Verification of de Broglie's hypothesis by electron diffraction from graphite

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1 Introduction

Is light a particle or a wave? The answer depends on how we choose to look at it. Some outcomes are explained with the particle or the photon model and some with the wave model. For example, light that is capable of knocking out electrons from a metal can also diffract from a grating. These complementary aspects are generally referred to as 'particle' and 'wave' aspects. We now accept that the true nature of light can not be described by a single picture and that both the models complement each other [1].

After the great success of dual particle-wave nature of light, de Broglie postulated that as photons exhibit both the wave and particle characteristics, perhaps all form of matter must obey the same wave-particle duality.

The current experiment is designed to verify de Broglie's hypothesis through diffraction of electrons from polycrystalline graphite. The main emphasis is on image processing using digital photography and ultimately students will be able to determine the inter-planar spacing of the graphite crystal.

KEYWORDS de Broglie relationship · Bragg's law · electron diffraction · Fluorescent material · Thermionic emission · crystal structure ·

2 Conceptual Objectives

In this experiment we will,

1. learn about the wave-particle duality of matter in particular, electrons,
2. understand and verify de Broglie's relationship,
3. get familiar with Bragg's law and learn about its implications,
4. learn about the functional principle of electron tubes,

5. learn about image processing using a freely available software **ImageJ**,
6. practice digital photography and learn its implementation in experimental physics,
7. practice manipulating error bars and weighted fit of a straight line, and,
8. determine the values of inter-planar spacing of a hexagonal crystal structure, i.e., graphite.

3 Theoretical background

3.1 De Broglie's waves

In the world around us, we usually consider cricket balls solely as particles and sound waves as waves. In case of photons or electrons such characteristics are not necessarily and immediately drawn in a straight manner.

In 1923 de Broglie suggested that the dual wave-particle nature of light must have its counterpart in matter. Thus under some circumstances particles must behave as waves and perhaps all forms of matter have the dual wave-particle nature [1]. De Broglie suggested that the wavelength of a wave associated with the material particle is given by the relationship,

$$\lambda = \frac{h}{p}, \tag{1}$$

where h is the Planck's constant and p is the momentum of the particle. Such a wave is called a matter wave, pilot wave or quite aptly, a de Broglie wave.

De Broglie's work attracted much attention and several scientists came up with the idea that the verification of de Broglie's work could be obtained through an experiment that could diffract electrons.

3.2 Davisson and Germer's classic experiment

The first experimental confirmation of de Broglie's hypothesis was given by C. J. Davisson and L. H. Germer, who succeeded in measuring the wavelength of the electron through diffraction from a crystal structure [1].

The experiment utilizes the bombardment of an electron beam emanating from a filament in an electron gun to a nickel target at normal incidence. The schematic of their setup is shown in Figure (1a). The electrons are accelerated towards the anode by a large potential. The experiment was conducted in a vacuum chamber minimizing collisions of the electrons with other atoms. An electron detector that could move in a circular path around the crystal measured the number of electrons scattered at different angles.

The experiment took a fruitful turn in what appears to be a lucky accident. Mishandling broke the vacuum vessel, air entered the chamber oxidizing the nickel target. Now in order to remove the covering oxide layer, the sample was slowly heated. Davisson and Germer found that due to prolonged heating, large single crystals in the presence of a hydrogen atmosphere were nucleated in the polycrystalline sample. These regions provided the perfect crystalline

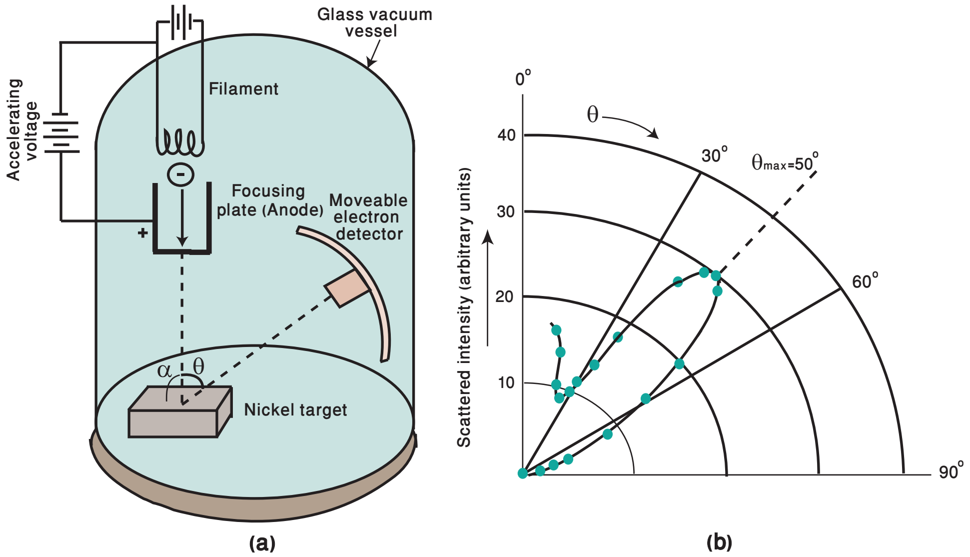

Figure 1: (a) A schematic illustration of Davisson-Germer's experimet. The scattering angle θ is the angle between incident and scattered beam while α corresponds to possible orientations of the nickel target, and, (b) experimental results with a polar plot of scattered intensity versus scattering angle θ. A polar plot is a plot in which the distance of each point on a plane is taken from a fixed point and plotted against angle from a certain direction.

arrangement necessarily required for electron diffraction patterns. You can read more about this historic, Nobel-Prize winning experiment in [1].

Davisson and Germer went on to calculate the wavelength of the electrons through the diffraction condition and compared it with de Broglie's wavelength. The results were tested out for different orientations of the detector and electron energies. A polar plot of the scattered intensity versus scattering angle is shown in Figure (1b). Here is the reproduction of their calculations.

The speed of non-relativistic electrons v can be calculated through the energy relation,

$$\frac{1}{2}mv^2 = eV_o, \tag{2}$$

implying,

$$v = \sqrt{\frac{2eV_o}{m}}. \tag{3}$$

Substituting the above expression in Equation (1) yields,

$$\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2meV_o}}. \tag{4}$$

Here m is the mass of the electron, e is its charge and V_o is the anode voltage.

For example, consider the parameters $V_o = 54\,\mathrm{V}$, $\alpha = 90^\circ$ and $\theta = 50^\circ$. The wavelength corresponding to this anode voltage is,

$$\lambda = \sqrt{\frac{6.63\times10^{-34}}{2(54)(1.6\times10^{-19})(9.11\times10^{-31})}} = 0.167\,\mathrm{nm}. \tag{5}$$

This wavelength should match the prediction from the diffraction condition. The Ni crystal's surface in in fact acting like a diffraction grating. The low energy electron beam can not penetrate deep inside the crystal and scatters only from the surface layer. Constructive interference takes place when the path difference between the two scattered rays is an integral multiple of the wavelength,

$$n\lambda = 2d\sin(\theta),$$

where d is the interplanar spacing.

This equation is called the Bragg condition and we will say more about it in the next subsection. Since the inter-planar spacing for nickel crystal is known to be 0.215 nm (from prior X-ray diffraction measurements), the wavelength corresponding to a diffraction maximum is,

$$\lambda = (0.215\,\text{nm})(\sin 50^\circ) = 0.165\,\text{nm}, \tag{6}$$

which is in excellent agreement with theoretically calculated value of de Broglie's wavelength given in Equation (5). Hence in Figure (1b), if a potential difference of 54 V is applied, a diffraction peak is observed at a scattering angle of $\theta \approx 50^\circ$, confirming both the de Broglie wavelength calculated in Equation (5) and the wavelength popping out of the Bragg condition in Equation (6) are a good match.

Q 1. An object is said to have wavelike properties if it exhibits interference or diffraction. Both of these phenomenon require that the size of the scattering object or aperture is the same as the wavelength of the wave.

(a) Why don't we see the wave-like properties of a cricket ball?

(b) Calculate the de Broglie's wavelength of a cricket ball of mass 140 g traveling at a speed of 27 m/s. Compare this wavelength with the size of the nucleus (~ 1 fm$= 10^{-15}$ m).

3.3 Bragg's law

Now let's look at the Bragg condition with a deeper eye. Refer to Figure 2. A crystal can be represented by a set of parallel planes which correspond to atomic planes. When a monochromatic X-ray beam or beam of electrons is incident on the surface of a crystal it is reflected. The reflection can only take place when the angle of incidence depends in a certain precise way on the wavelength and the lattice constants of the crystal.

Consider a crystal in which atoms are arranged in a periodic manner with interatomic spacing d shown in Figure (2). A monochromatic X-ray beam of wavelength λ falls on the crystal at a glancing angle θ. The planes act as mirrors and the beam is partially reflected from each of the planes. The reflected rays are then collected at a detector. At the detector position, the rays interfere and constructive interference occurs when the path difference between two adjacent rays is an integral multiple of the wavelength,

$$\text{Path difference } \Delta = n\lambda, \qquad n = 1, 2, 3, \ldots \tag{7}$$

Suppose AB and DE are the incident rays and BC and EF are the reflected rays. The two incident rays are labeled as ray 1 and 2. The ray 2 travels an extra distance $(GE + EH)$ when

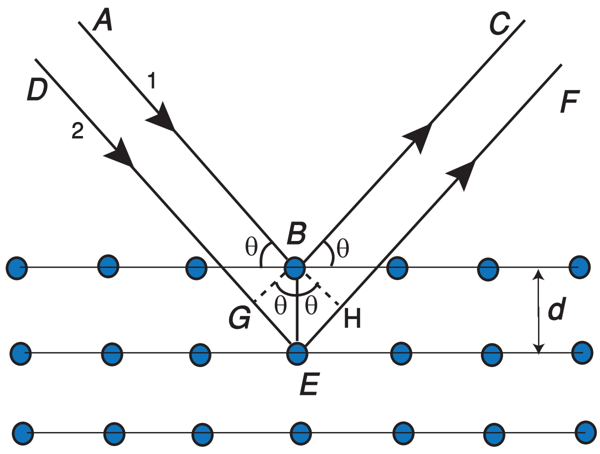

Figure 2: Bragg diffraction from a crystal with interatomic spacing d. Rays AB and DE represent the incident rays while BC and EF are reflected (diffracted) rays and θ is called the glancing angle.

compared with the distance traversed by ray 1. This extra distance is the the path difference between two rays and given as,

$$\Delta = \overline{GE} + \overline{EH}. \tag{8}$$

Since,

$$\overline{GE} = d\sin(\theta), \quad \text{and} \quad \overline{GE} = \overline{EH}, \tag{9}$$

Δ becomes,

$$d\sin(\theta) + d\sin(\theta) = 2d\sin(\theta). \tag{10}$$

Comparing Equations (7) and (10) gives the Bragg law,

$$2d\sin(\theta) = n\lambda. \tag{11}$$

This relationship is a consequence of the periodicity of the crystal and the wave nature of radiation.

Q 2. At what angles must an X-ray beam with $\lambda = 0.110\,\text{nm}$ fall on the family of planes if a diffracted beam is to exist? Assume the material to be sodium chloride ($a_o = 0.563\,\text{nm}$ and $d = a_o/\sqrt{5}$) [2].

Q 3. In comparing the wavelengths of two monochromatic X-ray lines, it is noted that line A gives a first-order reflection maximum at a glancing angle of 23.2° to the face of a crystal. Line B, known to have a wavelength of 96.7 pm, gives a third-order reflection maximum at an angle of 58.0° from the same face of the same crystal [2].

(a) Calculate the inter-planar spacing.

(b) Find the wavelength producing the reflection maximum at position A.

4 Experimental objectives

The experiment utilizes an electron diffraction tube in which an electron beam is produced by thermionic emission of the heated filament and accelerated towards the graphite target by applying a very high potential (2-5 kV). The scattered beam produces two diffracted rings on the fluorescent screen that correspond to different planes of the hexagonal crystal structure of the carbon atoms. The diameter of the ring is obtained from digital photographs taken of the screen. These photographs are brought into the computer and processed and analyzed using a freely available image analysis software **ImageJ** [4]. From the visible rings, one can calculate the inter-planar spacing, while the de Broglie's wavelength can be calculated through the accelerating anode voltage.

5 Apparatus

- Electron diffraction tube 1013885 (3B Scientific),
- High voltage DC power supply 0-5 kV U33010 (3B Scientific),
- Tube holder 1008507 (3B Scientific),
- Ammeter,
- Meter rule,
- Connecting wires,
- ImageJ software. [4] It is a freely available software used for image processing and analysis written in Java. Possessing a rich plugin infrastructure, it is widely used in the life sciences community.

5.1 Electron diffraction tube

The electron diffraction tube is a highly evacuated tube designed to study the wavelike properties of electrons by observing interference fringes. These fringes appear when a beam of electrons passes through the graphite crystal and strikes on a fluorescent screen. The wiring has already been setup for yo. Please don't change the connections as this may lead to damage to the tube.

Here is a brief description of some of the components of electron diffraction tube.

5.1.1 The electron gun

The heater and the electrodes (cathode and anode) make up an electron gun. The electrons are emitted by thermionic emission from a heated filament. The emitted electrons are accelerated by applying an adjustable high potential (V_o = 2000-4500 V) between the cathode and anode that results in a fine accelerated electron beam. The energy of the electrons is given by Equation (2).

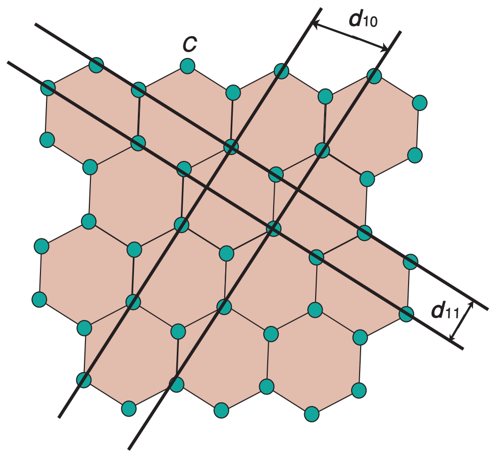

Figure 3: The hexagonal crystal structure of graphite. Only one layer is shown. The interplanar spacing are d_{10} and d_{11}, corresponding to two different distinguishable sets of carbon atoms. The circles represent carbon atoms.

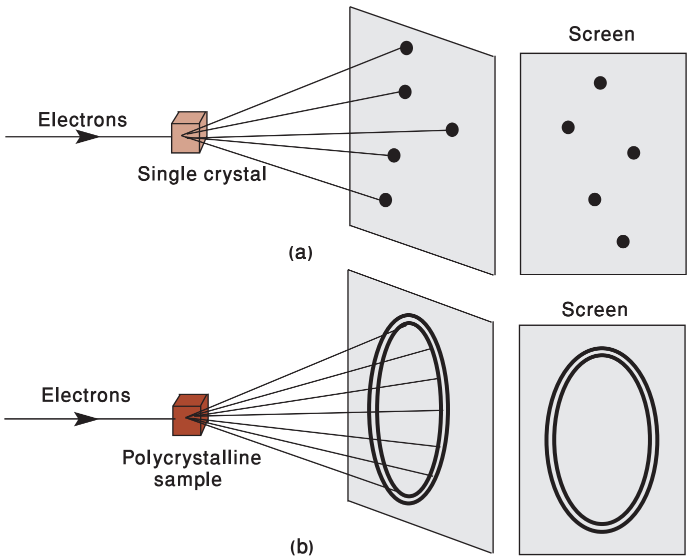

Figure 4: Electrons scattered from a (a) single crystal, and, (b) from a polycrystalline sample.

5.1.2 Graphite target

The electron beam, after passing through the anode strikes a micro-meshed nickel grid on which a thin layer of graphite (carbon) has been deposited. The graphite crystal is our analog to the Ni target used by Davisson and Germer. The graphite target has a hexagonal crystal structure as shown in Figure (3).

Our target is a polycrystal with large number of microcrystals that are randomly oriented. The main advantage of using polycrystalline carbon is that all possible angle of incidences can be accessed without changing the direction of the electron beam. This results in scattering along a cone instead of scattering in the form of scattered discrete rays. The comparison between diffraction patterns from single and polycrystalline samples is shown in Figure (4).

5.1.3 Fluorescent screen

Diffracted electrons image on a screen made up of a fluorescent material. This results in a visible diffraction pattern. The pattern comprises rings. One set of planes produces one ring, at an angle satisfying the Bragg condition. The two planes have inter-planar spacings of d_10 and d_11 which we seek to determine.

5.1.4 Geometry of the diffracted rings

The geometrical relation between diameter of the ring D, the distance between the graphite target and fluorescent screen L, radius of curvature R and the scattering angle 2θ can be obtained by considering the triangle OAB [3], shown in Figure 5

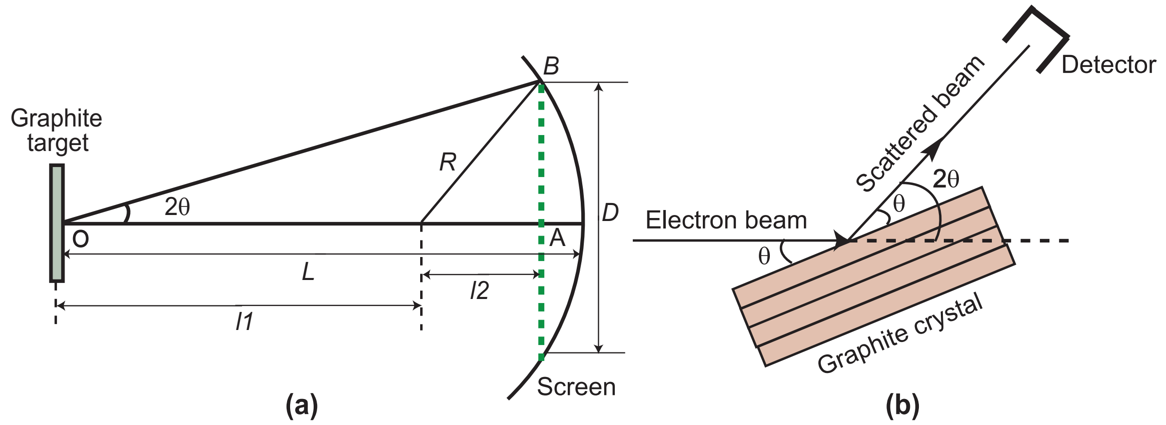

Figure 5: (a) Diagram showing the geometry of the vacuum tube, D is the diameter of the ring, R is the radius of curvature, 2θ is the scattering angle and L is the distance of the graphite target to the screen, and, (b) diffraction geometry for graphite crystal.

$$\tan(2\theta) = \frac{D/2}{(l_1 + l_2)}, \tag{12}$$

The lengths l_1 and l_2 are defined as,

$$\begin{aligned} l_1 &= L - R, \\ l_2 &= \sqrt{R^2 - (D/2)^2}, \end{aligned}$$

where R is the screen's radius of curvature. Therefore Equation (12) yields,

$$\tan(2\theta) = \frac{D/2}{(L - R + \sqrt{R^2 - (D/2)^2})}. \tag{13}$$

Now, for the small angle approximation, we can write,

$$\sin(\theta) \approx \tan(\theta), \tag{14}$$

and

$$\tan(2\theta) = \frac{2\tan(\theta)}{1 - \tan^2(\theta)} \approx 2\tan(\theta) = 2\sin(\theta). \tag{15}$$

Q 4. Use the geometry of the diffraction rings and the Bragg condition to find an expression relating the wavelength with the geometrical parameters and lattice spacing. You should arrive to the expression,

$$\lambda = d\left[\frac{D/2}{(L - R + \sqrt{R^2 - (D/2)^2})}\right]. \tag{16}$$

Finally, equating λ to the de Broglie's formula, Equation 11 results in.

$$\left[\frac{D}{(L - R + \sqrt{R^2 - (D/2)^2})}\right] = \left[\frac{2(1.23 \times 10^{-9}\,\mathrm{m})}{d}\right]\frac{1}{\sqrt{V_o}}. \tag{17}$$

Q 5. Verify that you understand and have derived the above equation, Figure 5 will help.

The values of some important parameters are given in Table (1).

Parameter	**Value**
Inter-planar spacing d_{10}	0.213 nm
Inter-planar spacing d_{11}	0.123 nm
Distance from target to fluorescent screen (L)	(125 ± 2) mm
Radius of curvature of the screen (R)	65 mm

Table 1: Values of some important parameters required in this experiment.

In the experiment, we will find the ring diameter D for varying anode voltage V_o and will plot the L.H.S. of Equation 10 with respect to $\frac{1}{\sqrt{V_o}}$ to determine the inter-planar spacing d. After familiarizing ourselves with the geometry of the tube and the diffraction, we now proceed to the experiment.

6 The experiment

6.1 Safety precautions

- Don't touch the electron diffraction tube during operation because the hot cathode tubes are thin-walled, highly evacuated and there is a risk of implosion.
- Make sure all power is switched off before you make electrical connections.
- As you are dealing with very high voltages, don't touch the connection wires when the unit is functional.
- The tube may be damaged if the voltage or current is too high. Monitor the ammeter reading continuously and care should be taken that the **current value should not exceed 0.17 mA**.
- The bright spot in the center of the tube may damage the fluorescent screen. To avoid this, turn the voltage knob to zero after taking each reading.
- When the tube is in operation, always allow it to cool down before dismantling.

6.2 Procedure

Note: The connections have already been set up. Do not change them..

Q 6. Set the voltage regulator knob to zero. Now switch on the Power button of the high voltage power supply and wait for a minute so that the filament warms up.

Q 7. Slowly increase the accelerating voltage by rotating the voltage knob clockwise until the rings are visible on the luminescent screen. Note down this voltage reading.

Q 8. You are provided with a high resolution digital camera. Taka a photograph of the rings formed at different voltages. The camera's ISO (Image sensitivity option) should be set at 400. Take images for different shutter speed that depends on the brightness of your image. If the image is quite bright, reduce the shutter speed.

Q 9. Quote uncertainties in the voltage values. Which type of probability distribution is associated with voltage values?

Q 10. Take readings for eight different voltages ranging from 2500-4500 V with equal increments. A small strip has already been pasted on the tube, measure the length of the strip. This will act as the calibration for image processing.

Q 11. Place a magnet near the beam path and close to the heater filament. What do you observe?

Q 12. Explain why two rings are appearing on the fluorescent screen instead of discrete points.

6.3 Image processing in ImageJ

We now wish to find out the diameter D and the scattering angle 2θ. For this we will use a specific third-party plugin that works in **ImageJ**. It identifies these points and makes a circle by going through these points and returns the radius as well as the center of curvature for the circle.

Q 13. Bring photographs of the rings from your camera to the computer. A USB cable will help you to do that. These are simple 'jpg' files.

Q 14. Open the **ImageJ** application by selecting it from the Desktop or the Windows menu. Go to **file**→ **open** and select the 'jpg' file of the rings.

Q 15. Convert your image into gray scale. You can do it by selecting **image**→ **type**→ **8 bit** as shown in Figure (6a & b). Go to **Process** and select **Enhance contrast**. Select a value by which the visibility of the rings is reasonably enhanced.

Q 16. First measure the length of the reference line that had already been pasted on the electron diffraction tube. Select **line** from the selection box. This selection will show you the length value in pixels. Divide this value with the manually measured value. This is the calibration that is required for converting pixels to absolute values.

Q 17. Go to **Plugins**→ **threepoint**→ **ThreepointcircularROI**. This selection will ask you to select three points for making a circle. Select three points such that those will completely cover up a circle. When a circle is drawn, a **log window** will be opened showing the value of

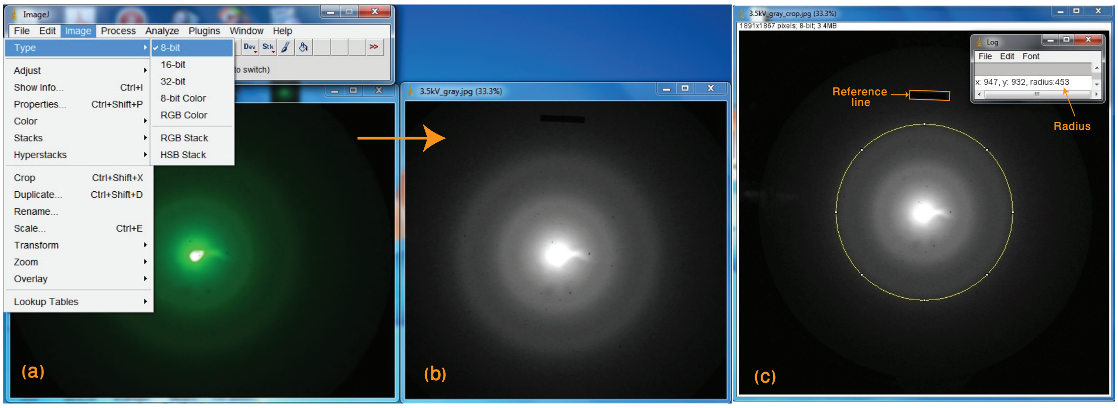

Figure 6: (a), (b) Image processing in ImageJ, and, (c) a way of finding the radius of rings using the plugin **threepoint**.

radius of the circle in pixels. A screenshot of the outcome after applying this plugin is shown in Figure (6c).

Q 18. Measure the radii of the inner and outer rings. It will give you the center of mass (centroid) value.

6.4 Calculations

Q 19. Find the value of the radiuses in millimeters (mm).

Q 20. Plot a graph of the expression on the L.H.S of Equation (17) versus $1/\sqrt{V}$. Perform weighted fit of the straight line and find the values of inter-planar spacing for both rings. Compare your results with the known values given in Table (1).

Q 21. What is uncertainty in your calculated values of d_{10} and d_{11}.

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