Imaging electron trajectories: e/m measurement with a "magic eye" and ImageJ

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When electrons are allowed to move freely in a magnetic field, they behave as charged particles following curved trajectories due to the Lorentz force. By making these trajectories "visible" and then measuring their curvature, the charge-to-mass ratio of the electrons can be determined.

In this experiment, we use a "magic eye" and digital photography to track the path of electrons. Digital images are then processed in a freeware named ImageJ to measure the curvature.

KEYWORDS

Anode · filament · phosphorescence · Lorentz force · magic eye · calibration · ImageJ · image processing · digital data · collision cross section · radius of curvature.

1 Objectives

In this experiment, we will:

1. learn to appreciate how subatomic particles can be tracked from their effects,
2. see how the energy of a free electron can be controlled by maintaining an electric field,
3. record and analyze observations in the form of digital images,
4. calculate the charge-to-mass ratio of the electron using a simple arrangement, and
5. understand the importance and method of calculating uncertainties associated with experimental measurements.

References and Essential Reading

[1] D. Prutchi, and S. Prutchi, *Exploring Quantum Physics through Hands-on Projects*, John Wiley, Inc., pp. 77-79 (2003).

[2] W.S. Rasband, ImageJ, U. S. National Institutes of Health, Bethesda, Maryland, USA, http://imagej.nih.gov/ij/, 1997-2012.

2 Introduction

Foresee: Does a stationary electron experience a force in a magnetic field? Provide a physical justification for your answer.

When an electron is accelerated by an electric field, in the presence of a magnetic field, it experiences what we know as the Lorentz force, $\mathbf{F}_\text{L}$, given by:

$$\mathbf{F}_\text{L} = q\ (\mathbf{E} + \mathbf{v} \times \mathbf{B}), \tag{1}$$

where $\mathbf{E}$ is the electric field strength, $\mathbf{v}$ is the velocity of the electron and $\mathbf{B}$ is the strength of the magnetic field. The Lorentz force makes the electron follow a curved path where the curvature depends on the strength of the magnetic field and electron's kinetic energy, which in turn depends on the electronic charge and the applied electric potential. This means that the charge-to-mass ratio of the electron can be derived from the curvature.

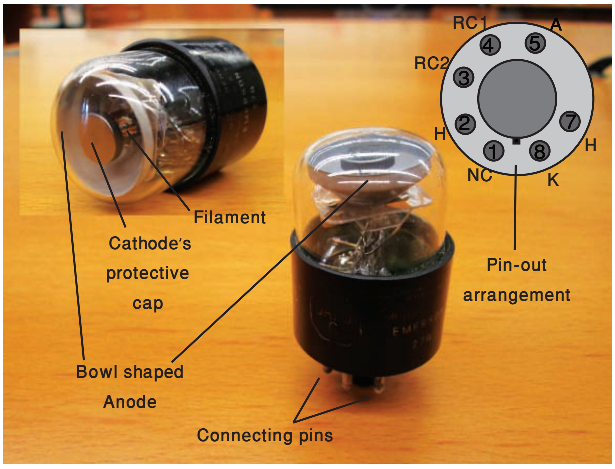
Predict: If the electrons are free to move, can you predict the shape of the trajectory if the electric field is parallel to the magnetic field?

3 Experimental setup: the magic eye

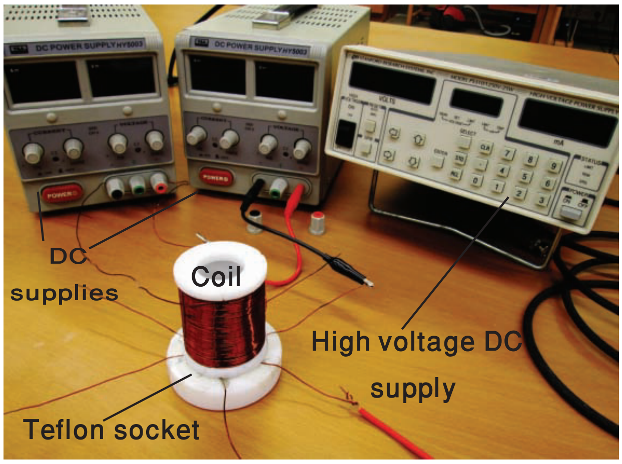
The so-called magic eye is a simple and compact triode tube (Figure 1a) from the times of the second World War. It was originally used as a tuning indicator on radio receivers, producing a bright glow for tuned stations. It is fitted with a filament thermionically emitting electrons that pass through a cathode and travel towards an anode painted with a phosphorescent material, producing a green glow as the electrons fall onto it (Figure 2a). We place the tube inside a current carrying coil, such that the direction of the magnetic field is perpendicular to the trajectory of the electrons. This makes the electrons follow a curved path (Figure 2b) the curvature of which can be controlled either by varying the current in the coil or the electric potential difference between the cathode and the anode.

In the given scenario, the radius of curvature of the trajectories is related to the charge-to-mass ratio by equation (2) [1]:

$$\frac{e}{m} = \frac{2V}{B^2\, r^2}. \tag{2}$$



(a)



(b)

Figure 1: (a) Different views of the magic eye tube and (b) the experimental setup for e/m measurement.

Identify: Identify the independent and dependent quantities in Eq. (2).

Derive: Derive equation (2).

On re-arranging equation (2), we obtain:

$$r^2 = \frac{m}{e}\frac{2}{B^2}V, \quad (3)$$

which suggests that the graph between the accelerating voltage and the square of radius should be a straight line. From the slope of this line, charge-to-mass ratio of the electron can be found conveniently.

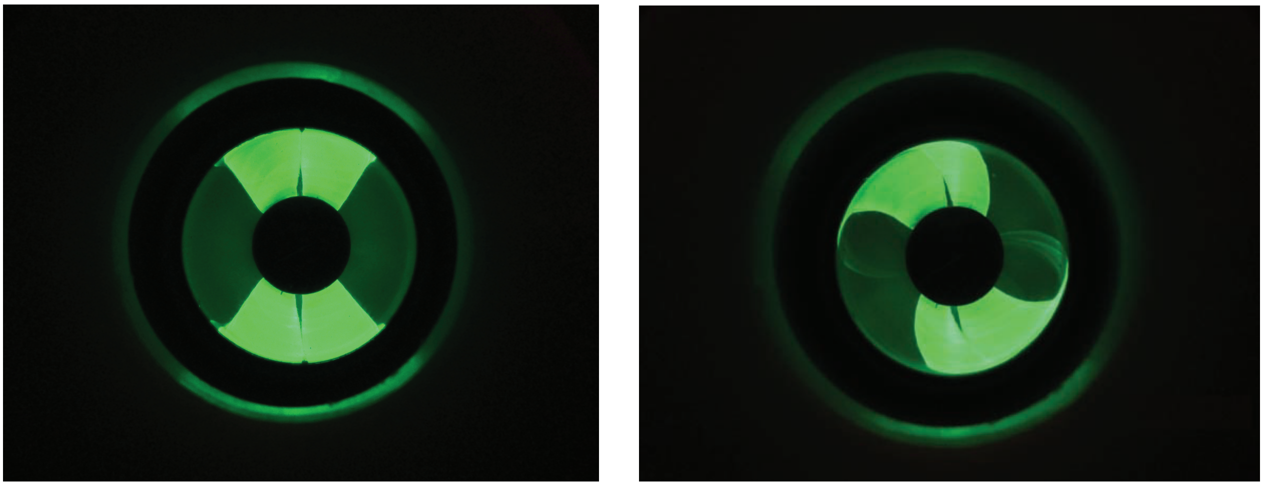
Grasp: Why is it better observe the relationship of voltage with r^2 instead of r? How is linearization important in analysis of data?

A finite value of e/m demonstrates the fact that electrons are massive particles. All of this is done using an extremely simple setup that consists of a hand-made coil, a homemade teflon socket for the tube and variable DC supplies (Figure 1b).

4 Measurements in ImageJ

Innovate: Make a list of any possible methods of measuring the radii of curvature in a magic eye.

We will use software to measure the curvature. ImageJ is an open source image processor with a user-friendly interface (Figure 3), available with a wide variety of basic and advanced methods that usually come as additional packages called "plugins". Basic operations like sharpening an image, conversion to grayscale, contrast enhancement and crop region are easy to apply and are very helpful in making the measurements more precise. After the image has been opened



(a) (b)

Figure 2: (a) Top view of the glow from an operating magic eye tube and (b) when it is placed inside a transverse magnetic field.

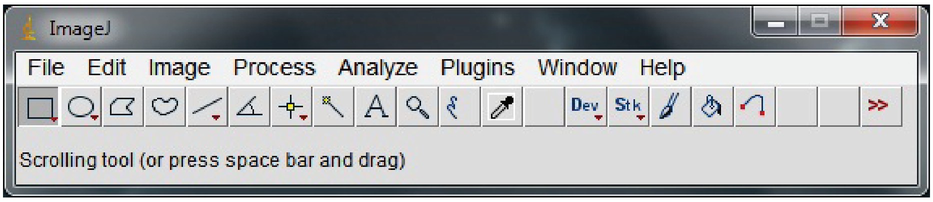


Figure 3: ImageJ GUI

with ImageJ, it can be transformed, enhanced and analyzed in several ways. The choice of the tools to use depends on the kind of analysis we want to do. Many nice and basic image processing tools can be found in "Process", "Analyze" and "Plugins" menus in the ImageJ launcher. Prior to any measurement, a calibration has to be performed to calculate the number of pixels per unit length. This is done by including an object of known physical dimensions in the image. In the case of a magic eye, we know the diameter of the shielding disc inside the tube (the dark area at the center) to be 10 mm.

Appreciate: What advantage does this technique of measurement have over other possible methods of curvature measurement?

5 The experiment

The basic component of this experiment is a magic-eye tube. Electrons are emitted from a heater carrying current produced by a DC power supply. They are accelerated by a potential applied between the anode and the cathode. This potential is supplied by a second power support, which is in fact a high voltage unit. Next, the tube is surrounded by a solenoid coil. A third power supply drives the current through this coil, which produces the magnetic field. Figure 4 shows the connection diagram.

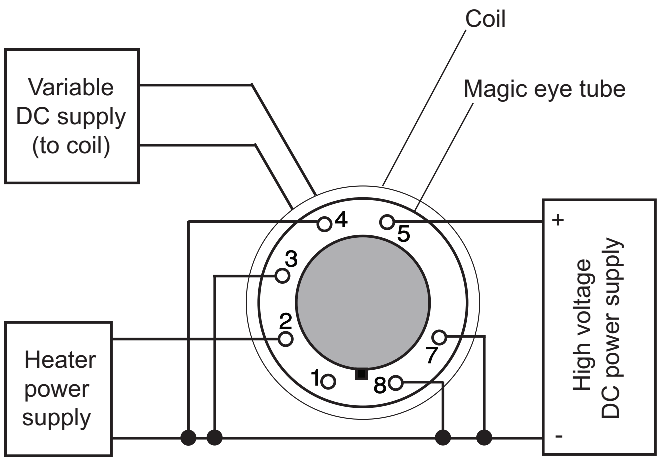


Figure 4: Connection diagram for the magic eye and coil. Connections are indicated by solid circles.

5.1 Apparatus

Following is the list of the needed apparatus for the experiment:

1. Magnetic field coil
2. High voltage DC supply (GwINSTEK; Model: GPR-30H10D)
3. High current DC supply for field coil (UNI-T; Model: UTP3315TF)
4. Battery eliminator DC supply with 220-110V transformer (V&A; Model: HY5003)
5. The magic eye tube (Model: 6AF6G)
6. Teflon base with tube holder and connectors
7. Digital multi-meter
8. Hall probe with USB cable
9. Desktop computer (running ImageJ)
10. Digital Camera

5.2 Procedure

Note: As always, you should write down every observation in your notebook along with the uncertainties wherever they are quantifiable.

1. Verify that the power supplies are in accordance with the diagram shown in Figure 4.(Before turning the supplies on, show the connections to the instructor.)

2. Set the voltage supply to the filament to 6 V. Keep the other supplies off and don't let the voltage exceed 6V, or you may melt the filament of the tube.

3. Now, press the power button to turn on the high voltage DC power supply and set the voltage to 90 V. You should be able to see a glow inside the magic eye, similar to shown in Figure 2a.

4. Now turn the supply to the magnetic-field generating coil on and set the current to 1.5 A. The glow inside the tube will be similar to what is shown in Figure 2b.

5. Connect the USB cable of the hall probe to the USB port of your computer and the output connectors to a voltmeter. Placing the hall probe inside the coil, you get a voltage reading. Use the following formula to convert this reading to a magnetic field measurement,

$$B = \left(\frac{V_B - V_o}{K}\right) \tag{4}$$

   where, B is the magnetic field strength in T, V_B is the reading on the voltmeter in the presence of the field, V_o is the zero-field voltage reading (2.5V in this case) and K is the sensitivity of the hall sensor. For this particular sensor, K=31.25V/T.

6. Use the digital camera to take a close up photo of the magic eye tube. Keep the camera as still as possible to avoid blurry images. Preferably, place the camera on a tripod stand.

7. Now increase the voltage from HVDC supply by 5 V and take a photo of the magic eye. Repeat the procedure up to a voltage of about 130 V.

8. Copy the images into the computer. Place them in a folder with the desired name.

9. Open images with ImageJ (or alternatively, launch ImageJ and use the "drag and drop" feature), and highlight the edges in the image using the "Find Edges" tool in the "Process" menu. The output should look similar to Figure 5b.

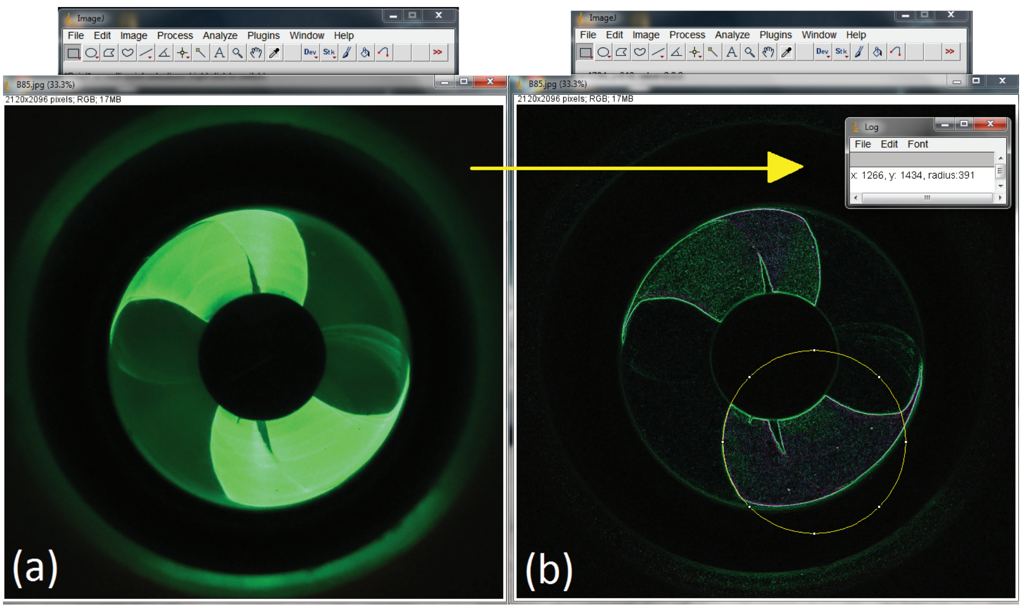


Figure 5: Finding edges and curvatures using ImageJ.

10. You will use a plugin called `ThreePointCircularROI` to measure the curvature radius of the emitting electrons. But first, you will have to calibrate for the number pixels per millimeter of length. For this purpose you will find the radius of an object of known dimension present in the photograph, which in this case is the circle in the center. In the "Plugins" menu, run `threepoint` >> `ThreePointCircularROI`. You'll be prompted to indicate three points on the circumference of the circle and a popup will show the center and radius in pixels. To obtain the number of pixels per millimeter, use the the physical diameter of the circle, which is 10 mm.

11. Use the same procedure to find the radius of curvature of the edges that represent the path of the electrons. Repeat every radius measurement in ImageJ at least 5 times.

12. Plot the data points for the corresponding values of V and r^2 in MATLAB.

13. After transferring all uncertainties to the y-axis, determine the uncertainty along y-axis, for each measurement. These values will be used to calculate the weight of each measurement.

14. Once you have the weights, you should be able to estimate the slope of the weighted fit line. From the slope, calculate the charge-to-mass ratio.

15. Determine the uncertainty in your measured charge-to-mass ratio, and present your results to the instructor. Compare the results to the accepted value and determine the accuracy of your measurement.