one of the many possible atomic arrangements exhibited by solids. The model represents the *unit cell* for sodium chloride. This is the smallest unit from which the crystal may be built up by repetition in three dimensions. You should verify that no smaller assembly of atoms possesses this property. For sodium chloride the length a_0 of the cube edge of the unit cell is 0.563 nm.

Each unit cell in sodium chloride has four sodium ions and four chlorine ions associated with it. In Fig. 15 the sodium ion in the center belongs entirely to the cell shown. Each of the other twelve sodium ions shown is shared with three adjacent unit cells so that each contributes one-fourth of an ion to the cell under consideration. The total number of sodium ions is then $1 + \frac{1}{4}(12) = 4$. By similar reasoning you can show that although there are fourteen chlorine ions in Fig. 15, only four are associated with the unit cell shown.

The unit cell is the fundamental repetitive diffracting unit in the crystal, corresponding to the slit (and its adjacent opaque strip) in the optical diffraction grating of Fig. 1. Figure 16*a* shows a particular plane in a sodium chloride crystal. If each unit cell intersected by this plane is represented by a small cube, Fig. 16*b* results. You may imagine each of these figures extended indefinitely in three dimensions.

Let us treat each small cube in Fig. 16*b* as an elementary diffracting center, corresponding to a slit in an optical grating. The *directions* (but not the intensities) of all the diffracted x-ray beams that can emerge from a sodium chloride crystal (for a given x-ray wavelength and a given orientation of the incident beam) are determined by the geometry of this three-dimensional lattice of diffracting centers. In exactly the same way the *directions* (but not the intensities) of all the diffracted beams that can emerge from a particular optical grating (for a given wavelength and orientation of the incident beam) are determined only by the geometry of the grating, that is, by the grating spacing d. Representing the unit cell by what is essentially

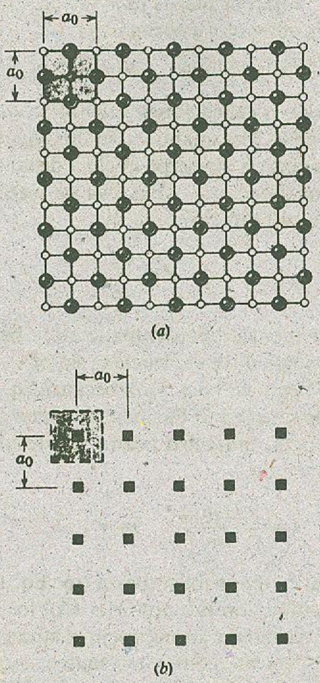

Figure 16 (a) A plane through a crystal of NaCl, showing the Na and Cl ions. (b) The corresponding unit cells in this section. Each cell is represented by a small black square.

a point, as in Fig. 16b, corresponds to representing the slits in a diffraction grating by lines, as we did in discussing the double-slit experiment in Section 45-1.

The *intensities* of the lines from an optical diffraction grating depend on the diffracting characteristics of a single slit, determined in particular by the slit width a; see, for example, Fig. 2 for a set of slits. The characteristics of actual optical gratings are determined by the profile of the grating rulings.

In exactly the same way the *intensities* of the diffracted beams emerging from a crystal depend on the diffracting characteristics of the unit cell. Fundamentally, the x rays are diffracted by electrons, diffraction by nuclei being negligible in most cases. Thus the diffracting characteristics of a unit cell depend on how the electrons are distributed throughout the volume of the cell. By studying the *directions* of diffracted x-ray beams, we can learn the basic symmetry of the crystal. By studying the *intensities* we can learn how the electrons are distributed in a unit cell. Figure 17 shows an example of this technique.

Bragg's Law

Bragg's law predicts the conditions under which diffracted x-ray beams from a crystal are possible. In deriving it, we ignore the structure of the unit cell, which is related only to the intensities of these beams. The dashed sloping lines in Fig. 18a represent the intersection with the

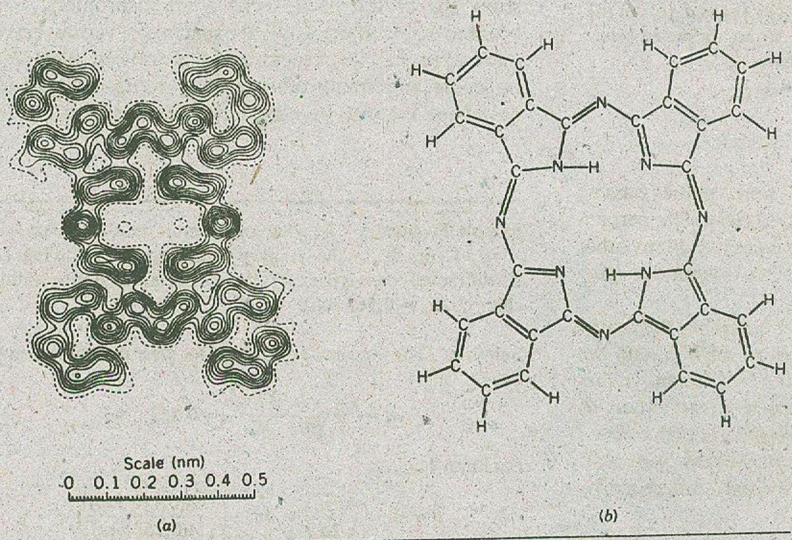

Figure 17 (a) Electron density contours for phthalocyanine ($C_{32}H_{18}N_8$) determined from the intensity distribution of scattered x rays. The dashed curves represent a density of one electron per 0.01 nm^2, and each adjacent curve represents an increase of one electron per 0.01 nm^2. (b) A structural representation of the molecule. Note that the greatest electron density occurs in (a) near the N atoms, which have the largest number of electrons (7). Note also that the H atoms, which contain only a single electron, are not prominent in (a).

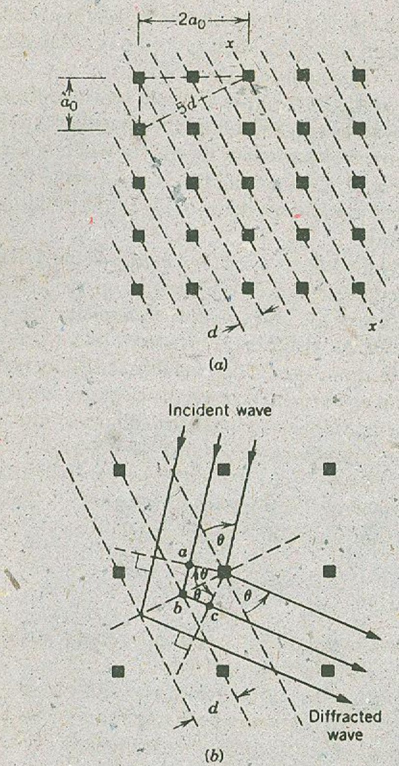

Figure 18 (*a*) A section through the NaCl lattice of Fig. 16. The dashed lines represent an arbitrary set of parallel planes connecting unit cells. The interplanar spacing is *d*. (*b*) An incident beam falls on a set of planes. A strong diffracted beam will be observed if Bragg's law is satisfied.

plane of the figure of an arbitrary set of planes passing through the elementary diffracting centers. The perpendicular distance between adjacent planes is *d*. Many other such families of planes, with different *interplanar spacings*, can be defined.

Figure 18*b* shows an incident wave striking the *family* of planes, the incident rays making an angle θ with the plane.* For a single plane, mirror-like "reflection" occurs for *any* value of θ. To have a constructive interference in the beam diffracted from the entire family of planes in the direction θ, the rays from the separate planes must reinforce each other. This means that the path difference for rays from adjacent planes (*abc* in Fig. 18*b*) must be an integral number of wavelengths or

$$2d \sin \theta = m\lambda \qquad m = 1, 2, 3, \ldots . \tag{12}$$

This relation is called *Bragg's law* after W. L. Bragg who first derived it. The quantity *d* in this equation (the interplanar spacing) is the perpendicular distance between the planes. For the planes of Fig. 18*a* we see that *d* is related to the unit cell dimension a_0 by

$$d = \frac{a_0}{\sqrt{5}}. \tag{13}$$

If an incident monochromatic x-ray beam falls at an arbitrary angle θ on a particular set of atomic planes, a diffracted beam will *not* result because Eq. 12 will not, in general, be satisfied. If the incident x rays are *continuous* in wavelength, diffracted beams will result when wavelengths given by

$$\lambda = \frac{2d \sin \theta}{m} \qquad m = 1, 2, 3, \ldots$$

are present in the incident beam (see Eq. 12).

X-ray diffraction is a powerful tool for studying both x-ray spectra and the arrangement of atoms in crystals. To study the spectrum of an x-ray source, a particular set of crystal planes, having a known spacing *d*, is chosen. Diffraction from these planes locates different wavelengths at different angles. A detector that can discriminate one angle from another can be used to determine the wavelength of radiation reaching it. On the other hand, we can study the crystal itself, using a monochromatic x-ray beam to determine not only the spacings of various crystal planes but also the structure of the unit cell. The DNA molecule and many other equally complex structures have been mapped by x-ray diffraction methods.

Sample Problem 5 At what angles must an x-ray beam with $\lambda = 0.110$ nm fall on the family of planes represented in Fig. 18*b* if a diffracted beam is to exist? Assume the material to be sodium chloride ($a_0 = 0.563$ nm).

Solution The interplanar spacing *d* for these planes is given by Eq. 13, or

$$d = \frac{a_0}{\sqrt{5}} = \frac{0.563 \text{ nm}}{\sqrt{5}} = 0.252 \text{ nm}.$$

Equation 12 gives

$$\theta = \sin^{-1}\left(\frac{m\lambda}{2d}\right) = \sin^{-1}\left(\frac{(m)(0.110 \text{ nm})}{(2)(0.252 \text{ nm})}\right).$$

Diffracted beams are possible for $\theta = 12.6°$ ($m = 1$); $\theta = 25.9°$ ($m = 2$), $\theta = 40.9°$ ($m = 3$), and $\theta = 60.9°$ ($m = 4$). Higher order beams cannot exist because they require that $\sin \theta > 1$.

Actually, the unit cell in cubic crystals such as NaCl has sym-

* In x-ray diffraction it is customary to specify the direction of a wave by giving the angle between the ray and the plane (the *glancing angle*) rather than the angle between the ray and the normal.

metry properties that require the intensity of diffracted x-ray beams corresponding to odd values of m to be zero. (See Problem 42.) Thus the only beams that are expected are $\theta = 25.9°$ ($m = 2$) and $\theta = 60.9°$ ($m = 4$).

47-5 HOLOGRAPHY *(Optional)*

The light emitted by an object contains the complete information on the size and shape of the object. We can consider that information to be stored in the wavefronts of the light from the object, specifically in the variation of intensity and phase of the electromagnetic fields. If we could record this information, we could reproduce a complete three-dimensional image of the object. However, photographic films record only the intensity variations; the films are not sensitive to phase variations. It is therefore not possible to use a photographic negative to reconstruct a three-dimensional image.

One exception to this restriction occurs in the case of x-ray diffraction from a crystal. Because of the regular spacing of the atoms of a crystal, we can easily deduce the relative phases of the diffracted waves reaching the film from different atoms. This possibility was realized by W. L. Bragg, who illuminated a photographic negative of an x-ray diffraction pattern and so reconstructed the image of a crystal. In this "double diffraction" method, diffraction of radiation from a diffraction pattern gives an image of the original object. For objects whose atoms are not arranged in such a periodic array, this simple method of image reconstruction does not work.

A scheme for recording the intensity *and* phase of the waves from objects was developed in 1948 by Dennis Gabor, who was awarded the 1971 Nobel Prize in physics for this discovery. This type of image formation is called *holography*, from the Greek words meaning "entire picture," and the image is called a *hologram*. The procedure is illustrated in Fig. 19. A wave diffracted from an object interferes on the photographic film with a reference wave. The interference between the two waves serves as the means for storing on the film information on the phase of the wave from the object. When the photographic image is viewed using light identical with the reference beam, a three-dimensional virtual image of the original object is reconstructed (Fig. 20). A second image (a real image), not shown in Fig. 20, is also produced by the hologram.

Because the film is illuminated uniformly by the diffracted light from the object and the reference beam, every piece of the film contains the information necessary to reproduce the three-dimensional image. The hologram itself (Fig. 21) shows only the interference fringes; in general, it is necessary to use a suitable monochromatic and coherent beam to reconstruct the image. For this reason, active development of holography did not occur until the early 1960s, when lasers became commonly available.

Some holograms can be viewed in white light. White-light holograms use a thick photographic emulsion, in which light is reflected by successive layers of grains in the film. Constructive interference occurs in the reflected light for the wavelength of the original reference beam, and destructive interference occurs for

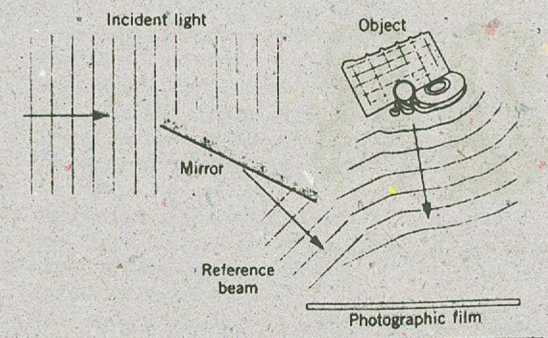

Figure 19 Apparatus for producing holograms. A portion of the beam from a source of coherent light (a laser, for instance) illuminates an object. The light diffracted by the object interferes on the film with a portion of the original beam, which serves as the reference.

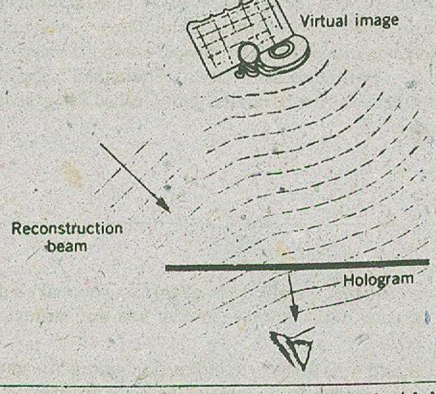

Figure 20 To view a hologram, it is illuminated with light identical to the reference beam. A three-dimensional virtual image can be seen, at the location of the original object.

Figure 21 A close-up view of a hologram, showing the interference pattern.

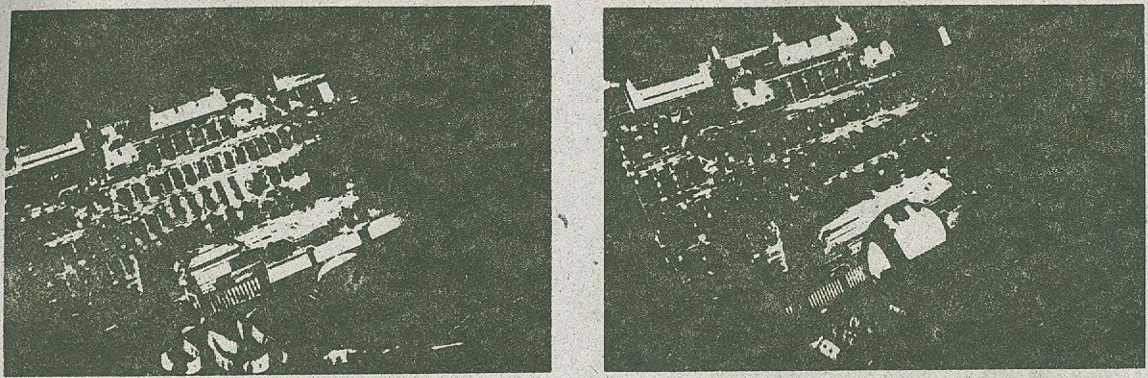

Figure 22 Two different views of the same hologram, taken from different directions. Note the relative movement of the objects in the images.

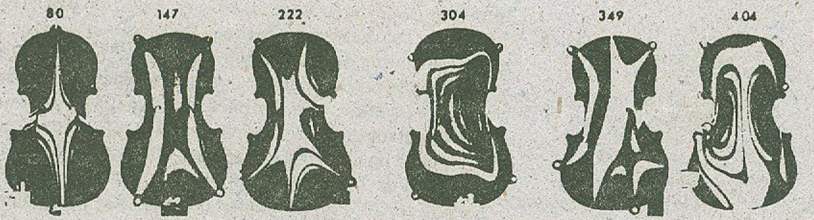

Figure 23 A holographic interference pattern of a top violin plate vibrating at different frequencies. The frequencies (in Hz) are shown above the plates.

other wavelengths. By using reference beams of several different colors, a full-color image can be produced.*

The hologram reconstructs a true three-dimensional image; for example, nearby objects appear "in front of" more distant objects, and by moving your head from side to side you can change the relative spatial orientation of the objects. Figure 22 shows two different views of the same hologram, illustrating the *parallax* effect of viewing the hologram from two different directions.

Holograms have a variety of applications in basic and applied science. For example, in producing holograms the object must be kept absolutely still while the film is exposed; any small movement would change the relative phase between the diffracted and reference beams and thereby change the interference pattern stored on the film. If a hologram is made by superimposing on the film two successive exposures of a vibrating object, such as the top or bottom plate of a violin, locations on the object that moved between the exposures by an integral number of wavelengths will show constructive interference, while parts of the object that moved by a half-integral number of wavelengths ($\lambda/2$, $3\lambda/2, \ldots$) will show destructive interference. Figure 23 shows an example of the use of this technique, called *holographic interferometry*. ■

* See "White-Light Holograms," by Emmett N. Leith, *Scientific American*, October 1976, p. 80.

QUESTIONS

1. Discuss this statement: "A diffraction grating can just as well be called an interference grating."
2. How would the spectrum of an enclosed source that is formed by a diffraction grating on a screen change (if at all) when the source, grating, and screen are all submerged in water?
3. (*a*) For what kind of waves could a long picket fence be considered a useful grating? (*b*) Can you make a diffraction grating out of parallel rows of fine wire strung closely together?
4. Could you construct a diffraction grating for sound? If so, what grating spacing is suitable for a wavelength of 0.5 m?
5. A crossed diffraction grating is ruled in two directions, at right angles to each other. Predict the pattern of light inten-

sity on the screen if light is sent through such a grating. Is there any practical value to such a grating?

6. Suppose that, instead of a slit, a small circular aperture were placed in the focal plane of the collimating lens in the telescope of a spectrometer. What would be seen when the telescope is illuminated by sodium light? Why then do we usually call spectra *line* spectra?

7. In a grating spectrograph, several lines having different wavelengths and formed in different orders might appear near a certain angle. How could you distinguish between their orders?

8. You are given a photograph of a spectrum on which the angular positions and the wavelengths of the spectrum lines are marked. (*a*) How can you tell whether the spectrum was taken with a prism or a grating instrument? (*b*) What information could you gather about either the prism or the grating from studying such a spectrum?

9. A glass prism can form a spectrum. Explain how. How many "orders" of spectra will a prism produce?

10. For the simple spectroscope of Fig. 8 show (*a*) that θ increases with λ for a grating and (*b*) that θ decreases with λ for a prism.

11. According to Eq. 6 the principal maxima become wider (that is, $\delta\theta$ increases) the higher the order m (that is, the larger θ becomes). According to Eq. 11 the resolving power becomes greater the higher the order m. Explain this apparent paradox.

12. Explain in your own words why increasing the number of slits N in a diffraction grating sharpens the maxima. Why does decreasing the wavelength do so? Why does increasing the grating spacing d do so?

13. How much information can you discover about the structure of a diffraction grating by analyzing the spectrum it forms of a monochromatic light source? Let $\lambda = 589$ nm, for example.

14. Assume that the limits of the visible spectrum are 430 and 680 nm. How would you design a grating, assuming that the incident light falls normally on it, such that the first-order spectrum barely overlaps the second-order spectrum?

15. (*a*) Why does a diffraction grating have closely spaced rulings? (*b*) Why does it have a large number of rulings?

16. Two light beams of nearly equal wavelengths are incident on a grating of N rulings and are not quite resolvable. However, they become resolved if the number of rulings is increased. Formulas aside, is the explanation of this that: (*a*) more light can get through the grating? (*b*) the principal maxima become more intense and hence resolvable? (*c*) the diffraction pattern is spread more and hence the wavelengths become resolved? (*d*) there is a large number of orders? or (*e*) the principal maxima become narrower and hence resolvable?

17. The relation $R = Nm$ suggests that the resolving power of a given grating can be made as large as desired by choosing an arbitrarily high order of diffraction. Discuss this possibility.

18. Show that at a given wavelength and a given angle of diffraction the resolving power of a grating depends only on its width $W (= Nd)$.

19. How would you experimentally measure (*a*) the dispersion D and (*b*) the resolving power R of a grating spectrograph?

20. For a given family of planes in a crystal, can the wavelength of incident x rays be (*a*) too large or (*b*) too small to form a diffracted beam?

21. If a parallel beam of x rays of wavelength λ is allowed to fall on a randomly oriented crystal of any material, generally no intense diffracted beams will occur. Such beams appear if (*a*) the x-ray beam consists of a continuous distribution of wavelengths rather than a single wavelength or (*b*) the specimen is not a single crystal but a finely divided powder. Explain each case.

22. Does an x-ray beam undergo refraction as it enters and leaves a crystal? Explain your answer.

23. Why cannot a simple cube of edge $a_0/2$ in Fig. 15 be used as a unit cell for sodium chloride?

24. In some respects Bragg reflection *differs* from plane grating diffraction. Of the following statements, which one is true for Bragg reflection but not true for grating diffraction? (*a*) Two different wavelengths may be superposed. (*b*) Radiation of a given wavelength may be sent in more than one direction. (*c*) Long waves are deviated more than short waves. (*d*) There is only one grating spacing. (*e*) Diffraction maxima of a given wavelength occur only for particular angles of incidence.

25. In Fig. 24*a* we show schematically the Debye–Scherrer experimental arrangement and in Fig. 24*b* a corresponding x-ray diffraction pattern. (*a*) Keeping in mind that the Laue method uses a large single crystal and an x-ray beam continuously distributed in wavelength, explain the origin of the spots in Fig. 14. (*Hint*: Each spot corresponds to the direction of scattering from a family of planes.) (*b*) Keeping in mind that the Debye–Scherrer method uses a large number of small single crystals randomly oriented and a monochromatic beam of x rays, explain the origin of the rings. (*Hint*: Because the small crystals are randomly oriented, all possible angles of incidence are obtained.)

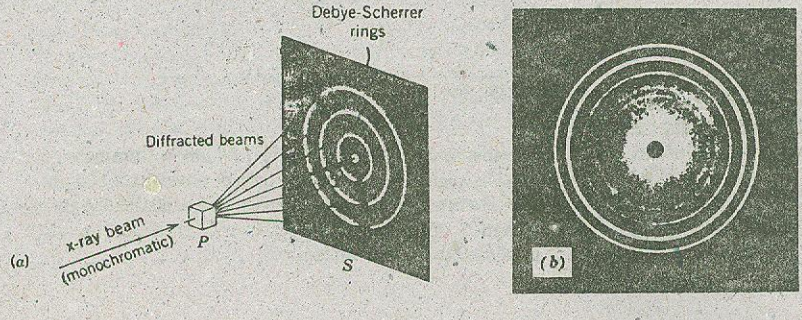

Figure 24 Question 25.

PROBLEMS

Section 47-1 Multiple Slits

1. A diffraction grating 21.5 mm wide has 6140 rulings. (*a*) Calculate the distance d between adjacent rulings. (*b*) At what angles will maximum-intensity beams occur if the incident radiation has a wavelength of 589 nm?

2. A diffraction grating 2.86 cm wide produces a deviation of 33.2° in the second order with light of wavelength 612 nm. Find the total number of rulings on the grating.

3. With light from a gaseous discharge tube incident normally on a grating with a distance 1.73 μm between adjacent slit centers, a green line appears with sharp maxima at measured transmission angles $\theta = \pm 17.6°, 37.3°, -37.1°, 65.2°$, and $-65.0°$. Compute the wavelength of the green line that best fits the data.

4. A narrow beam of monochromatic light strikes a grating at normal incidence and produces sharp maxima at the following angles from the normal: 6° 40′, 13° 30′, 20° 20′, 35° 40′. No other maxima appear at any angle between 0° and 35° 40′. The separation between adjacent ruling centers in the grating is 5040 nm. Find the wavelength of light used.

5. Light of wavelength 600 nm is incident normally on a diffraction grating. Two adjacent principal maxima occur at $\sin\theta = 0.20$ and $\sin\theta = 0.30$. The fourth order is missing. (*a*) What is the separation between adjacent slits? (*b*) What is the smallest possible individual slit width? (*c*) Name all orders actually appearing on the screen with the values derived in (*a*) and (*b*).

6. A diffraction grating is made up of slits of width 310 nm with a 930-nm separation between centers. The grating is illuminated by monochromatic plane waves, $\lambda = 615$ nm, the angle of incidence being zero. (*a*) How many diffraction maxima are there? (*b*) Find the width of the spectral lines observed in first order if the grating has 1120 slits.

7. Derive this expression for the intensity pattern for a three-slit "grating":

$$I = \tfrac{1}{9}I_m(1 + 4\cos\phi + 4\cos^2\phi),$$

where

$$\phi = \frac{2\pi d \sin\theta}{\lambda}.$$

Assume that $a \ll \lambda$ and be guided by the derivation of the corresponding double-slit formula (Eq. 17 of Chapter 46).

8. (*a*) Using the result of Problem 7, show that the halfwidth of the fringes for a three-slit diffraction pattern, assuming θ small enough so that $\sin\theta \approx \theta$, is

$$\Delta\theta \approx \frac{\lambda}{3.2d}.$$

(*b*) Compare this with the expression derived for the two-slit pattern in Problem 25, Chapter 45 and show that these results support the conclusion that for a fixed slit spacing the interference maxima become sharper as the number of slits is increased.

9. (*a*) Using the result of Problem 7, show that a three-slit "grating" has only one secondary maximum. Find (*b*) its location and (*c*) its relative intensity.

10. A three-slit grating has separation d between adjacent slits. If the middle slit is covered up, will the halfwidth of the intensity maxima become broader or narrower and by what factor? See Problem 8.

11. A diffraction grating has a large number N of slits, each of width a. Let I_{max} denote the intensity at some principal maximum, and let I_k denote the intensity of the kth adjacent secondary maximum. (*a*) If $k \ll N$, show from the phasor diagram that, approximately, $I_k/I_{max} = 1/(k+\frac{1}{2})^2\pi^2$. (Compare this with the single-slit formula.) (*b*) For those secondary maxima that lie roughly midway between two adjacent principal maxima, show that roughly $I_k/I_{max} = 1/N^2$. (*c*) Consider the central principal maximum and those adjacent secondary maxima for which $k \ll N$. Show that this part of the diffraction pattern quantitatively resembles that for one single slit of width Na.

Section 47-2 Diffraction Gratings

12. A diffraction grating has 200 rulings/mm and a principal maximum is noted at $\theta = 28°$. (*a*) What are the possible wavelengths of the incident visible light? (*b*) What colors are they?

13. A grating has 315 rulings/mm. For what wavelengths in the visible spectrum can fifth-order diffraction be observed?

14. Show that in a grating with alternately transparent and opaque strips of equal width, all the even orders (except $m = 0$) are absent.

15. Given a grating with 400 rulings/mm, how many orders of the entire visible spectrum (400–700 nm) can be produced?

16. Assume that light is incident on a grating at an angle ψ as shown in Fig. 25. Show that the condition for a diffraction maximum is

$$d(\sin\psi + \sin\theta) = m\lambda \qquad m = 0, 1, 2, \ldots .$$

Only the special case $\psi = 0$ has been treated in this chapter (compare with Eq. 1).

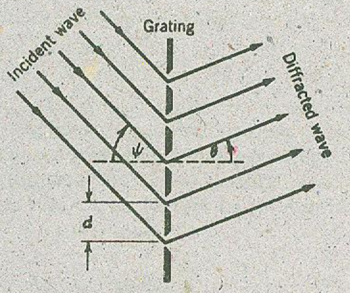

Figure 25 Problem 16.

17. A transmission grating with $d = 1.50$ μm is illuminated at various angles of incidence by light of wavelength 600 nm. Plot as a function of angle of incidence (0 to 90°) the angular deviation of the first-order diffracted beam from the incident direction. See Problem 16.

18. Assume that the limits of the visible spectrum are arbitrarily chosen as 430 and 680 nm. Calculate the number of rulings per mm of a grating that will spread the first-order spectrum through an angular range of 20.0°.

19. White light (400 nm $< \lambda <$ 700 nm) is incident on a grating. Show that, no matter what the value of the grating spacing d, the second- and third-order spectra overlap.

20. A grating has 350 rulings/mm and is illuminated at normal incidence by white light. A spectrum is formed on a screen 30 cm from the grating. If a 10-mm square hole is cut in the screen, its inner edge being 50 mm from the central maximum and parallel to it, what range of wavelengths passes through the hole?

Section 47-3 Dispersion and Resolving Power

21. The "sodium doublet" in the spectrum of sodium is a pair of lines with wavelengths 589.0 and 589.6 nm. Calculate the minimum number of rulings in a grating needed to resolve this doublet in the second-order spectrum.

22. A grating has 620 rulings/mm and is 5.05 mm wide. (*a*) What is the smallest wavelength interval that can be resolved in the third order at $\lambda = 481$ nm? (*b*) How many higher orders can be seen?

23. A source containing a mixture of hydrogen and deuterium atoms emits light containing two closely spaced red colors at $\lambda = 656.3$ nm whose separation is 0.180 nm. Find the minimum number of rulings needed in a diffraction grating that can resolve these lines in the first order.

24. (*a*) How many rulings must a 4.15-cm-wide diffraction grating have to resolve the wavelengths 415.496 nm and 415.487 nm in the second order? (*b*) At what angle are the maxima found?

25. In a particular grating the sodium doublet (see Problem 21) is viewed in third order at 10.2° to the normal and is barely resolved. Find (*a*) the ruling spacing and (*b*) the total width of the grating.

26. Show that the dispersion of a grating can be written

$$D = \frac{\tan\theta}{\lambda}.$$

27. A grating has 40,000 rulings spread over 76 mm. (*a*) What is its expected dispersion D in °/nm for sodium light ($\lambda = 589$ nm) in the first three orders? (*b*) What is its resolving power in these orders?

28. Light containing a mixture of two wavelengths, 500 nm and 600 nm, is incident normally on a diffraction grating. It is desired (1) that the first and second principal maxima for each wavelength appear at $\theta \leq 30°$, (2) that the dispersion be as high as possible, and (3) that the third order for 600 nm be a missing order. (*a*) What should be the separation between adjacent slits? (*b*) What is the smallest possible individual slit width? (*c*) Name all orders for 600 nm that actually appear on the screen with the values derived in (*a*) and (*b*).

29. A diffraction grating has a resolving power $R = \lambda/\Delta\lambda = Nm$. (*a*) Show that the corresponding frequency range $\Delta\nu$ that can just be resolved is given by $\Delta\nu = c/Nm\lambda$. (*b*) From Fig. 1, show that the "times of flight" of the two extreme rays differ by an amount $\Delta t = (Nd/c)\sin\theta$. (*c*) Show that $(\Delta\nu)(\Delta t) = 1$, this relation being independent of the various grating parameters. Assume $N \gg 1$.

Section 47-4 X-Ray Diffraction

30. X rays of wavelength 0.122 nm are found to reflect in the second order from the face of a lithium fluoride crystal at a Bragg angle of 28.1°. Calculate the distance between adjacent crystal planes.

31. A beam of x rays of wavelength 29.3 pm is incident on a calcite crystal of lattice spacing 0.313 nm. Find the smallest angle between the crystal planes and the beam that will result in constructive reflection of the x rays.

32. Monochromatic high-energy x rays are incident on a crystal. If first-order reflection is observed at Bragg angle 3.40°, at what angle would second-order reflection be expected?

33. An x-ray beam, containing radiation of two distinct wavelengths, is scattered from a crystal, yielding the intensity spectrum shown in Fig. 26. The interplanar spacing of the scattering planes is 0.94 nm. Determine the wavelengths of the x rays present in the beam.

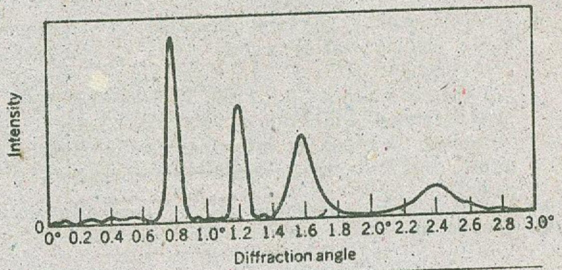

Figure 26 Problem 33.

34. In comparing the wavelengths of two monochromatic x-ray lines, it is noted that line A gives a first-order reflection maximum at a glancing angle of 23.2° to the face of a crystal. Line B, known to have a wavelength of 96.7 pm, gives a third-order reflection maximum at an angle of 58.0° from the same face of the same crystal. (*a*) Calculate the interplanar spacing. (*b*) Find the wavelength of line A.

35. Monochromatic x rays are incident on a set of NaCl crystal planes whose interplanar spacing is 39.8 pm. When the beam is rotated 51.3° from the normal, first-order Bragg reflection is observed. Find the wavelength of the x rays.

36. Show that, in Bragg diffraction by a monochromatic beam of x rays, no intense maxima will be obtained if the wavelength of the x rays is greater than twice the largest crystal plane separation. See Question 20.

37. Prove that it is not possible to determine both wavelength of radiation and spacing of Bragg reflecting planes in a crystal by measuring the angles for Bragg reflection in several orders.

38. Assume that the incident x-ray beam in Fig. 27 is not monochromatic but contains wavelengths in a band from 95.0 to 139 pm. Will diffracted beams, associated with the planes

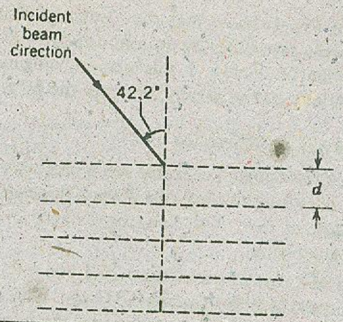

Figure 27 Problems 38 and 40.

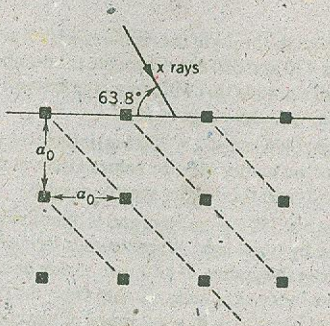

Figure 28 Problem 39.

shown, occur? If so, what wavelengths are diffracted? Assume $d = 275$ pm.

39. First-order Bragg scattering from a certain crystal occurs at an angle of incidence of 63.8°; see Fig. 28. The wavelength of the x rays is 0.261 nm. Assuming that the scattering is from the dashed planes shown, find the unit cell size a_0.

40. Monochromatic x rays ($\lambda = 0.125$ nm) fall on a crystal of sodium chloride, making an angle of 42.2° with the reference line shown in Fig. 27. The planes shown are those of Fig. 18*a*, for which $d = 0.252$ nm. Through what angles must the crystal be turned to give a diffracted beam associated with the planes shown? Assume that the crystal is turned about an axis that is perpendicular to the plane of the page.

41. Consider an infinite two-dimensional square lattice as in Fig. 16*b*. One interplanar spacing is obviously a_0 itself. (*a*) Calculate the next five smaller interplanar spacings by sketching figures similar to Fig. 18*a*. (*b*) Show that the general formula is

$$d = a_0/\sqrt{h^2 + k^2},$$

where h and k are both relatively prime integers that have no common factors other than unity.

42. In Sample Problem 5 the $m = 1$ beam, permitted by interference considerations, has zero intensity because of the diffracting properties of the unit cell for this geometry of beams and crystal. Prove this. (*Hint*: Show that the "reflection" from an atomic plane through the top of a layer of unit cells is canceled by a "reflection" from a plane through the middle of this layer of cells. All odd-order beams prove to have zero intensity.)