# Verification of de Broglie's Hypothesis by Electron Diffraction from Graphite

**Hibah Faisal**[a]

[a] Physlab, Lahore University of Management Sciences (LUMS)

**This experiment successfully verified de Broglie's hypothesis by showing that electrons behave like waves when diffracted by a polycrystalline graphite target. We accelerated electrons using voltages between 2800 V and 4200 V, which produced concentric diffraction rings. We used ImageJ software to precisely measure the ring diameters. By linearizing the data based on the geometric-diffraction theory, we performed fits to determine the interplanar spacings ($d$). Our results were $d_{\text{inner}} = (0.294 \pm 0.026)$ nm and $d_{\text{outer}} = (0.162 \pm 0.008)$ nm. The strong linear trend confirmed the physical principle. The differences between our final results and the accepted values are mainly attributed to systematic errors in the equipment's setup.**

Electron diffraction | de Broglie wavelength | graphite | Bragg's law | lattice spacing | systematic error

## Introduction and Theory

The de Broglie hypothesis is the cornerstone of the matter-wave theory, stating that every particle has a corresponding wavelength $\lambda$ inversely proportional to its momentum $p$: $\lambda = h/p$. For an electron of mass $m_e$ and charge $e$ accelerated through a potential difference $V_0$, its momentum is $p = \sqrt{2m_e e V_0}$ . The electron's wavelength is thus given by:

$$\lambda = \frac{h}{\sqrt{2m_e e V_0}} = \frac{K}{\sqrt{V_0}},$$

where $K = 1.23 \times 10^{-9}\,\text{m} \cdot \text{V}^{1/2}$ is the collection of fundamental constants. This relationship was first experimentally confirmed by Davisson and Germer in their classic 1927 electron diffraction experiment [2].

The electron wave is scattered by the periodic structure of the polycrystalline graphite foil. Diffraction occurs when the Bragg condition is satisfied: $2d\sin\theta = n\lambda$. Since the target is polycrystalline, for any given $\lambda$, a subset of crystals is correctly aligned to produce constructive interference at an angle $2\theta$. This creates the sharp, circular, concentric rings observed on the fluorescent screen [3]. The two prominent rings correspond to the reflections from the ($d10$) and ($d11$) crystal planes (Figure 1) [1].

The geometry of the apparatus is shown in Figure 2 (spherical screen radius $R$, ring diameter $D$, and target distance $L$) [1]. Equating this with the de Broglie relation and rearranging into a linear form gives expression (1):

$$\frac{D}{L - R + \sqrt{R^2 - (D/2)^2}} = \frac{2K}{d} \cdot \frac{1}{\sqrt{V_0}}. \qquad [1]$$

The resultant linear fit of the LHS versus the RHS has a slope $m$, which is used to calculate the interplanar spacing $d = 2K/m$. The goal of this experiment is to verify de Broglie's hypothesis by confirming the linear dependence, and to use the slope $m$ to determine the characteristic interplanar spacings of graphite ($d_{10}$ and $d_{11}$).

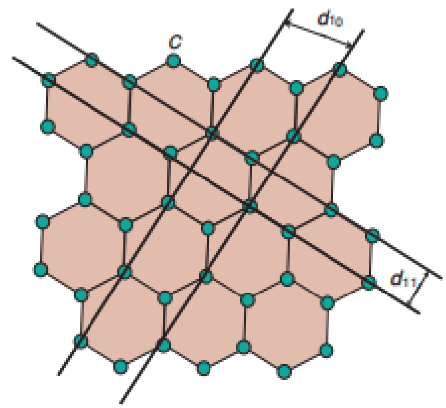


**Fig. 1.** The hexagonal crystal structure of graphite showing the interplanar spacings $d_{10}$ and $d_{11}$ for the diffraction rings [1].

**Table 1. Accepted Interplanar Spacings for Graphite [1]**

| Crystal Plane Index | Spacing Symbol | Accepted Value (nm) |
|---|---|---|
| (10) Reflection | $d_{10}$ | 0.213 |
| (11) Reflection | $d_{11}$ | 0.123 |

## Methods

**Apparatus.** The apparatus (shown in Figure 3) used in this experiment included the electron diffraction tube 1013885 (3B Scientific), a high voltage DC power supply 0-5 kV U33010 (3B Scientific), a tube holder 1008507 (3B Scientific), an ammeter, a meter rule, connecting wires, and the ImageJ software.

**Procedure.** The accelerating voltage was varied from 2800 V to 4200 V in increments of 200 V. At each setting, digital

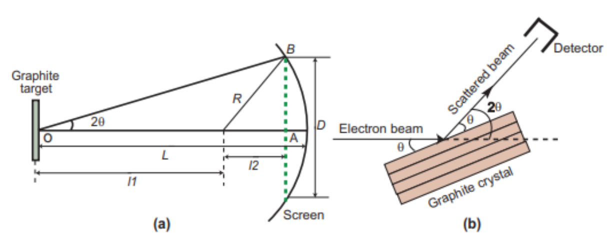


**Fig. 2.** Diagram showing the geometry of the vacuum tube. $D$ is the ring diameter, $R$ is the screen's radius of curvature, $L$ is the target-to-screen distance, and $2\theta$ is the scattering angle [1].

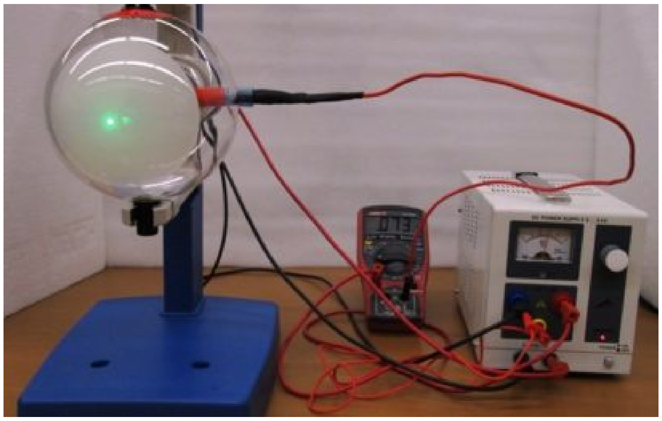

**Fig. 3.** Photograph of the electron diffraction tube apparatus (taken from Physlab website), showing the electron gun, graphite target and fluorescent screen.

photographs of the diffraction pattern were taken after setting up a tripod with a phone camera. Five manual measurements of the inner and outer ring diameters ($D$) were recorded using ImageJ. The raw data was then linearly fitting using expression (1) in python to plot the graph (Figure 5).

## Results

**Table 2. Inner-ring diameter readings (cm) and their mean**

| Voltage (V) | Diameters (cm) | | | | | Mean (cm) |
|---|---|---|---|---|---|---|
| $2800 \pm 41$ | 2.45 | 2.42 | 2.49 | 2.40 | 2.44 | 2.44 |
| $3000 \pm 41$ | 2.42 | 2.44 | 2.38 | 2.42 | 2.41 | 2.41 |
| $3200 \pm 41$ | 2.39 | 2.35 | 2.36 | 2.38 | 2.39 | 2.37 |
| $3400 \pm 41$ | 2.35 | 2.33 | 2.33 | 2.32 | 2.32 | 2.33 |
| $3600 \pm 41$ | 2.28 | 2.29 | 2.26 | 2.28 | 2.29 | 2.28 |
| $3800 \pm 41$ | 2.20 | 2.22 | 2.22 | 2.21 | 2.23 | 2.22 |
| $4000 \pm 41$ | 2.17 | 2.11 | 2.13 | 2.15 | 2.15 | 2.14 |
| $4200 \pm 41$ | 2.11 | 2.07 | 2.10 | 2.09 | 2.10 | 2.09 |

**Table 3. Outer-ring diameter readings (cm) and their mean**

| Voltage (V) | Diameters (cm) | | | | | Mean (cm) |
|---|---|---|---|---|---|---|
| $2800 \pm 41$ | 4.59 | 4.62 | 4.53 | 4.66 | 4.53 | 4.59 |
| $3000 \pm 41$ | 4.40 | 4.42 | 4.50 | 4.42 | 4.43 | 4.43 |
| $3200 \pm 41$ | 4.37 | 4.35 | 4.39 | 4.35 | 4.36 | 4.36 |
| $3400 \pm 41$ | 4.28 | 4.28 | 4.27 | 4.26 | 4.26 | 4.27 |
| $3600 \pm 41$ | 4.22 | 4.22 | 4.23 | 4.24 | 4.24 | 4.23 |
| $3800 \pm 41$ | 4.15 | 4.15 | 4.12 | 4.15 | 4.14 | 4.14 |
| $4000 \pm 41$ | 4.03 | 4.02 | 4.00 | 4.01 | 4.03 | 4.02 |
| $4200 \pm 41$ | 3.96 | 3.96 | 3.93 | 3.95 | 3.96 | 3.95 |

### Raw Data.

**Analysis and Results.** The ring diameters presented in the tables were measured from digital image processing. A plugin ('threepoint') was applied to digitally fit a circle to the concentric rings. These pixel measurements were then converted to physical units (cm) using a known reference distance (2cm, measured using a piece of tape on the apparatus) to establish a scale factor. The mean diameter $\bar{D}$ for each voltage was calculated and used to compute the linearised variables on the LHS and RHS for the plot.

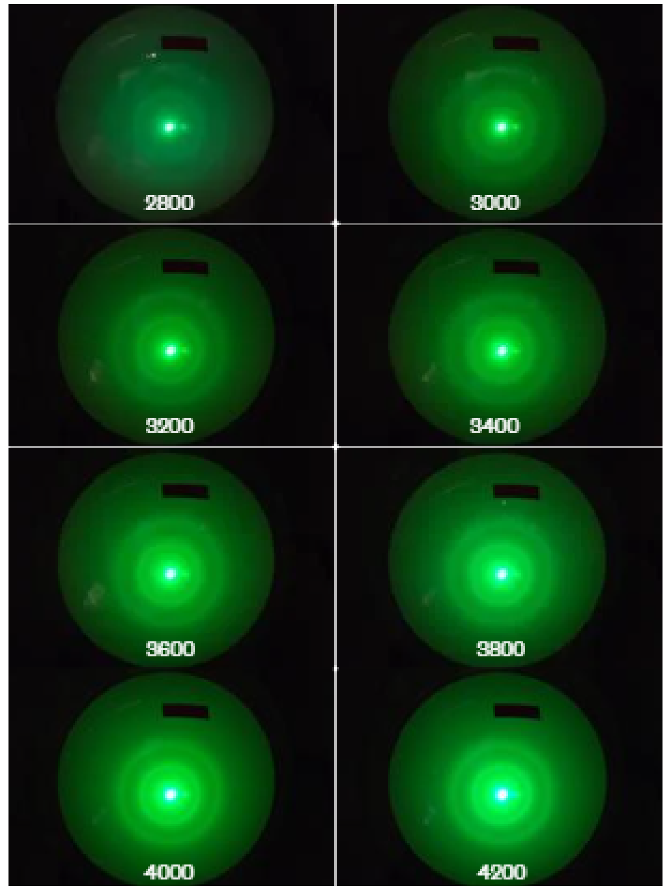


**Fig. 4.** Concentric electron diffraction rings produced by electrons scattering off the polycrystalline graphite target at different voltages (mentioned on each photograph). The rings are used to measure $d$.

Linear fits to $y$ vs. $x = 1/\sqrt{V_0}$ yielded the following results. For the inner ring, the slope was $m_{\text{in}} = 8.378787 \pm 0.746146$, which corresponds to an interplanar spacing of $d_{\text{inner}} = (0.294 \pm 0.026)$ nm. For the outer ring, the slope was $m_{\text{out}} = 15.199480 \pm 0.751350$, which corresponds to an interplanar spacing of $d_{\text{outer}} = (0.162 \pm 0.008)$ nm.

## Discussion

The linear correlation in Figure 5 confirms the fundamental principle of matter waves: electron wavelength is proportional to $1/\sqrt{V_0}$, verifying de Broglie's hypothesis. We also compared our measured interplanar spacings to accepted values for graphite's (10) and (11) crystal planes: $d_{10} = 0.213$ nm and $d_{11} = 0.123$ nm [1].

Our results show: the inner ring result is $0.162 \pm 0.008$ nm (4.94% uncertainty) versus the accepted $d_{11} = 0.123$ nm, and the outer ring result is $0.294 \pm 0.026$ nm (8.91% uncertainty) versus the accepted $d_{10} = 0.213$ nm.

**Uncertainties** We calculated total uncertainty by combining Type A and Type B errors: Type B sources were voltage ($\Delta V = 200/(2\sqrt{6})$ V) and screen distance ($\Delta L = \pm 2$ mm), while Type A uncertainty came from the standard deviation of

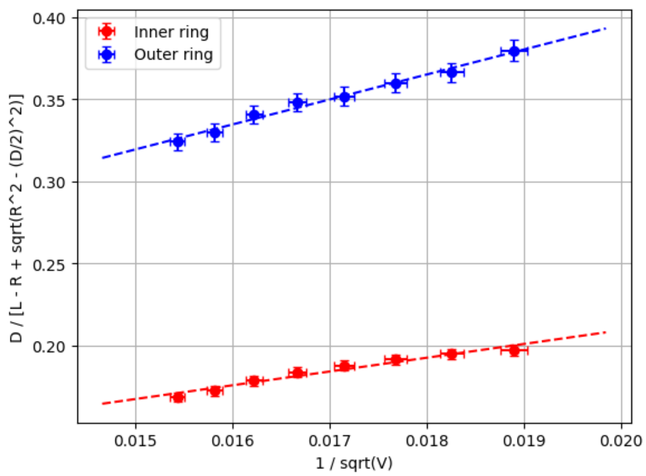


**Fig. 5.** Plot of the geometric diffraction term $(y)$ vs. $1/\sqrt{V_0}$ $(x)$ for the inner and outer rings with linear fits.

five diameter measurements at each voltage. We propagated these through our analysis, where diameter errors became $y$-axis error bars, which gave us the slope uncertainty $\Delta m$ from the fit; since $d = 2K/m$, we found $\Delta d = d \cdot (\Delta m/m)$.

**Sources of Error** The biggest systematic error came from uncertainties in $R$ (screen radius) and $L$ (screen distance), as small errors in these values shift all our calculated spacings by a similar amount. Random errors came from the fuzzy diffraction rings, which made measuring diameters difficult even with ImageJ; the outer ring's larger uncertainty (8.91%) reflects its less distinct appearance, and although we tried to take multiple measurements of this, having clearer rings would improve our accuracy.

## Conclusion

The experiment successfully confirmed de Broglie's hypothesis by demonstrating the wave nature of electrons. The linear relationship between diffraction pattern and $1/\sqrt{V_0}$ verified the predicted behavior. Our measured interplanar spacings were $d_{\text{inner}} = 0.162 \pm 0.008$ nm and $d_{\text{outer}} = 0.294 \pm 0.026$ nm. While the core wave-particle relationship was established, differences from parameter values stem mainly from systematic uncertainties in the apparatus and measurement limitations.

## References


[1] A. Shaheen and M. S. Anwar, "Verification of de Broglie's hypothesis by electron diffraction from graphite," LUMS School of Science and Engineering, Lab Manual Version 2015-2, September 2015. [1]

[2] C. J. Davisson and L. H. Germer, "Diffraction of Electrons by a Crystal of Nickel," *Physical Review*, vol. 30, pp. 705-740, 1927. [2]

[3] M. A. Asadabad and M. J. Eskandari, "Electron diffraction," in *Transmission Electron Microscopy in Physical and Life Sciences*, IntechOpen, 2016. [3]