2.5 THE HALL EFFECT AND HALL DEVICES

An important phenomenon that we can comfortably explain using the "electron as a particle" concept is the Hall effect, which is illustrated in Figure 2.16. When we apply a magnetic field in a perpendicular direction to the applied field (which is driving the current), we find there is a transverse field in the sample that is perpendicular to the direction of both the applied field $\mathcal{E}_x$ and the magnetic field B_z, that is, in the y direction. Putting a voltmeter across the sample, as in Figure 2.16, gives a voltage reading V_H. The applied field $\mathcal{E}_x$ drives a current J_x in the sample. The electrons move in the $-x$ direction, with a drift velocity v_{dx}. Because of the magnetic field, there is a force (called the **Lorentz force**) acting on each electron and given by $F_y = -ev_{dx}B_z$. The direction of this Lorentz force is the $-y$ direction, which we can show by applying the corkscrew rule, because, in vector notation, the force **F** acting on a charge q moving with a velocity **v** in a magnetic field **B** is given through the vector product

$$\mathbf{F} = q\mathbf{v} \times \mathbf{B} \tag{2.29}$$

Lorentz force

All moving charges experience the Lorentz force in Equation 2.29 as shown schematically in Figure 2.17. In our example of a metal in Figure 2.16, this Lorentz force is the $-y$ direction, so it pushes the electrons downward, as a result of which there is a negative charge accumulation near the bottom of the sample and a positive charge near the top of the sample, due to exposed metal ions (*e.g.*, Cu^+).

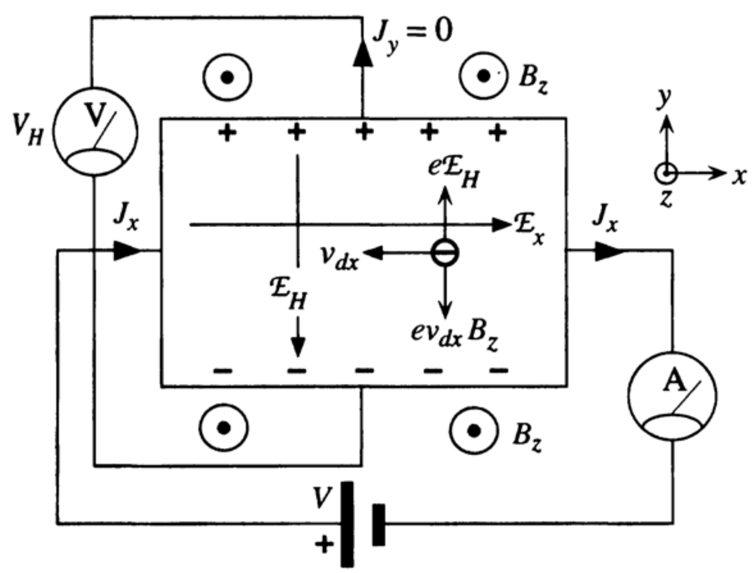


Figure 2.16 Illustration of the Hall effect.
The z direction is out of the plane of the paper. The externally applied magnetic field is along the z direction.

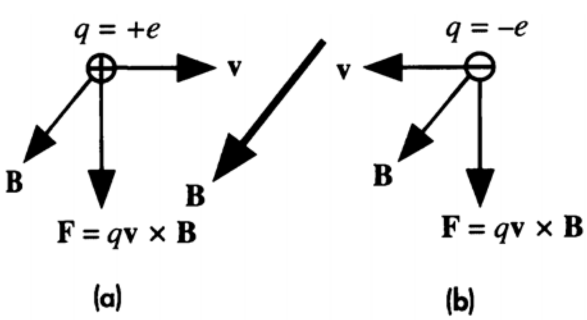


Figure 2.17 A moving charge experiences a Lorentz force in a magnetic field.
(a) A positive charge moving in the x direction experiences a force downward.
(b) A negative charge moving in the $-x$ direction also experiences a force downward.

The accumulation of electrons near the bottom results in an internal electric field $\mathcal{E}_H$ in the $-y$ direction. This is called the **Hall field** and gives rise to a Hall voltage V_H between the top and bottom of the sample. Electron accumulation continues until the increase in $\mathcal{E}_H$ is sufficient to stop the further accumulation of electrons. When this happens, the magnetic-field force $ev_{dx}B_z$ that pushes the electrons down just balances the force $e\mathcal{E}_H$ that prevents further accumulation. Therefore, in the steady state,

$$e\mathcal{E}_H = ev_{dx}B_z$$

However, $J_x = env_{dx}$. Therefore, we can substitute for v_{dx} to obtain $e\mathcal{E}_H = J_x B_z / n$ or

$$\mathcal{E}_H = \left(\frac{1}{en}\right) J_x B_z \qquad [2.30]$$

A useful parameter called the **Hall coefficient** R_H is defined as

Definition of Hall coefficient

$$R_H = \frac{\mathcal{E}_y}{J_x B_z} \qquad [2.31]$$

The quantity R_H measures the resulting Hall field, along y, per unit transverse applied current and magnetic field. The larger R_H, the greater $\mathcal{E}_y$ for a given J_x and B_z. Therefore, R_H is a gauge of the magnitude of the Hall effect. A comparison of Equations 2.30 and 2.31 shows that for metals,

Hall coefficient for electron conduction

$$R_H = -\frac{1}{en} \qquad [2.32]$$

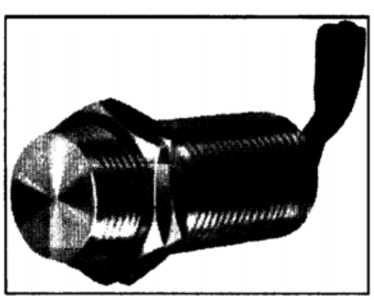
The reason for the negative sign is that $\mathcal{E}_H = -\mathcal{E}_y$, which means that $\mathcal{E}_H$ is in the $-y$ direction.

Inasmuch as R_H depends inversely on the free electron concentration, its value in metals is much less than that in semiconductors. In fact, Hall-effect devices (such as magnetometers) always employ a semiconductor material, simply because the R_H is larger. Table 2.4 lists the Hall coefficients of various metals. Note that this is negative

Table 2.4 Hall coefficient and Hall mobility ($\mu_H = |\sigma R_H|$) of selected metals

Metal	n [m^{-3}] ($\times 10^{28}$)	R_H (Experimental) [$m^3\,A^{-1}\,s^{-1}$] ($\times 10^{-11}$)	$\mu_H = \lvert\sigma R_H\rvert$ [$m^2\,V^{-1}\,s^{-1}$] ($\times 10^{-4}$)
Ag	5.85	−9.0	57
Al	18.06	−3.5	13
Au	5.90	−7.2	31
Be	24.2	+3.4	?
Cu	8.45	−5.5	32
Ga	15.3	−6.3	3.6
In	11.49	−2.4	2.9
Mg	8.60	−9.4	22
Na	2.56	−25	53

SOURCES: Data from various sources, including C. Nording and J. Osterman, *Physics Handbook*, Bromley, England: Chartwell-Bratt Ltd., 1982.



Magnetically operated Hall-effect position sensor as available from Micro Switch.

for most metals, although a few metals exhibit a positive Hall coefficient (see Be in Table 2.4). The reasons for the latter involve the band theory of solids, which we will discuss in Chapter 4.

Since the Hall voltage depends on the product of two quantities, the current density J_x and the transverse applied magnetic field B_z, we see that the effect naturally multiplies two independently variable quantities. Therefore, it provides a means of carrying out a multiplication process. One obvious application is measuring the power dissipated in a load, where the load current and voltage are multiplied. There are many instances when it is necessary to measure magnetic fields, and the Hall effect is ideally suited to such applications. Commercial Hall-effect magnetometers can measure magnetic fields as low as 10 nT, which should be compared to the earth's magnetic field of ~50 μT. Depending on the application, manufacturers use different semiconductors to obtain the desired sensitivity. Hall-effect semiconductor devices are generally inexpensive, small, and reliable. Typical commercial, linear Hall-effect sensor devices are capable of providing a Hall voltage of ~10 mV per mT of applied magnetic field.

The Hall effect is also widely used in magnetically actuated electronic switches. The application of a magnetic field, say from a magnet, results in a Hall voltage that is amplified to trigger an electronic switch. The switches invariably use Si and are readily available from various companies. Hall-effect electronic switches are used as noncontacting keyboard and panel switches that last almost forever, as they have no mechanical contact assembly. Another advantage is that the electrical contact is "bounce" free. There are a variety of interesting applications for Hall-effect switches, ranging from ignition systems, to speed controls, position detectors, alignment controls, brushless dc motor commutators, etc.

EXAMPLE 2.16

HALL-EFFECT WATTMETER The Hall effect can be used to implement a wattmeter to measure electrical power dissipated in a load. The schematic sketch of the Hall-effect wattmeter is shown in Figure 2.18, where the Hall-effect sample is typically a semiconductor material (usually Si). The load current I_L passes through two coils, which are called current coils and are shown as C in Figure 2.18. These coils set up a magnetic field B_z such that $B_z \propto I_L$. The Hall-effect sample is positioned in this field between the coils. The voltage V_L across the load drives a current

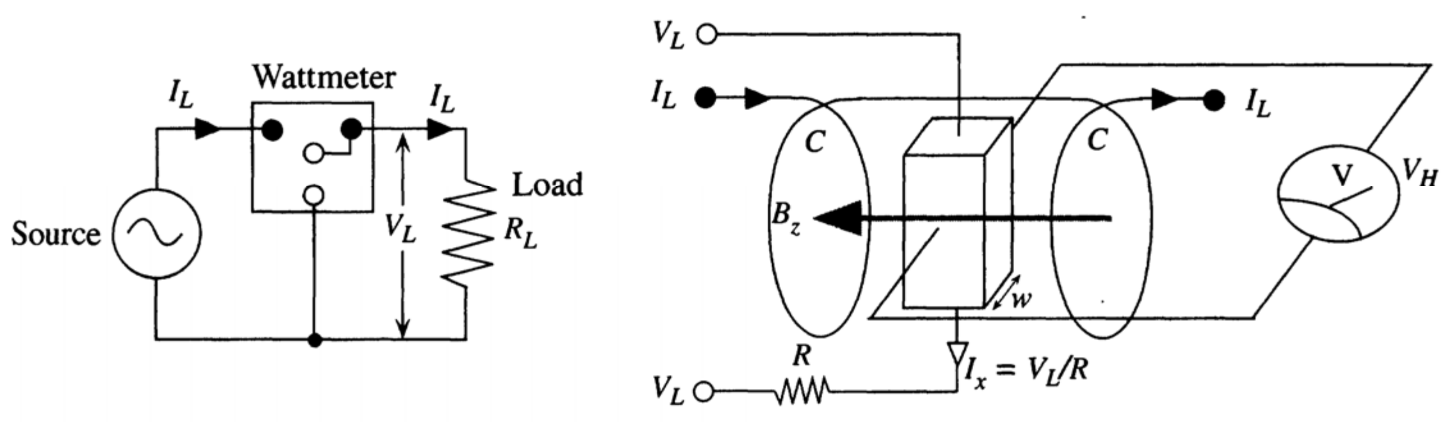


Figure 2.18 Wattmeter based on the Hall effect.
Load voltage and load current have L as subscript; C denotes the current coils for setting up a magnetic field through the Hall-effect sample (semiconductor).

$I_x = V_L/R$ through the sample, where R is a series resistance that is much larger than the resistance of the sample and that of the load. Normally, the current I_x is very small and negligible compared to the load current. If w is the width of the sample, then the measured Hall voltage is

$$V_H = w\mathcal{E}_H = wR_H J_x B_z \propto I_x B_z \propto V_L I_L$$

which is the electrical power dissipated in the load. The voltmeter that measures V_H can now be calibrated to read directly the power dissipated in the load.

EXAMPLE 2.17

HALL MOBILITY Show that if R_H is the Hall coefficient and σ is the conductivity of a metal, then the drift mobility of the conduction electrons is given by

$$\mu_d = |\sigma R_H| \qquad [2.33]$$

The Hall coefficient and conductivity of copper at 300 K have been measured to be -0.55×10^{-10} m^3 A^{-1} s^{-1} and 5.9×10^7 Ω^{-1} m^{-1}, respectively. Calculate the drift mobility of electrons in copper.

SOLUTION

Consider the expression for

$$R_H = \frac{-1}{en}$$

Since the conductivity is given by $\sigma = en\mu_d$, we can substitute for en to obtain

$$R_H = \frac{-\mu_d}{\sigma} \qquad \text{or} \qquad \mu_d = -R_H\sigma$$

which is Equation 2.33. The drift mobility can thus be determined from R_H and σ.

The product of σ and R_H is called the **Hall mobility** μ_H. Some values for the Hall mobility of electrons in various metals are listed in Table 2.4. From the expression in Equation 2.33, we get

$$\mu_d = -(-0.55 \times 10^{-10} \text{ m}^3 \text{ A}^{-1} \text{ s}^{-1})(5.9 \times 10^7 \ \Omega^{-1} \text{ m}^{-1}) = 3.2 \times 10^{-3} \text{ m}^2 \text{ V}^{-1} \text{ s}^{-1}$$

It should be mentioned that Equation 2.33 is an oversimplification. The actual relationship involves a numerical factor that multiplies the right term in Equation 2.33. The factor depends on the charge carrier scattering mechanism that controls the drift mobility.

EXAMPLE 2.18

CONDUCTION ELECTRON CONCENTRATION FROM THE HALL EFFECT Using the electron drift mobility from Hall-effect measurements (Table 2.4), calculate the concentration of conduction electrons in copper, and then determine the average number of electrons contributed to the free electron gas per copper atom in the solid.

SOLUTION

The number of conduction electrons is given by $n = \sigma/e\mu_d$. The conductivity of copper is 5.9×10^7 Ω^{-1} m^{-1}, whereas from Table 2.4, the electron drift mobility is 3.2×10^{-3} m^2 V^{-1} s^{-1}. So,

$$n = \frac{(5.9 \times 10^7 \ \Omega^{-1} \text{ m}^{-1})}{[(1.6 \times 10^{-19} \text{ C})(3.2 \times 10^{-3} \text{ m}^2 \text{ V}^{-1} \text{ s}^{-1})]} = 1.15 \times 10^{29} \text{ m}^{-3}$$

Since the concentration of copper atoms is 8.5×10^{28} m^{-3}, the average number of electrons contributed per atom is $(1.15 \times 10^{29} \text{ m}^{-3})/(8.5 \times 10^{28} \text{ m}^{-3}) \approx 1.36$.