CHAPTER 49

LIGHT AND QUANTUM PHYSICS

Thus far we have studied radiation — including not only light but all of the electromagnetic spectrum — through the phenomena of reflection, refraction, interference, diffraction, and polarization, all of which can be understood by treating radiation as a wave. The evidence in support of this wave behavior is overwhelming.

We now move off in a new direction and consider experiments that can be understood only by making quite a different assumption about electromagnetic radiation, namely, that it behaves like a stream of particles.

The concepts of wave and particle are so different that it is hard to understand how light (and other radiation) can be both. In a wave, for example, the energy and momentum are distributed smoothly over the wavefront, while they are concentrated in bundles in a stream of particles. We delay a discussion of this dual nature until Chapter 50. In the meantime, we ask that you not worry about this puzzle and that you consider the compelling experimental evidence that radiation has this particlelike nature. This begins our study of quantum physics, which leads eventually to our understanding of the fundamental structure of matter.

49-1 THERMAL RADIATION

We see most objects by the light that is reflected from them. At high enough temperatures, however, bodies become self-luminous, and we can see them glow in the dark. Incandescent lamp filaments and bonfires (see Fig. 1) are familiar examples. Although we see such objects by the visible light that they emit, we do not have to linger too long near a bonfire to believe that it also emits copiously in the infrared region of the spectrum. It is a curious fact that quantum physics, which dominates our modern view of the world around us, arose from the study — under controlled laboratory conditions — of the radiations emitted by hot objects.

Radiation given off by a body because of its temperature is called *thermal radiation*. All bodies not only emit such radiation but also absorb it from their surroundings. If a body is hotter than its surroundings it emits more radiation than it absorbs and tends to cool. Normally, it will come to thermal equilibrium with its surroundings, a condition in which its rates of absorption and emission of radiation are equal.

The spectrum of the thermal radiation from a hot solid body is continuous, its details depending strongly on the temperature. If we were steadily to raise the temperature of such a body, we would notice two things: (1) the higher the temperature, the more thermal radiation is emitted — at first the body appears dim, then it glows brightly; and (2) the higher the temperature, the shorter is the wavelength of that part of the spectrum radiating most intensely — the predominant color of the hot body shifts from dull red through bright yellow-orange to bluish "white heat." Since the characteristics of its spectrum depend on the temperature, we can estimate the temperature of a hot body, such as a glowing steel ingot or a star, from the radiation it emits. The eye sees chiefly the color corresponding to the most intense emission in the visible range.

The radiation emitted by a hot body depends not only on the temperature but also on the material of which the body is made, its shape, and the nature of its surface. For example, at 2000 K a polished flat tungsten surface emits radiation at a rate of 23.5 W/cm^2; for molybdenum, however, the corresponding rate is 19.2 W/cm^2. In each case the rate increases somewhat if the surface is roughened.

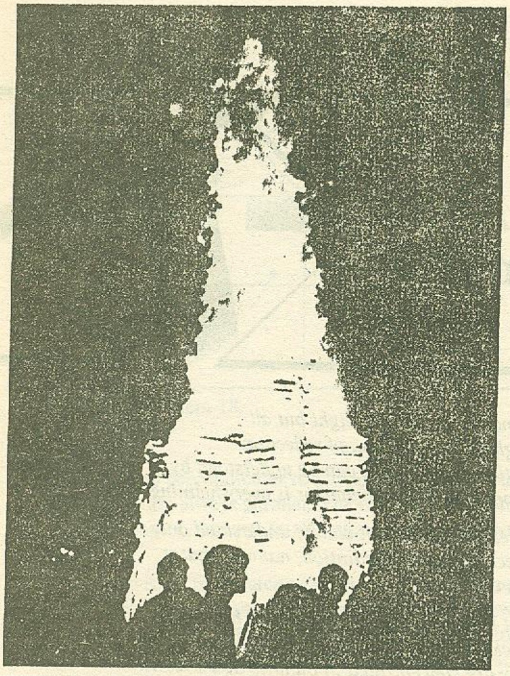

Figure 1 Students contemplating thermal radiation. The study of such radiation, under controlled laboratory conditions, laid the foundations for modern quantum mechanics.

Other differences appear if we measure the distribution in wavelength of the emitted radiation. Such details make it hard to understand thermal radiation in terms of simpler physical ideas; it reminds us of the complications that arise in trying to understand the properties of real gases in terms of a simple atomic model. The "gas problem" was managed by introducing the notion of an ideal gas. In much the same spirit, the "radiation problem" can be made manageable by introducing an "ideal radiator" for which the spectrum of the emitted thermal radiation depends *only* on the temperature of the radiator and not on the material, the nature of the surface, or other factors.

We can make such an ideal radiator by forming a cavity within a body, the walls of the cavity being held at a uniform temperature. We must pierce a small hole through the wall so that a sample of the radiation inside the cavity can escape into the laboratory to be examined. It turns out that such thermal radiation, called *cavity radiation,** has a very simple spectrum whose nature is indeed determined only by the temperature of the walls and not in any way by the material of the cavity, its shape, or its size. Cavity radiation (radiation in a box) helps us to understand the nature of thermal radiation, just as the ideal gas (matter in a box) helped us to understand matter in its gaseous form.

Figure 2 shows a cavity radiator made of a thin-walled cylindrical tungsten tube about 1 mm in diameter and heated to incandescence by passing a current through it. A small hole has been drilled in its wall. It is clear from the figure that the radiation emerging from this hole is much more intense than that from the outer wall of the cavity, even though the temperatures of the outer and inner walls are more or less equal.

There are three interrelated properties of cavity radiation — all well verified in the laboratory — that any theory of cavity radiation must explain.

1. *The Stefan–Boltzmann law.* The total radiated power per unit area of the cavity aperture, summed over all wavelengths, is called its *radiant intensity* $I(T)$ and is related to the temperature by

$$I(T) = \sigma T^4, \tag{1}$$

in which $\sigma\,(= 5.670 \times 10^{-8}\ \mathrm{W/m^2 \cdot K^4})$ is a universal constant, called the *Stefan–Boltzmann constant.* Ordinary hot objects always radiate less efficiently than do cavity radiators. We express this by generalizing Eq. 1 to

$$I(T) = \epsilon \sigma T^4, \tag{2}$$

in which ϵ, a dimensionless quantity, is called the *emissivity* of the surface material. For a cavity radiator, $\epsilon = 1$, but for the surfaces of ordinary objects, the emissivity is always less than unity and is almost always a function of temperature.

2. *The spectral radiancy.* The *spectral radiancy* $R(\lambda)$ tells us how the intensity of the cavity radiation varies with

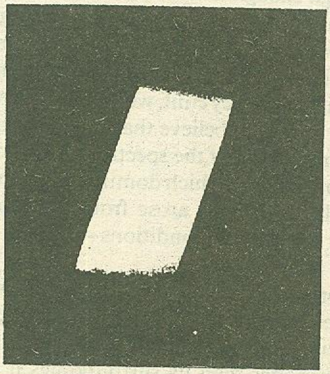

Figure 2 An incandescent tungsten tube with a small hole drilled in its wall. The radiation emerging from the hole is cavity radiation.

* Also known as *black-body radiation,* because an ideal black body (one that absorbs all radiation incident on it) would emit the same type of radiation. We assume that the dimensions of the cavity are much greater than the wavelength of the radiation.

wavelength for a given temperature. It is defined so that the product $R(\lambda)\,d\lambda$ gives the radiated power per unit area that lies in the wavelength band that extends from λ to $\lambda + d\lambda$. $R(\lambda)$ is a statistical distribution function of the same type we considered in Chapter 24. We can find the radiant intensity $I(T)$ for any temperature by adding up (that is, by integrating) the spectral radiancy over the complete range of wavelengths. Thus

$$I(T) = \int_0^\infty R(\lambda)\,d\lambda \qquad \text{(fixed } T\text{)}. \qquad (3)$$

Figure 3 shows the spectral radiancy for cavity radiation at four selected temperatures. Equation 3 shows that we can interpret the radiant intensity $I(T)$ as the area under the appropriate spectral radiancy curve. We see from the figure that, as the temperature increases, so does this area and thus the radiant intensity, as Eq. 1 predicts.

3. *The Wien displacement law.* We can see from the spectral radiancy curves of Fig. 3 that λ_{max}, the wavelength at which the spectral radiancy is a maximum, decreases as the temperature increases. Wilhelm Wien (German, 1864–1928) deduced that λ_{max} varies as $1/T$ and that the product $\lambda_{max}T$ is a universal constant. Its measured value is

$$\lambda_{max}T = 2898\ \mu\text{m}\cdot\text{K}. \qquad (4)$$

This relationship is called the *Wien displacement law;* Wien was awarded the 1911 Nobel prize in physics for his research into thermal radiation.

Sample Problem 1 How hot is a star? The "surfaces" of stars are not sharp boundaries like the surface of the Earth. Most of the radiation that a star emits is in thermal equilibrium with the hot gases that make up the star's outer layers. Without too much error, then, we can treat starlight as cavity radiation. Here are the wavelengths at which the spectral radiancies of three stars have their maximum values:

Star	λ_{max}	*Appearance*
Sirius	240 nm	Blue-white
Sun	500 nm	Yellow
Betelgeuse	850 nm	Red

(*a*) What are the surface temperatures of these stars? (*b*) What are the radiant intensities of these three stars? (*c*) The radius *r* of the Sun is 7.0×10^8 m and that of Betelgeuse is over 500 times larger, or 4.0×10^{11} m. What is the total radiated power output (that is, the *luminosity L*) of these stars?

Solution (*a*) From Eq. 4 we find, for Sirius,

$$T = \frac{2898\ \mu\text{m}\cdot\text{K}}{\lambda_{max}} = \left(\frac{2898\ \mu\text{m}\cdot\text{K}}{240\ \text{nm}}\right)\left(\frac{1000\ \text{nm}}{1\ \mu\text{m}}\right) = 12{,}000\ \text{K}.$$

The temperatures for the Sun and for Betelgeuse work out in the same way to be 5800 K and 3400 K, respectively. At 5800 K, most of the radiation from the Sun's surface lies within the visible region of the spectrum. This suggests that over ages of

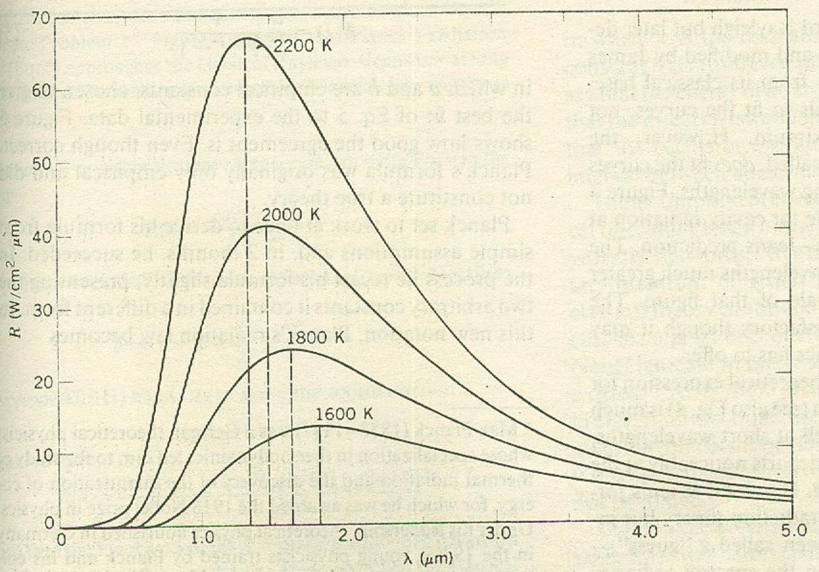

Figure 3 Spectral radiancy curves for cavity radiation at four selected temperatures. Note that as the temperature increases, the wavelength of the maximum spectral radiancy shifts to lower values.

evolution, eyes have adapted to the Sun to become most sensitive to those wavelengths that it radiates most intensely.

(*b*) For Sirius we have, from the Stefan–Boltzmann law (Eq. 1)

$$I = \sigma T^4 = (5.67 \times 10^{-8}\ \text{W/m}^2\cdot\text{K}^4)(12{,}000\ \text{K})^4 = 1.2 \times 10^9\ \text{W/m}^2.$$

The radiant intensities for the Sun and for Betelgeuse work out to be $6.4 \times 10^7\ \text{W/m}^2$ and $7.7 \times 10^6\ \text{W/m}^2$, respectively.

(*c*) We find the luminosity of a star by multiplying its radiant intensity by its surface area. Thus, for the Sun,

$$L = I(4\pi r^2) = (6.4 \times 10^7\ \text{W/m}^2)(4\pi)(7.0 \times 10^8\ \text{m})^2 = 3.9 \times 10^{26}\ \text{W}.$$

For Betelgeuse the luminosity works out to be 1.5×10^{31} W, about 38,000 times larger. The enormous size of Betelgeuse, which is classified as a "red giant," much more than makes up for the relatively low radiant intensity associated with its low surface temperature.

The colors of stars are not strikingly apparent to the average observer because the retinal cones, which are responsible for color vision, do not function well in dim light. If this were not so, the night sky would be spangled with color.

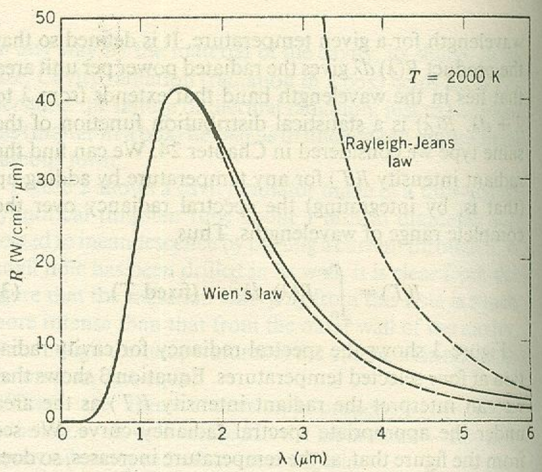

Figure 4 The solid curve shows the experimental spectral radiancy for radiation from a cavity at 2000 K. The predictions of the classical Rayleigh–Jeans law and Wien's law are shown as dashed lines. The shaded vertical bar represents the range of visible wavelengths.

49-2 PLANCK'S RADIATION LAW

Is there a simple formula, derivable from basic principles, that fits the experimental radiancy curves of Fig. 3? In September 1900 there were two suggested formulas, neither of which could fit the curves over the entire range of wavelengths.

The first, due originally to Lord Rayleigh but later derived independently by Einstein and modified by James Jeans, was developed rigorously from its classical base. Unfortunately, it completely fails to fit the curves, not even passing through a maximum. However, the Rayleigh–Jeans formula, as it is called, *does* fit the curves quite well in the limit of very long wavelengths. Figure 4 shows the spectral radiancy curve for cavity radiation at 2000 K, along with the Rayleigh–Jeans prediction. The good fit we speak of occurs for wavelengths much greater than 50 μm, far beyond the scale of that figure. The Rayleigh–Jeans formula, unsatisfactory though it may be, is the best that classical physics has to offer.

Wilhelm Wien also derived a theoretical expression for the spectral radiancy. His formula (see also Fig. 4) is much better. It fits the curves quite well at short wavelengths, passes through a maximum, but departs noticeably at the long-wavelength end of the scale. However, Wien's formula was not based on classical radiation theory but instead on a conjecture—it has been called a "guess"—that there is an analogy between the spectral radiancy curves and the Maxwell speed distribution curves for the molecules of an ideal gas.

Thus we have two formulas, one agreeing with experiment at long wavelengths and the other at short wavelengths. Max Planck,* seeking to reconcile these two radiation laws, made an inspired interpolation between them that turned out to fit the data at *all* wavelengths. Planck's radiation formula, announced to the Berlin Physical Society on October 19, 1900, is

$$R(\lambda) = \frac{a}{\lambda^5}\frac{1}{e^{b/\lambda T} - 1}, \qquad (5)$$

in which a and b are empirical constants, chosen to give the best fit of Eq. 5 to the experimental data. Figure 5 shows how good the agreement is. Even though correct, Planck's formula was originally only empirical and did not constitute a true theory.

Planck set to work at once to derive his formula from simple assumptions and, in 2 months, he succeeded. In the process he recast his formula slightly, presenting the two arbitrary constants it contained in a different form. In this new notation, Planck's radiation law becomes

* Max Planck (1858–1947) was a German theoretical physicist whose specialization in thermodynamics led him to the study of thermal radiation and the discovery of the quantization of energy, for which he was awarded the 1918 Nobel prize in physics. Under his leadership, theoretical physics flourished in Germany in the 1920s; young physicists trained by Planck and his colleagues produced a complete mathematical formulation of the quantum theory. In his later life, Planck wrote extensively on religious and philosophical issues.

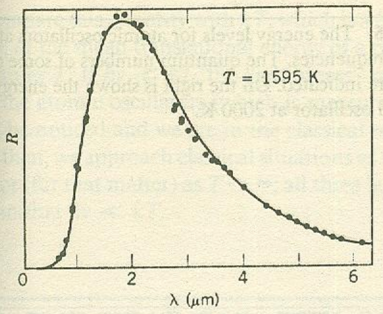

Figure 5 Planck's radiation law fitted to experimental data for a cavity radiator at 1595 K.

$$R(\lambda) = \frac{2\pi c^2 h}{\lambda^5} \frac{1}{e^{hc/\lambda kT} - 1}. \qquad (6)$$

The two adjustable constants a and b in Eq. 5 are here replaced by quantities involving two different constants, the *Boltzmann constant k* (see Section 23-1) and a new constant, now called the *Planck constant h;* the quantity c is the speed of light.

By fitting Eq. 6 to the experimental data, Planck could find values for k and h. His values were within a percent or so of their presently accepted values, which are

$$k = 1.381 \times 10^{-23}\ \text{J/K}$$

and

$$h = 6.626 \times 10^{-34}\ \text{J}\cdot\text{s}.$$

Sample Problem 2 Figure 4 suggests that Planck's radiation law (Eq. 6) approaches the classical Rayleigh–Jeans law at long wavelengths. To what expression does Planck's law reduce as $\lambda \to \infty$?

Solution For algebraic convenience, we can write Eq. 6 in the form

$$R(\lambda) = \frac{2\pi c^2 h}{\lambda^5} \frac{1}{e^x - 1},$$

in which $x = hc/\lambda kT$. As $\lambda \to \infty$, we see that $x \to 0$. Recalling that

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$

(see Appendix H) allows us to make the approximation

$$e^x - 1 \approx x.$$

Thus we have

$$R(\lambda) \approx \frac{2\pi c^2 h}{\lambda^5} \frac{1}{x} = \frac{2\pi c^2 h}{\lambda^5} \left(\frac{\lambda kT}{hc}\right) = \frac{2\pi c k T}{\lambda^4}.$$

Note that the Planck constant h, a sure identifier of a quantum formula, conveniently cancels out as we approach the classical long-wave limit. The above result, in fact, is precisely the classical Rayleigh–Jeans expression for the spectral radiancy.

49-3 THE QUANTIZATION OF ENERGY

We turn now to the assumptions made by Planck in deriving his radiation law and to the significance of the constant h that appears in it. These assumptions and their consequences were not immediately clear to Planck's contemporaries or for that matter (as he confirmed later) to Planck himself. In what follows we describe the situation as it appeared some 6 or 7 years after Planck first advanced his theory. It seems to be true that the basic premise underlying Planck's radiation law—the quantization of energy—was not understood at any earlier date.

Planck derived his radiation law by analyzing the interplay between the radiation in the cavity volume and the atoms that make up the cavity walls. He assumed that these atoms behave like tiny oscillators, each with a characteristic frequency of oscillation. These oscillators radiate energy into the cavity and absorb energy from it. It should be possible to deduce the characteristics of the cavity radiation from the characteristics of the oscillators that generate it.

Classically, the energy of these tiny oscillators is a smoothly continuous variable. We certainly assume this for large-scale oscillators such as pendulums or mass–spring systems. It turns out, however, that in order to derive Planck's radiation law it is necessary to make a radical assumption; namely, *atomic oscillators may not emit or absorb any energy E but only energies chosen from a discrete set, defined by*

$$E = nh\nu, \qquad n = 1, 2, 3, \ldots, \qquad (7)$$

in which ν is the oscillator frequency. Here the Planck constant h is introduced into physics for the first time. We say that the energy of an atomic oscillator is *quantized* and that the integer n is a *quantum number*. Equation 7 tells us that the oscillator energy levels are evenly spaced, the interval between adjacent levels being $h\nu$; see Fig. 6.

The assumption of energy quantization is indeed a radical one, and Planck himself resisted accepting it for many years. In his words, "My futile attempts to fit the elementary quantum of action [that is, h] somehow into the classical theory continued for a number of years, and they cost me a great deal of effort." Max von Laue, the 1914 Nobel laureate in physics and a student of Planck's, has written: "After 1900 Planck strove for many years to bridge, if not to close, the gap between the older and the quantum physics. The effort failed, but it had value in that it provided the most convincing proof that the two could not be joined."

Let us look at energy quantization in the context of a large-scale oscillator such as a swinging pendulum. Our experience suggests that a pendulum can oscillate with *any* reasonable total energy and not only with certain selected energies. As friction causes the pendulum ampli-