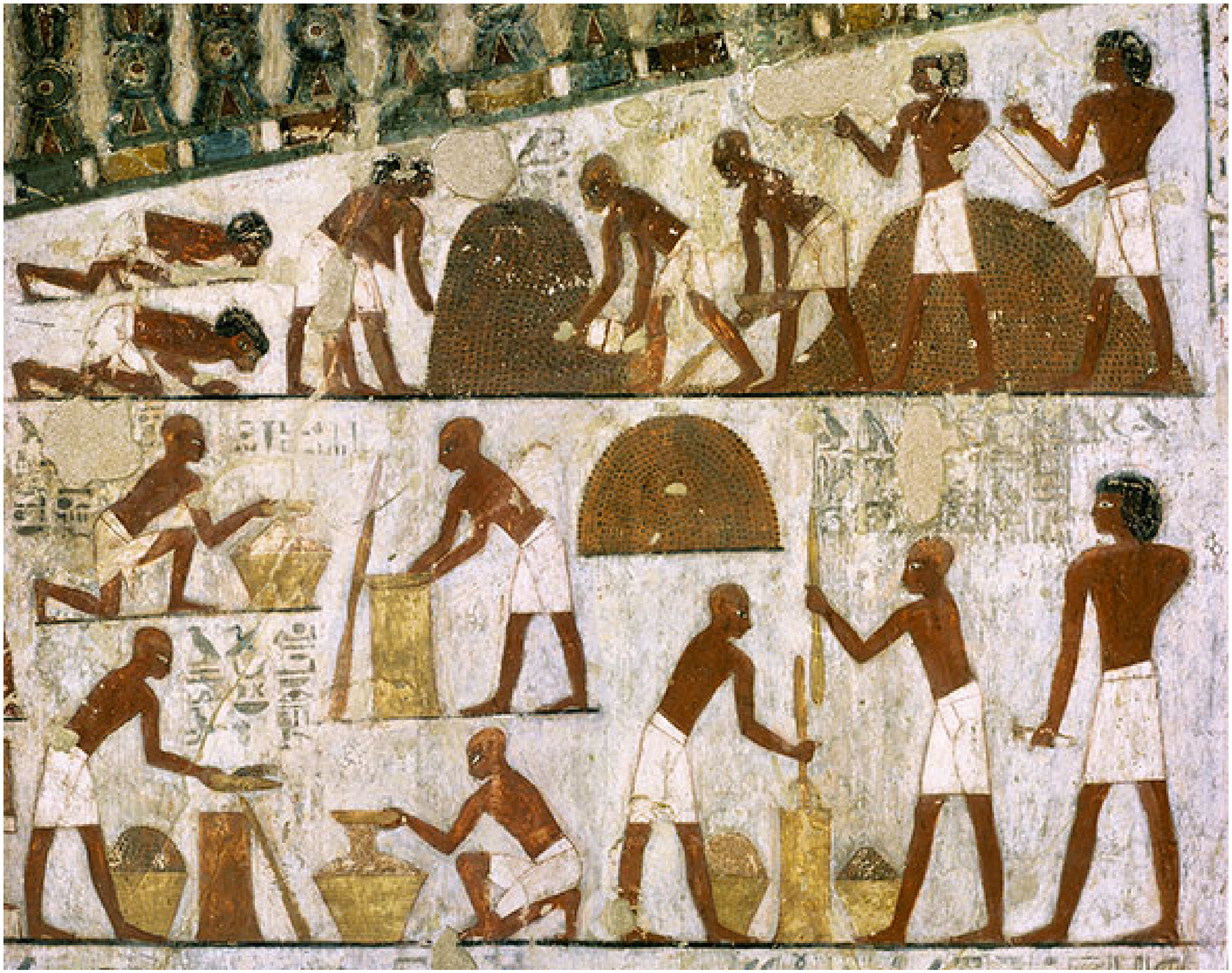
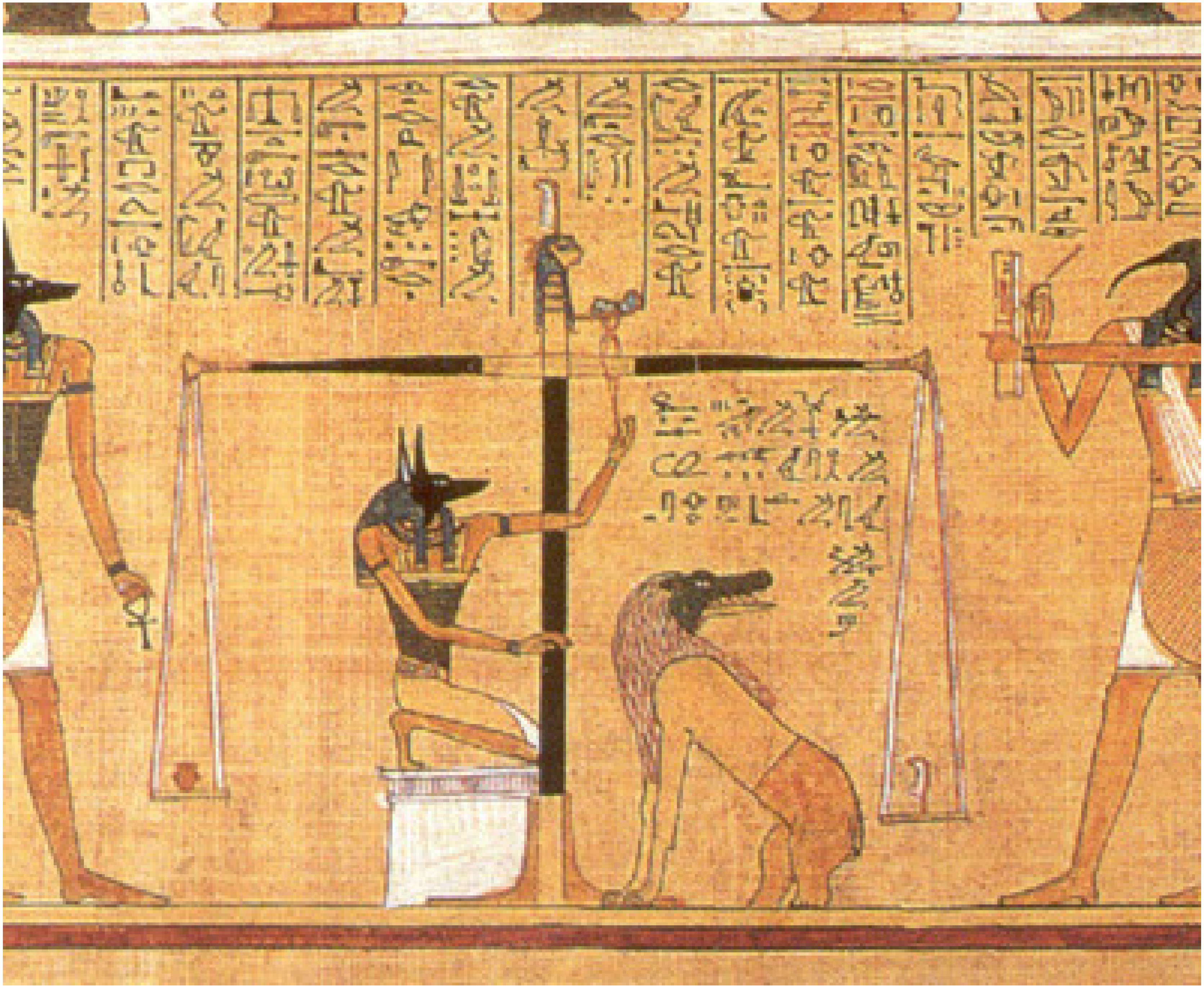
Measurement and uncertainties

Dr. Muhammad Sabieh Anwar
and lab colleagues

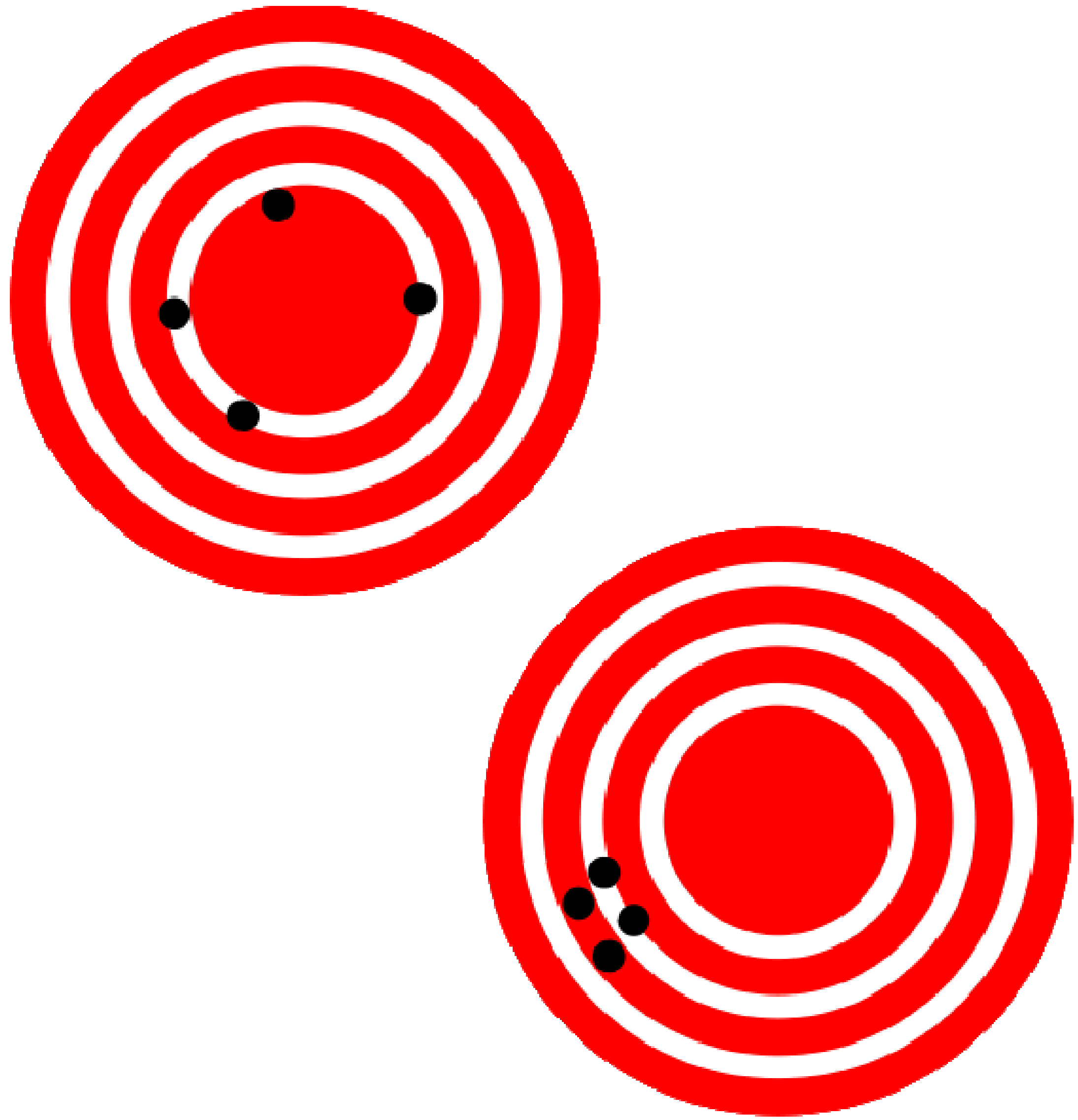
Measurement Matters

و اقيمو ا الوزن با لقسط ولا تخسروا الميزان





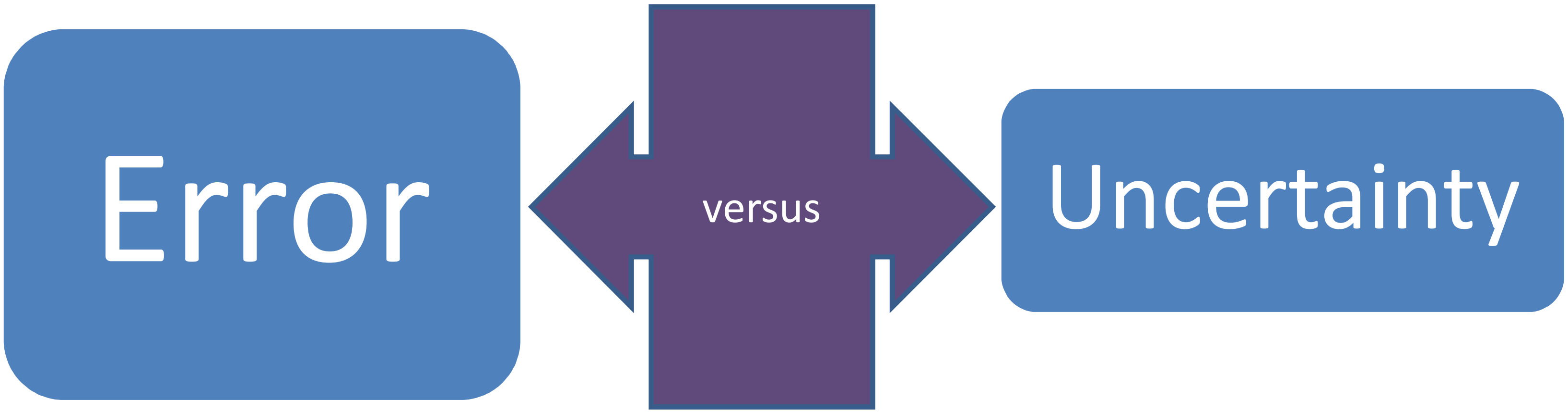
Some terminology



- Accuracy
- Precision
- Repeatability
- Reproducibility
- Reliability
- Traceability

Measure the GUM Way

http://www.bipm.org/en/publications/guides/gum.html



- Errors are mistakes while uncertainties are not.
- Errors are idealized and can never be known.

Uncertainties are:

- quantifiable
- There exist formal methods for their determination.
- They are of two kinds: A and B.
- They are transferrable.

Random Errors

Systematic Errors

versus

Type A Uncertainties

Type B Uncertainties

- Avoid the term “human error” – a human error is a mistake!

Type A	• Evaluated Statistically
Type B	• Evaluated by other means

Reading

versus

Measurement

From a reading(s) emerges data.

From a measurement, one infers physical information about the measurand.

Measurement is an inference about the measurand

- Best estimate
- Dispersion or spread
- Probability distribution
- Coverage or confidence
- Units

Rounding off

Observed value	Rounded value
3.05	3.0
3.15	3.2
3.25	3.2
3.35	3.4
3.33	3.3
3.36	3.4

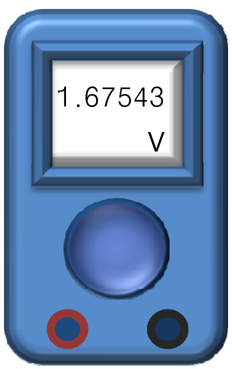
Concept of precision in numerical values

- How is 2 different from 2.0?
- How is 2 different from 2.00?

Rounding off a reading
Is all this an error?

2 V scale
- -1.99999 V
- +1.99999 V

20 V scale
- -19.9999 V
- +19.9999 V



Measurement #1

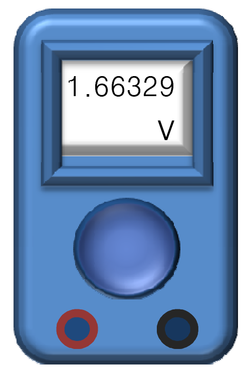
1.67543 V

1.68 V

Measurement #2

1.6633V

1.66 V



How does arithmetic affect precision?

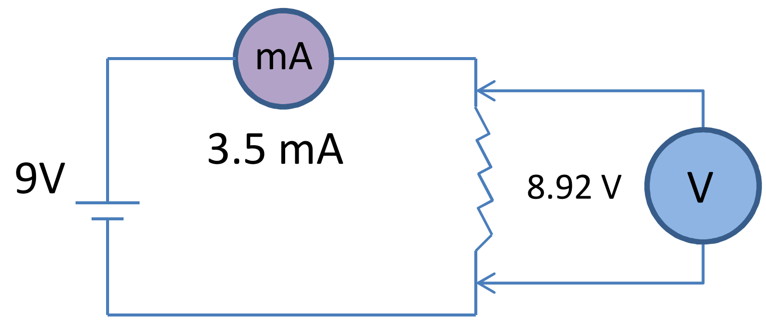
$R = 8.92/3.5 \times 10^{-3}$

$= 2548.571429\ \Omega$

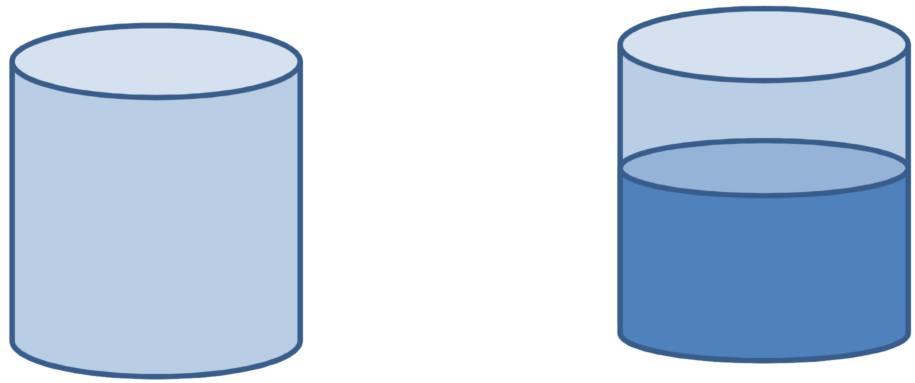
% precision in voltage = 0.06%

% precision in current = 0.01%

Reasonable answer= ?



Example question

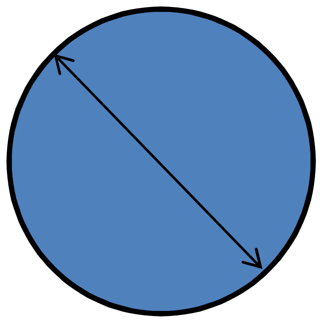


Mass of the can = 0.562 kg

Mass of the can with liquid = 1.5778 kg

Mass of the liquid= ?

Another example question



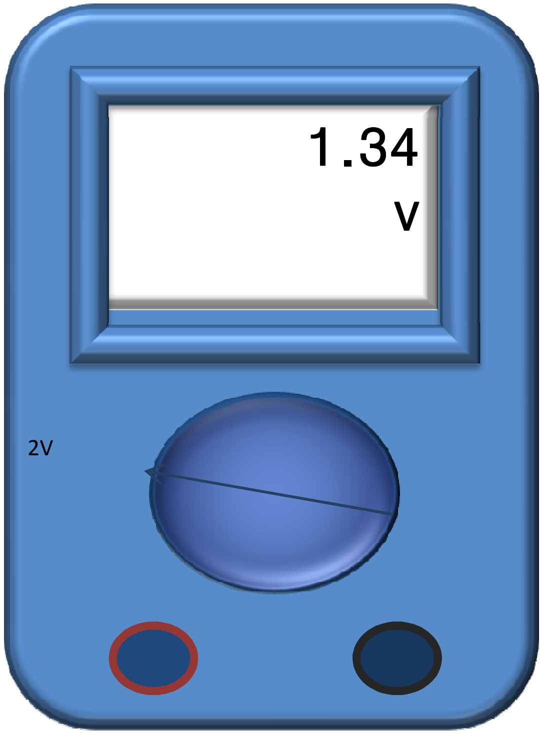
- D = 50.1 mm
- Volume = ?

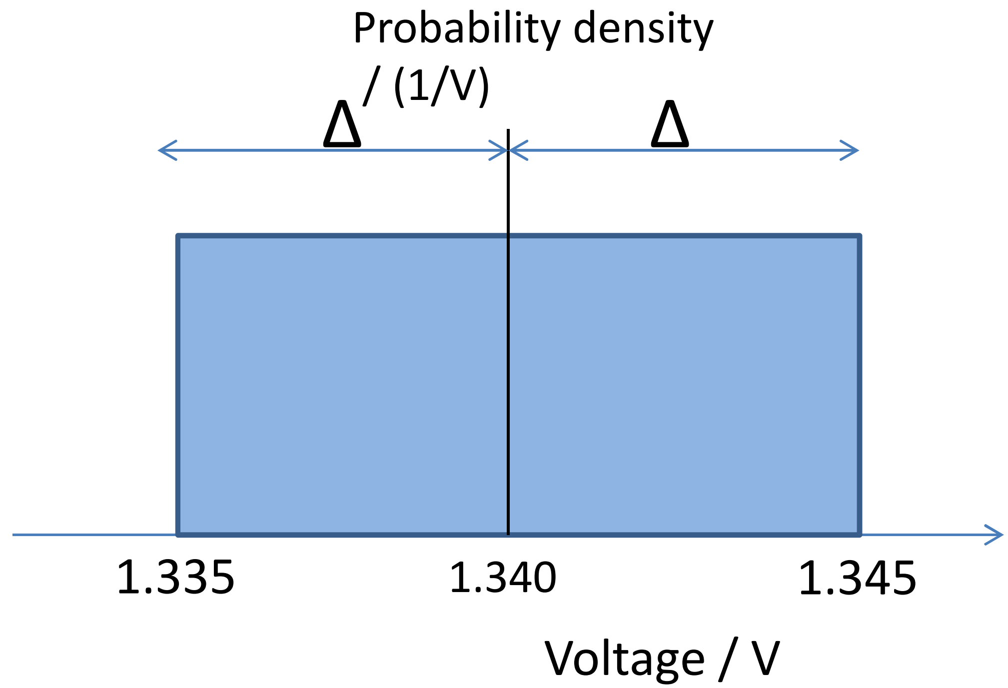
$V = \frac{1}{6} \pi D^3$

Precision in the ‘pth’ power of a measurand ‘n’ is |np|.

Type B Evaluations

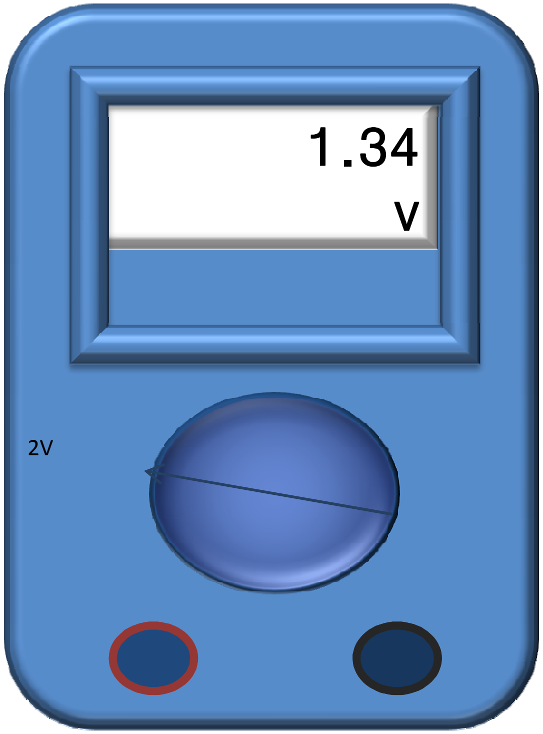
Probabilistic interpretation (Scenario 1)

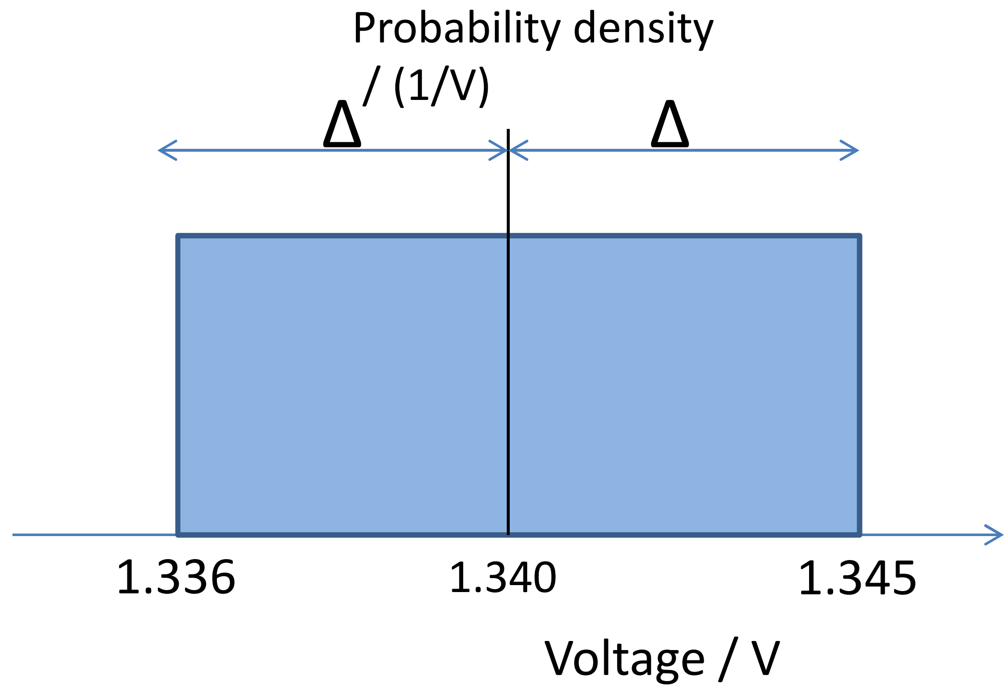




$(1.340 \pm \varepsilon)$ V

Probabilistic interpretation (Scenario 2)

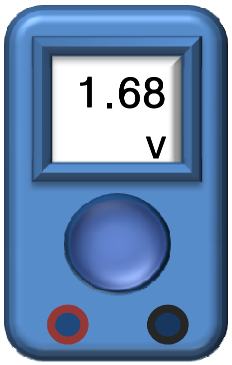


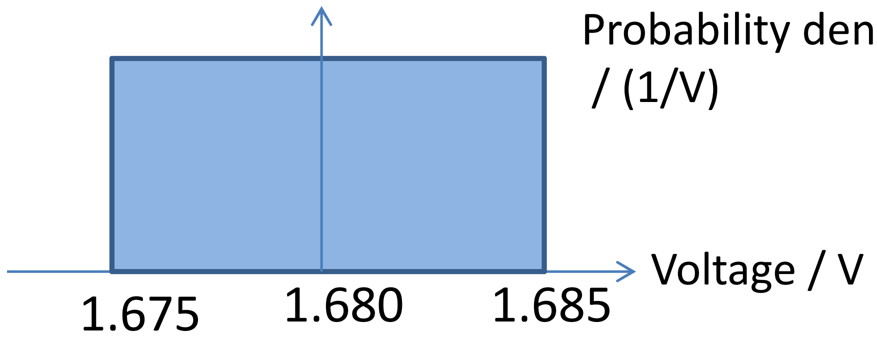


(1.340 ± ε) V

Need for simplicity!

Probabilistic interpretation of uncertainty in a single reading





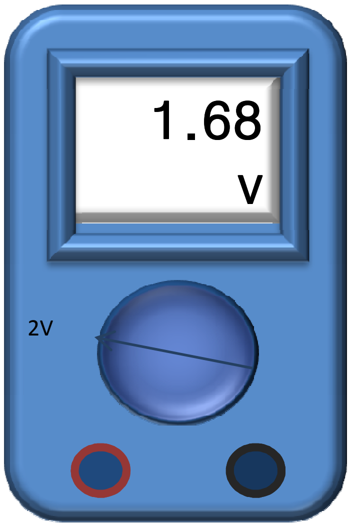
Uniform probability distribution

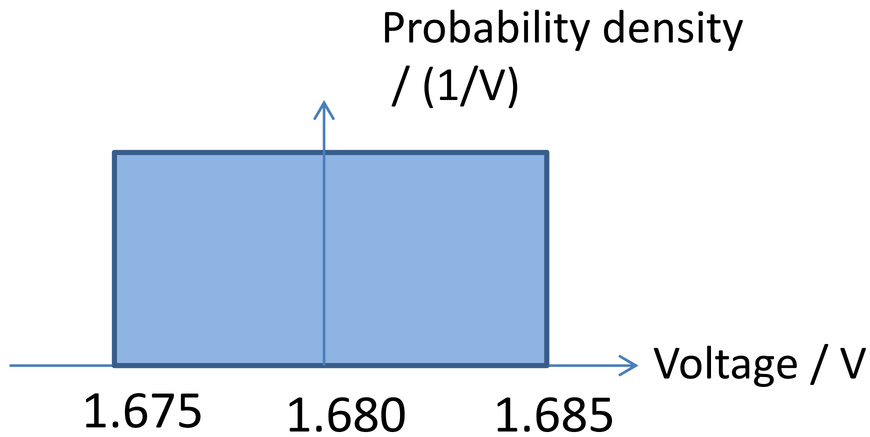
Length of the interval 2Δ = 1.685-1.675= 0.010 V

Half of the length Δ =0.010/2 =0.005 V

Standard uncertainty is $\pm \ \Delta / \sqrt{3} = 0.003\text{V}$

Inference about the measurand

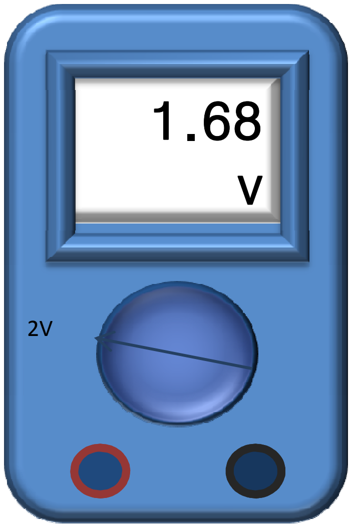




(1.680 ± 0.003) V

Uniform pdf

65% confidence

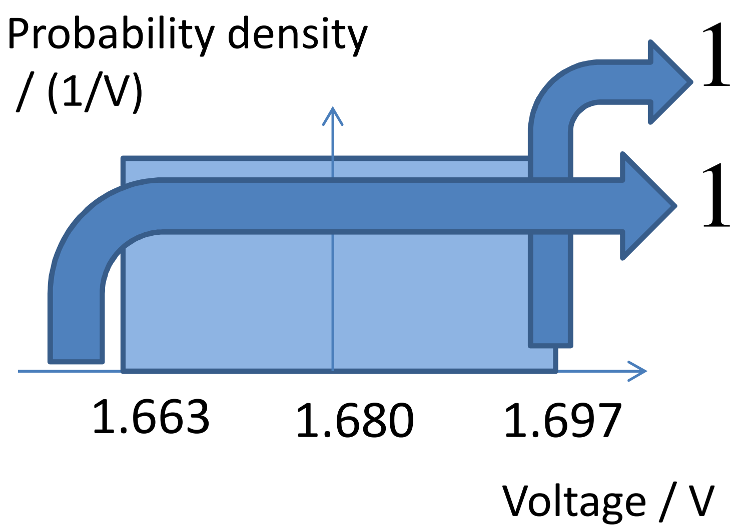


Sources of type B uncertainty

Digital resolution or scale or finite resolution

$$u_s = 0.0028\text{V}$$

Accuracy or rating ± 1%



$$1.680 + 0.01 \times 1.680 = 1.697$$

$$1.680 - 0.01 \times 1.680 = 1.663$$

$$2\Delta = 0.034$$

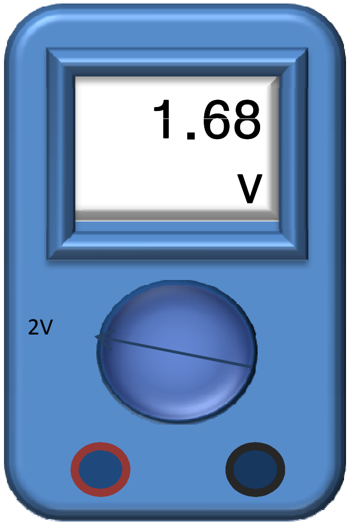
$$\Delta = 0.017$$

$$u_r = \Delta / \sqrt{3} = 0.0098\text{V}$$

Combined standard uncertainty

$$u_s = 0.0028\text{V} \qquad u_r = 0.0098\text{V}$$

$$u = \sqrt{u_s^{\,2} + u_r^{\,2}} = \sqrt{0.0028^2 + 0.0098^2} = 0.010\text{V}$$



(1.68± 0.01) V

(1.680± 0.010) V

Coverage probability 68%

Standard uncertainty = Gaussian distribution

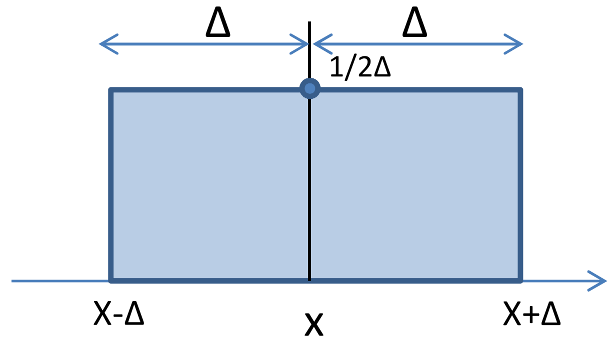
Erroneous recordings

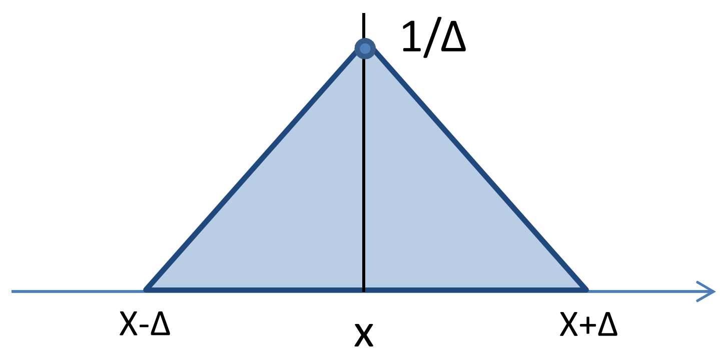
- (1.34 ± 0.005) V
- (1.34 ± 2.0) V
- (1.34 ± 0.1) V
- (1.34 ± 0.12) V
- (1.34 ± 0.02) V
- (1.340 ± 0.12) V

Common Type B Probability Distributions

Digital measuring instrument

$$u = \Delta / \sqrt{3}$$





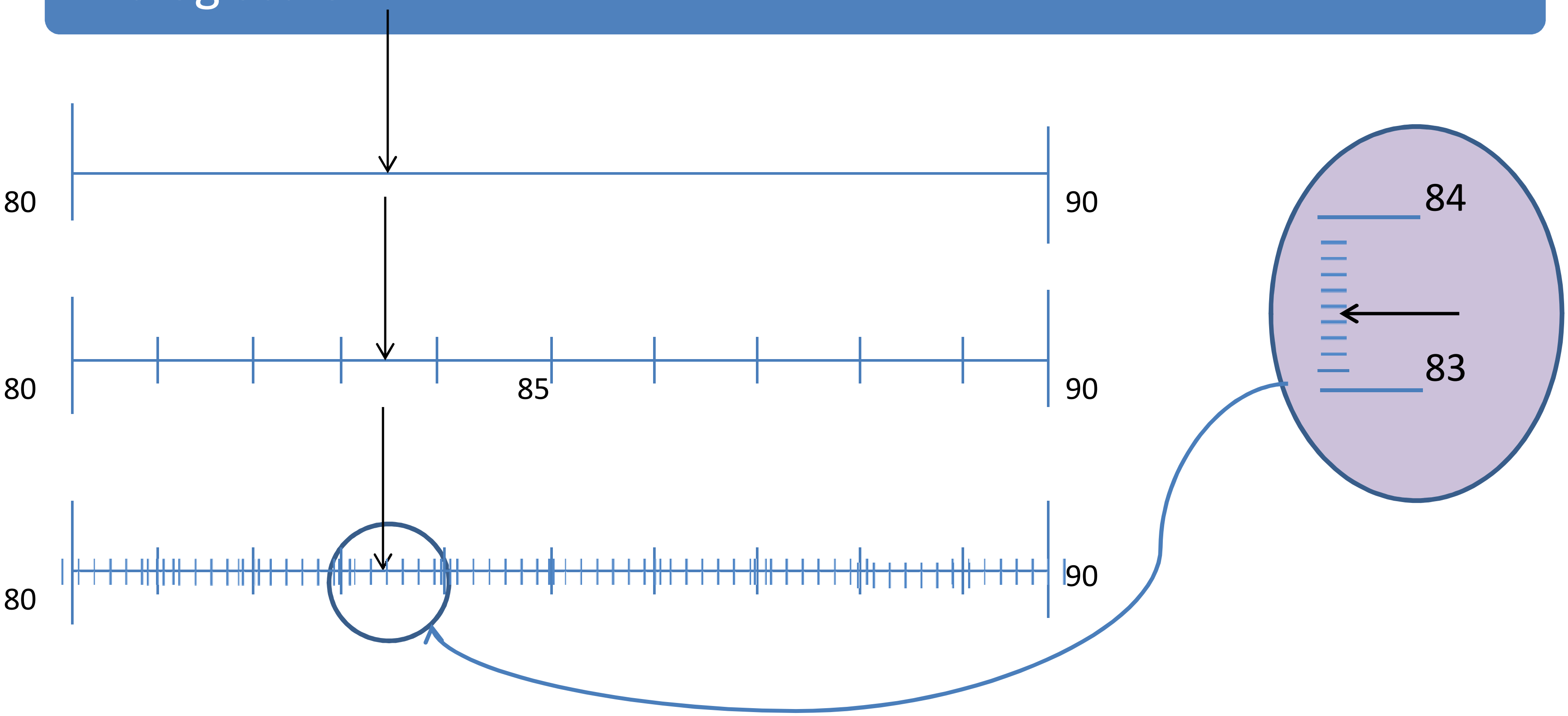
Analog instruments

$$u = \Delta / \sqrt{6}$$

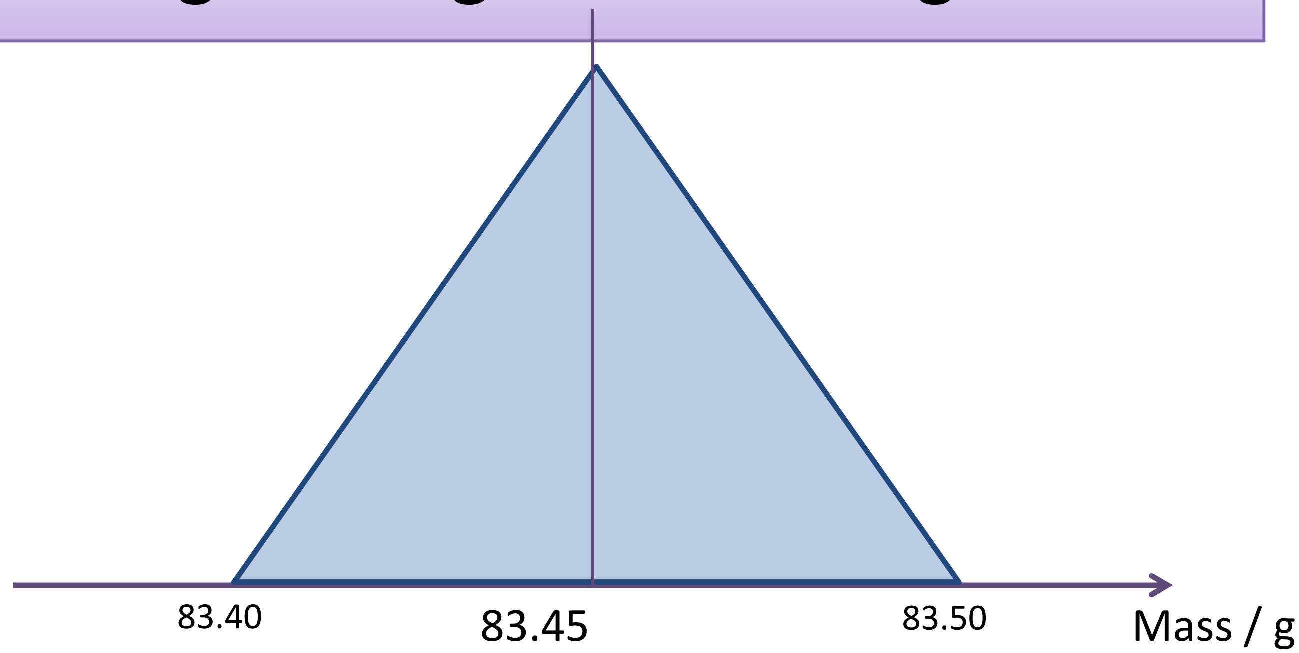
Digital scale

- 83.6 g
- 83.62 g
- 83.627 g

Analog scale



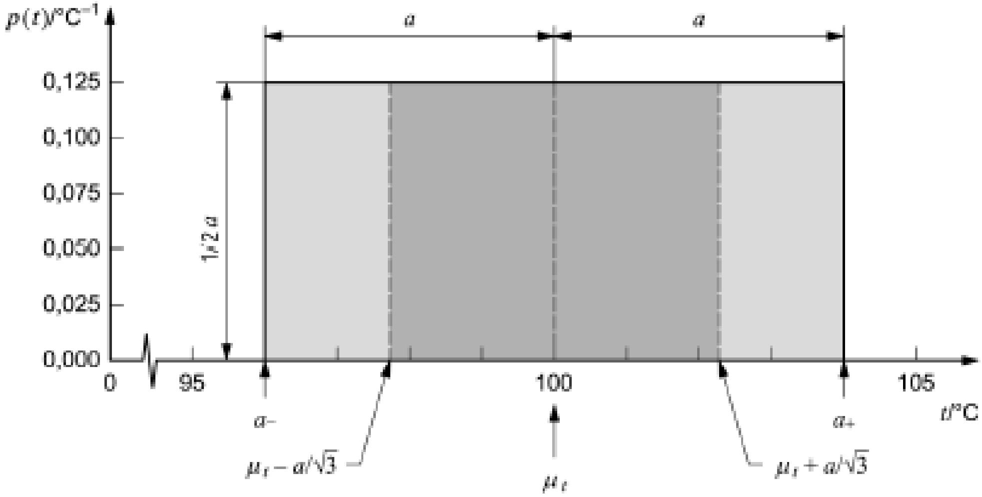
Reading 83.45 g on an analog scale



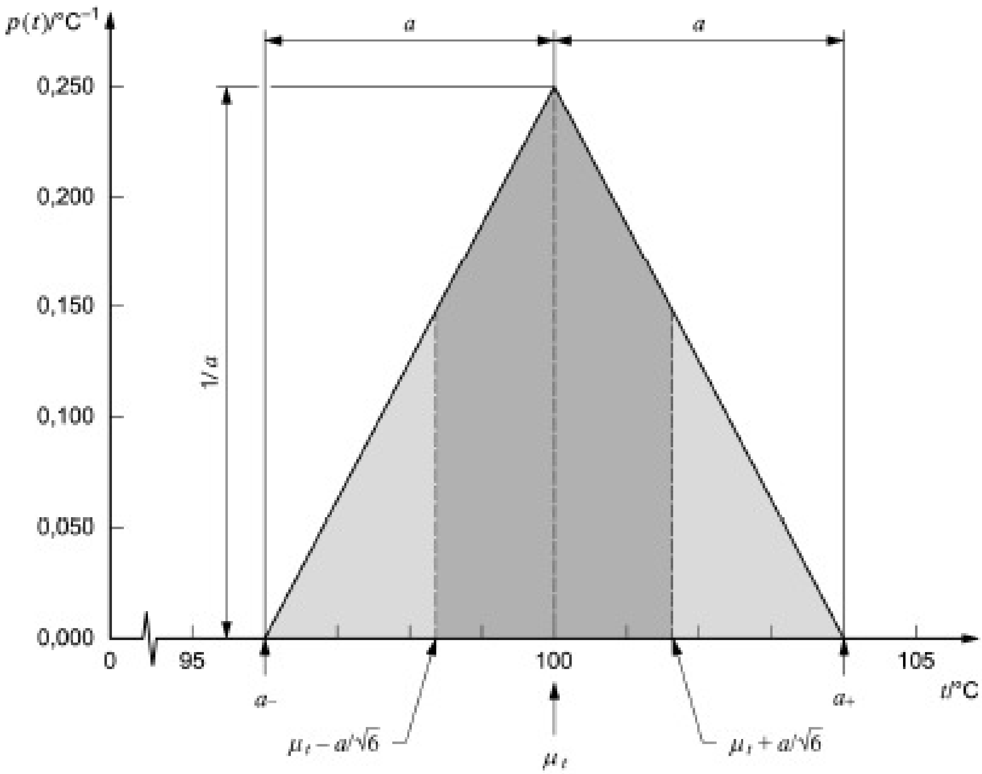
Δ = 0.05 g

u = 0.02 g

Estimate of the measurand is (83.45 $\pm$ 0.02) g assuming a triangular PDF



a)



b)

Second Moments

$$\text{variance } (v) = \int_{-\infty}^{\infty} (x-\overline{x})^2 p(x)dx$$

$$= \int_{\overline{x}-\Delta}^{\overline{x}+\Delta} (x-\overline{x})^2 \frac{1}{2\Delta} dx$$

Suppose $(x-\overline{x}) = \beta$

$$v = \frac{1}{2\Delta}\int_{-\Delta}^{\Delta} \beta^2 d\beta = \left.\frac{\beta^3}{6\Delta}\right|_{-\Delta}^{\Delta} = \frac{1}{6\Delta}(2\Delta^3)$$

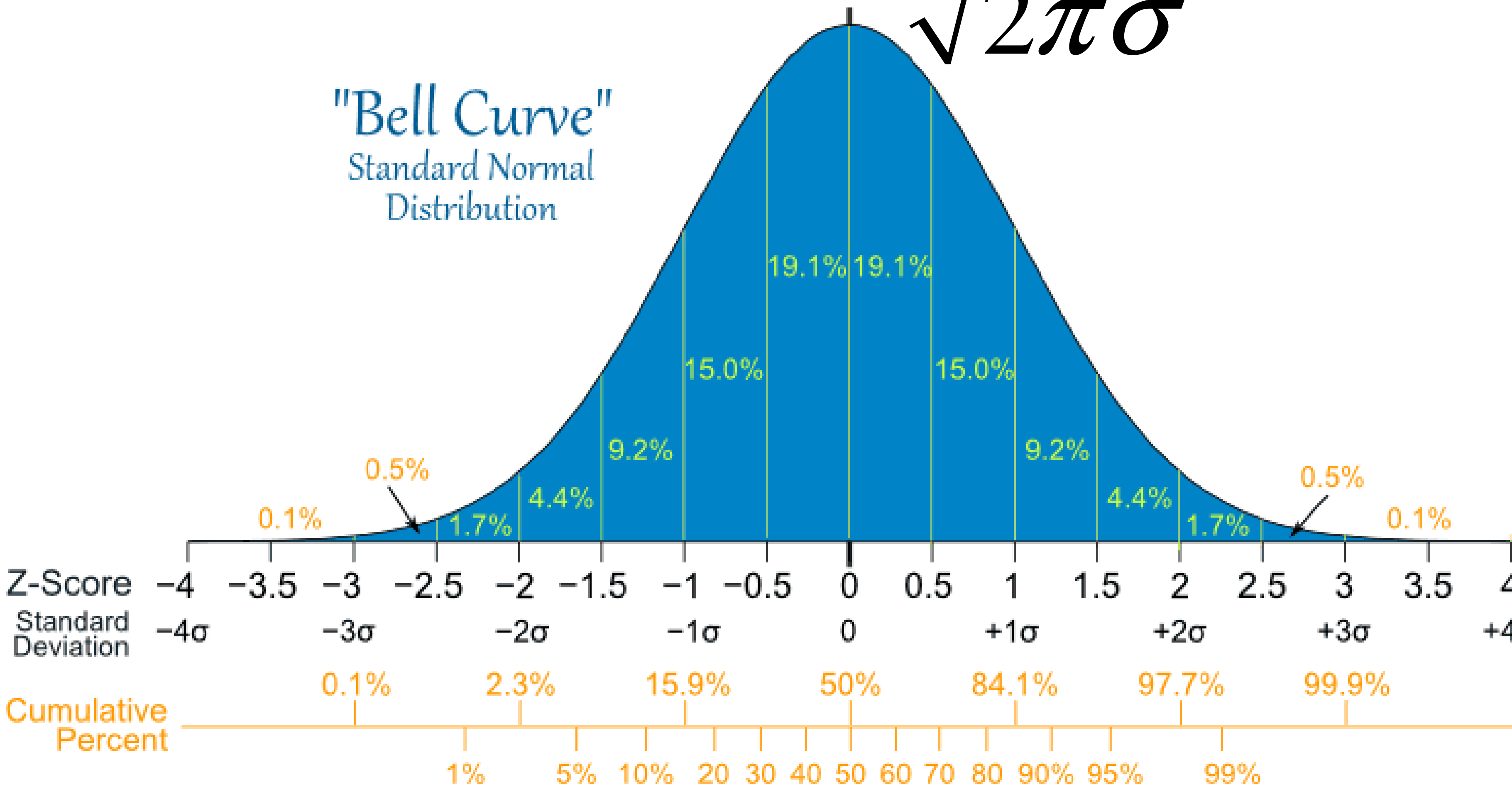
$$= \frac{\Delta^2}{3}$$

$$u = \frac{\Delta}{\sqrt{3}}$$

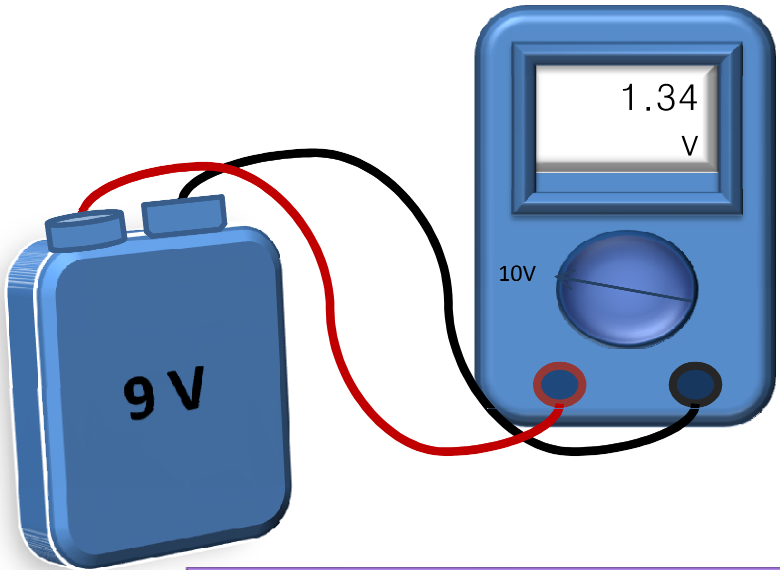
The Gaussian Distribution:

Mean and Standard deviation

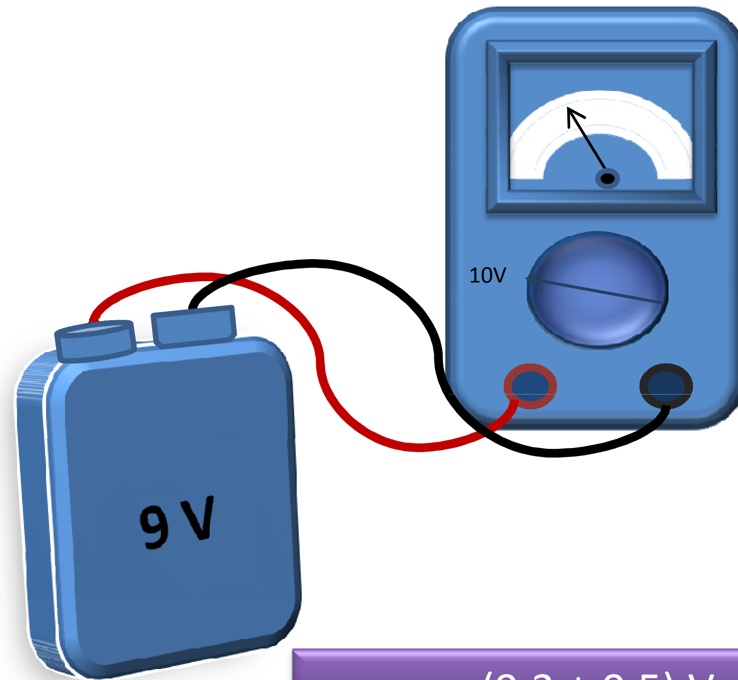
$$p(x) = \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{(x-\bar{x})^2}{2\sigma^2}}$$



Let’s try this out…



(9.38 ± 0.05) V



(9.3 ± 0.5) V

Absolute accuracy ratings of instruments

UT50B

Enlarge

Registered Design No.: 0211977.6M003

Download Manual

Modern Digital Multimeters

SPECIFICATIONS

Basic Functions	Range	Best Accuracy
DC Voltage	200mV/2V/20V/200V/1000V	±(0.5%+1)
AC Voltage	2V/20V/200V/750V	±(0.8%+3)
DC Current	2mA/20mA/200mA/20A	±(0.8%+1)
AC Current	20mA/200mA/20A	±(1%+3)
Resistance	200W/2kW/20kW/200kW/2MW/20MW/200MW	±(0.8%+1)
Capacitance	20nF/200nF/2mF/100mF	±(4%+3)
Temperature (°C)	-40°C ~ 1000°C	±(1%+3)
Temperature (°F)	-40°F ~ 1832°F	±(1%+4)
Special Functions		
Diode		✓
Continuity Buzzer		✓
Data Hold		✓
Display Backlight	Auto Sensor	✓
Full Icon Display		✓
Sleep Mode		✓
Low Battery Display		✓
Input Impedance for DC Voltage Measurement	Around 10MW	✓
Max. Display	1999	✓

GENERAL CHARACTERISTICS

Power	9V Battery (6F22)
LCD Size	59 x 25mm
Product Colour	Red and Grey
Product Net Weight	275g
Product Size	165 x 80 x 38.3mm
Standard Accessories	Test Lead, Battery, English Manual, Point Contact Temperature Probe, Test Clip
Standard Individual Packing	Gift Box
Standard Quantity Per Carton	40pcs
Standard Carton Measurement	598 x 418 x 340mm (Around 0.085 CBM Per Standard Carton)
Standard Carton Gross Weight	21kg

Back to main menu

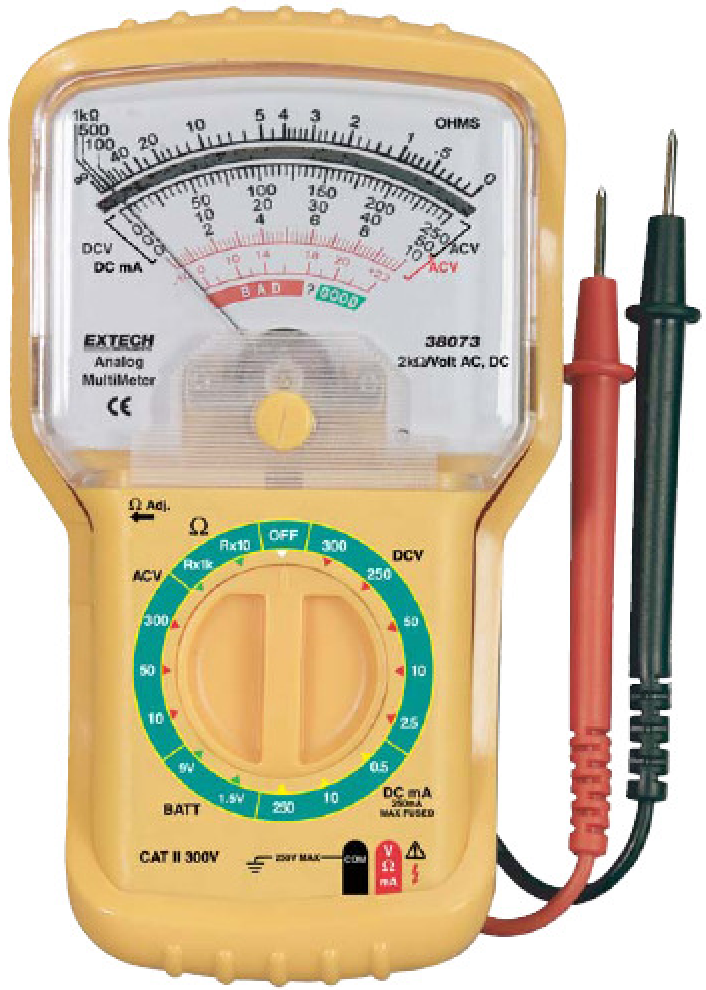
Mini Analog MultiMeter

Convenient pocket MultiMeter

With easy to read color coded analog display

Features:

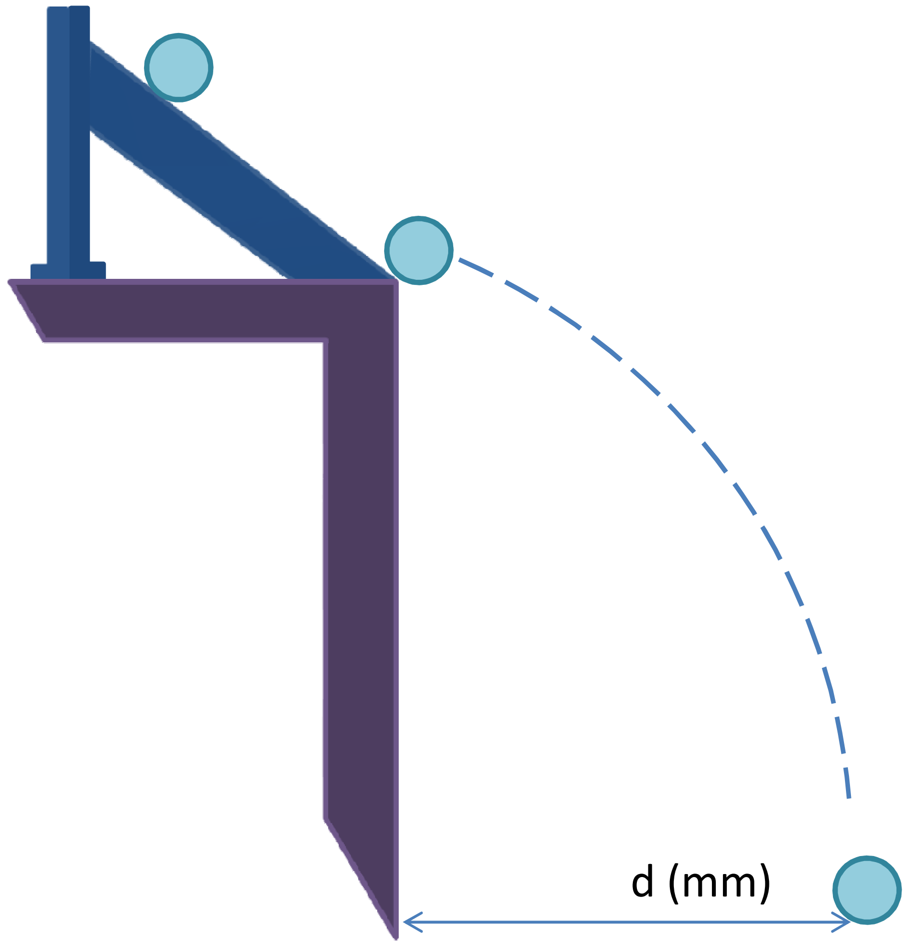
- Easy to read analog display
- Measure AC/DC Voltage, DC Current, Resistance and Decibel
- 5% full scale accuracy
- Battery test on 9V and 1.5V batteries
- Complete with protective holster, test leads and 1.5V AA battery

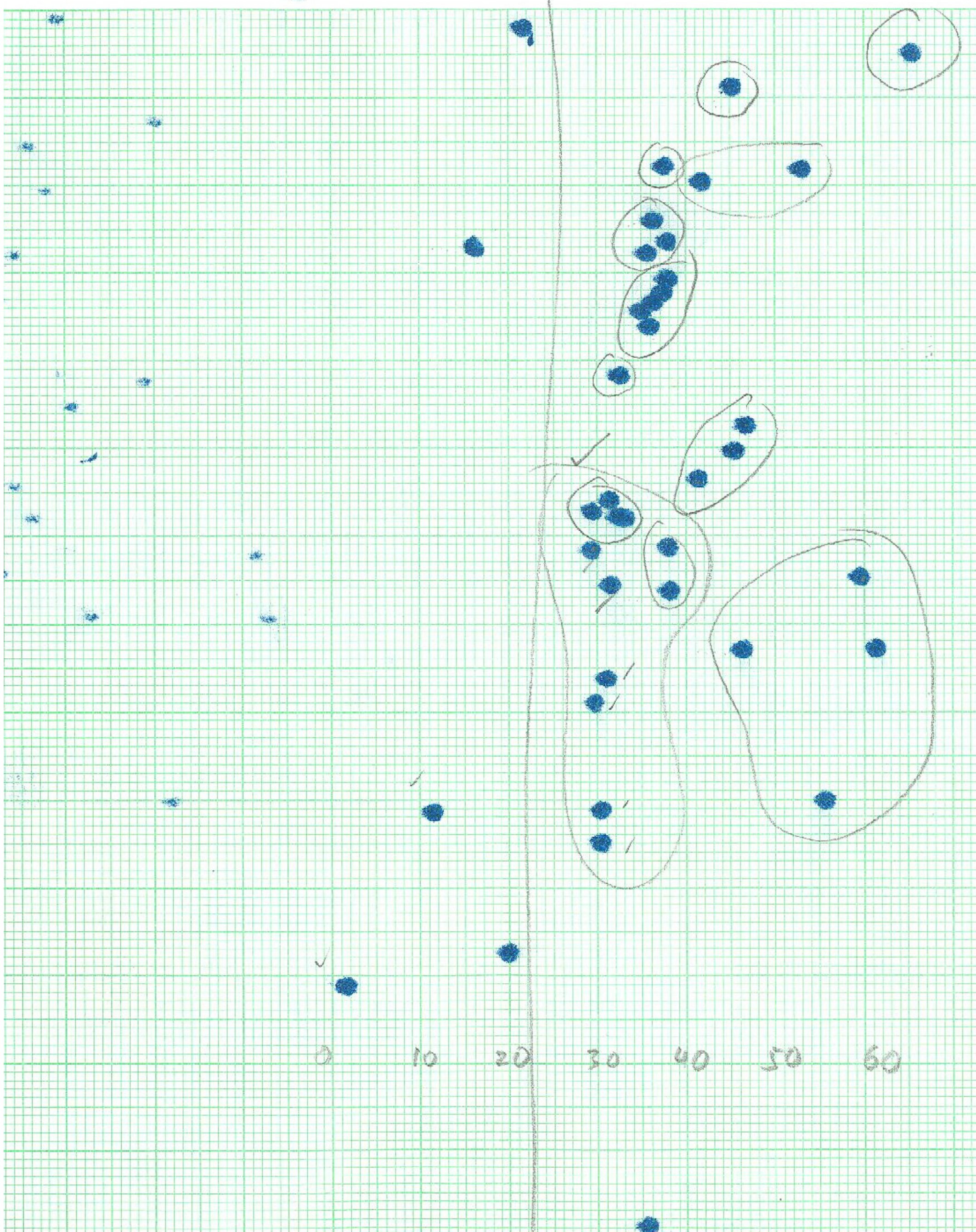


Easy to read color coded analog display

Type A Evaluations

Repeated experiment of sliding a ball down a plane





1.5	35.5
11.5	37.5
16	36
21.5	37
20	55.5
30.5	46.5
30.5	61.5
29.8	59.8
31	41
31.5	45.5
29	47
29.5	41.5
31.5	52.5
32	45
38	65
38	
32.5	
36	
35.5	
36	
37	
37.5	

Statistics of multiple readings

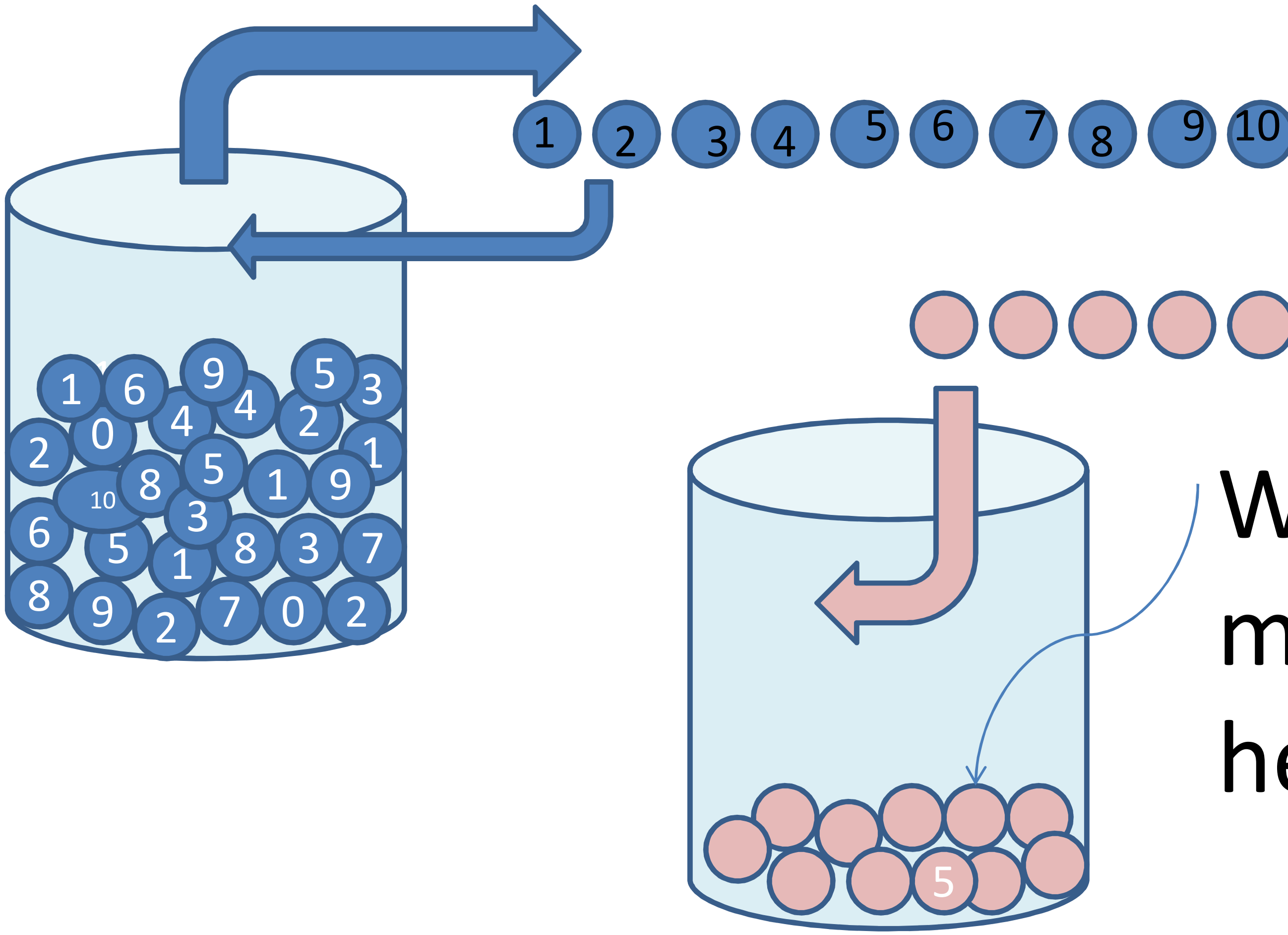
Mean or Average

$$\bar{x} = \frac{\sum_i x_i}{n}$$

standard deviation

$$s = \sqrt{\frac{\sum_i (x_i - \bar{x})^2}{n}}$$

Take 10 balls out at random.
Find their mean and put them back.



Write mean here

How does one calculate σ_m and σ?

$$\sigma_m^2 = \left(\frac{1}{n}\right)\sigma^2$$

$$\sigma^2 = \left(\frac{n}{n-1}\right)s^2$$

$$\sigma_m^2 = \left(\frac{1}{n-1}\right)s^2$$

Relation between σ_m and σ

$d_i = x_i - \bar{x}$ Deviation

$e_i = x_i - X$ Error

$$\sigma^2 = \frac{\sum e_i^{\,2}}{\text{n}}, \sigma_m^{\ 2} = \sum \frac{E^2}{n}$$

where $E = \bar{x} - X$

$$= \sum \frac{x_i}{n} - X$$

$$= \frac{1}{n}\left(\sum (x_i - X)\right)$$

$$E = \frac{1}{n}\sum e_i$$

$$E^2 = \frac{1}{n^2}\left(\sum e_i\right)^2$$

$$E^2 = \frac{1}{n^2}\left[\sum (e_i^2) + \sum_i \sum_j e_i e_j\right]$$

$$\langle E^2 \rangle = \frac{1}{n^2}\langle \sum (e_i^2) \rangle + \frac{1}{n}\left[\sum_i \sum_j e_i e_j\right]$$

$$\sigma_m^{\ 2} = \frac{1}{n}[\sigma^2]$$

Relation between σ_m, σ and s

$$s^2 = \sum \frac{d_i^{\,2}}{n} = \frac{1}{n}\sum (x_i - \overline{x})^2$$

$$= \frac{1}{n}\sum (e - E)^2$$

$$= \frac{1}{n}[\sum (e^2 + E^2 - 2eE)]$$

$$\frac{1}{n}[\sum e^2 + nE^2 - 2E\sum e]$$

$$= \frac{\sum e^2}{n} + E^2 - 2E(\frac{\sum e}{n})$$

$$s^2 = \frac{\sum e^2}{n} - E^2$$

Taking Average

$$\mathrm{s}^2 = \sigma^2 - \sigma^2{}_m$$

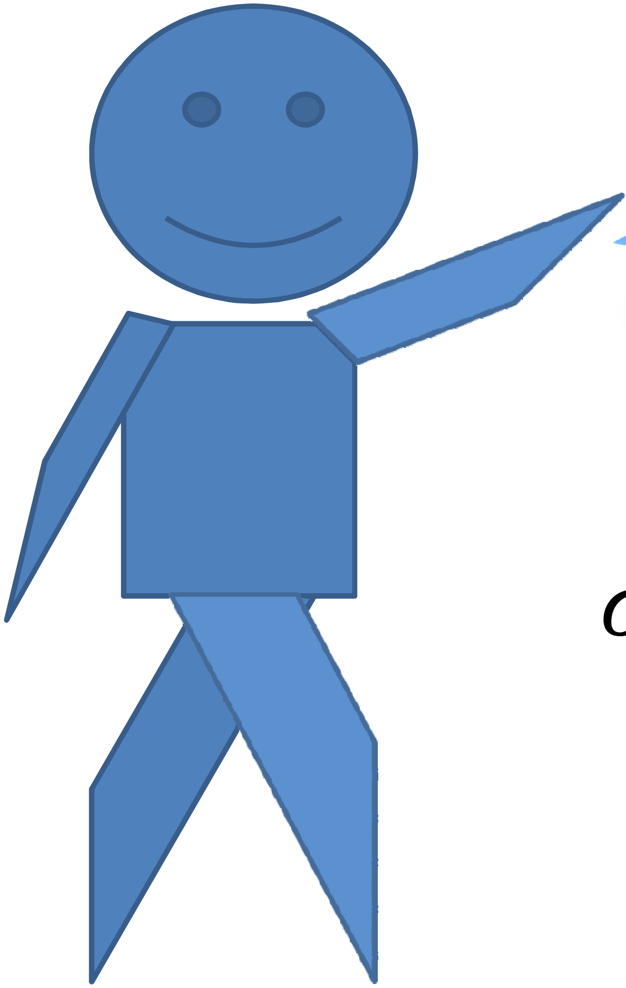
$$= \sigma^2 - \frac{\sigma^2}{n}$$

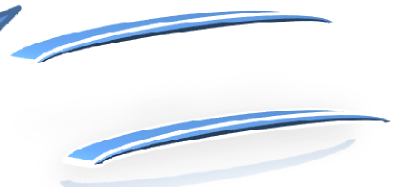
$$= (\frac{n-1}{n})\sigma^2$$

$$\sigma^2 = (\frac{n}{n-1})s^2$$

$$\sigma^2{}_m = (\frac{1}{n-1})s^2$$

Practical Example 1 (Falling Paper)





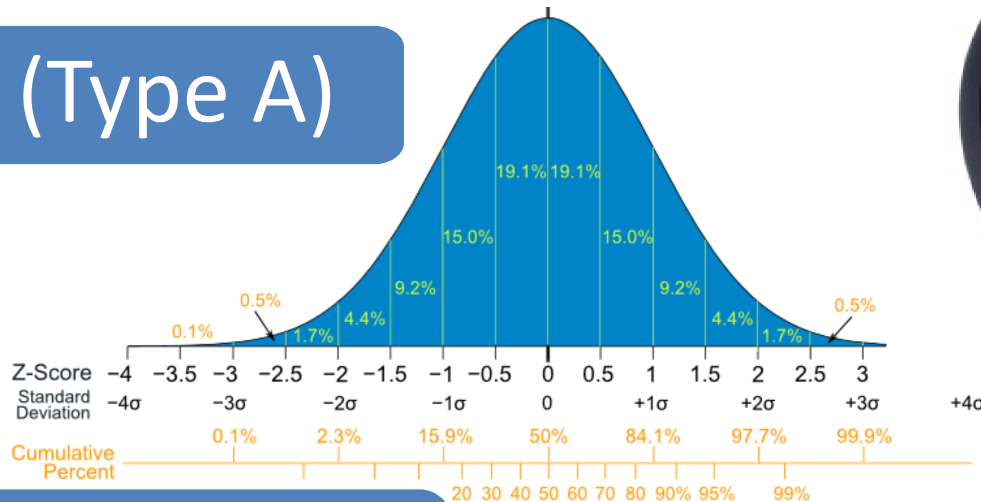
$$\sigma_m^2 = \left(\frac{1}{n-1}\right) s^2$$

$$s = \sqrt{\frac{\sum_i (x_i - \overline{x})^2}{n}}$$

Combining Uncertainties from different distributions

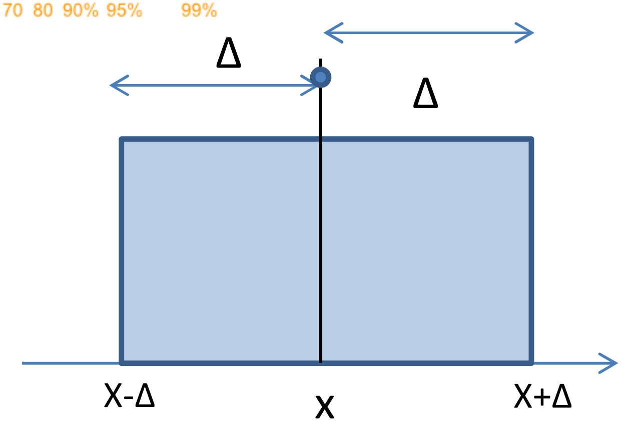
Scatter (Type A)

68%



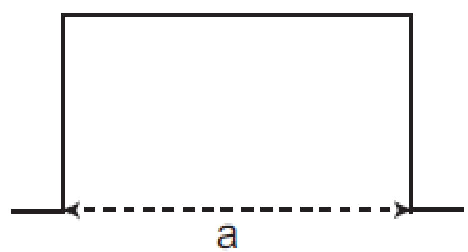
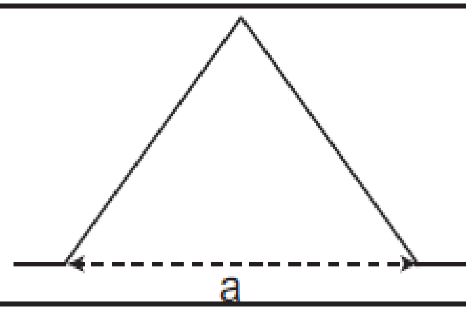
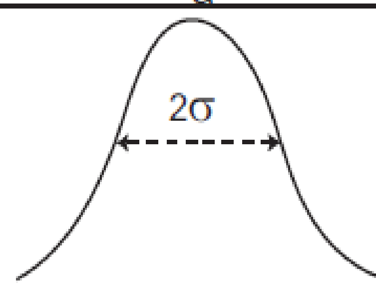
Reaction time (Type B)

65%

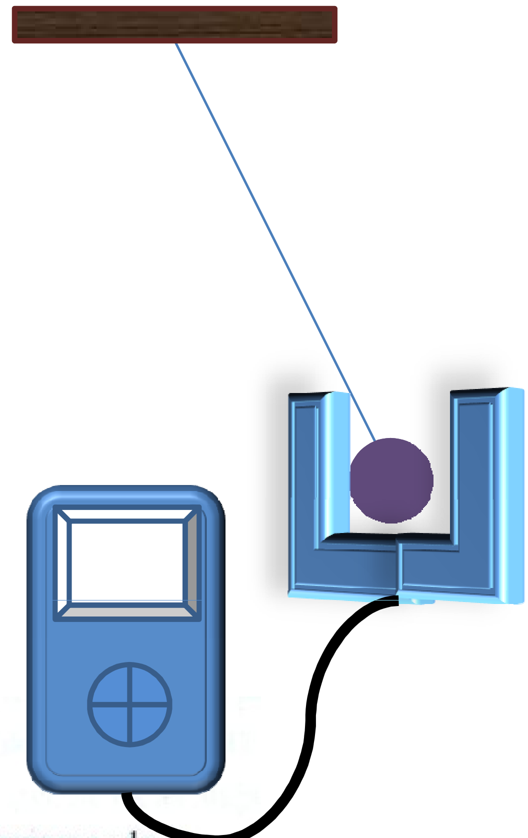


$$u = \sqrt{{u_{scatter}}^{2} + {u_{response}}^{2}}$$

Commonly used probability distribution functions associated with measurements

Evaluation type	Probability distribution function type		Standard uncertainty (u)	Extended Uncertainty
Type B A single digital reading	Rectangular	 	$u = \frac{a/2}{\sqrt{3}}$	1u = 58% 1.65u = 95% 1.73u = 100%
Type B A single analog reading	Triangular	 	$u = \frac{a/2}{\sqrt{6}}$	1u = 65% 1.81u = 95% 2.45u = 100%
Type A For a set of repeated measurements	Gaussian	 	$u = \frac{\sigma}{\sqrt{n}}$	1u = 68% 2u = 95% 3u = 99%

Practical Example 2 (Pendulum)



Specifications

Resolution: The basic timing resolution of the Smart Timer is 100 microseconds in all modes except Stopwatch, which is 10 ms.

Calculated Values: Calculated values are displayed to one or two decimal places with typical accuracy being +/- 1 in the least significant digit. For extremely high speeds (such as might be generated by hand spinning a Super Pulley), accuracy is degraded for calculated parameters because of the very short timing intervals involved.

Worked example of dispersion related uncertainty

Resistance R/ Ω	Deviation d /mΩ	d^2 / $(m\Omega)^2$
4.615	-10	100
4.638	13	169
4.597	-28	784
4.634	9	81
4.613	-12	144
4.623	-2	4
4.659	34	1156
4.623	-2	4
Mean=4.625		**Σ=2442**

Best estimate is 4.625 Ω

SEM is 0.017 Ω

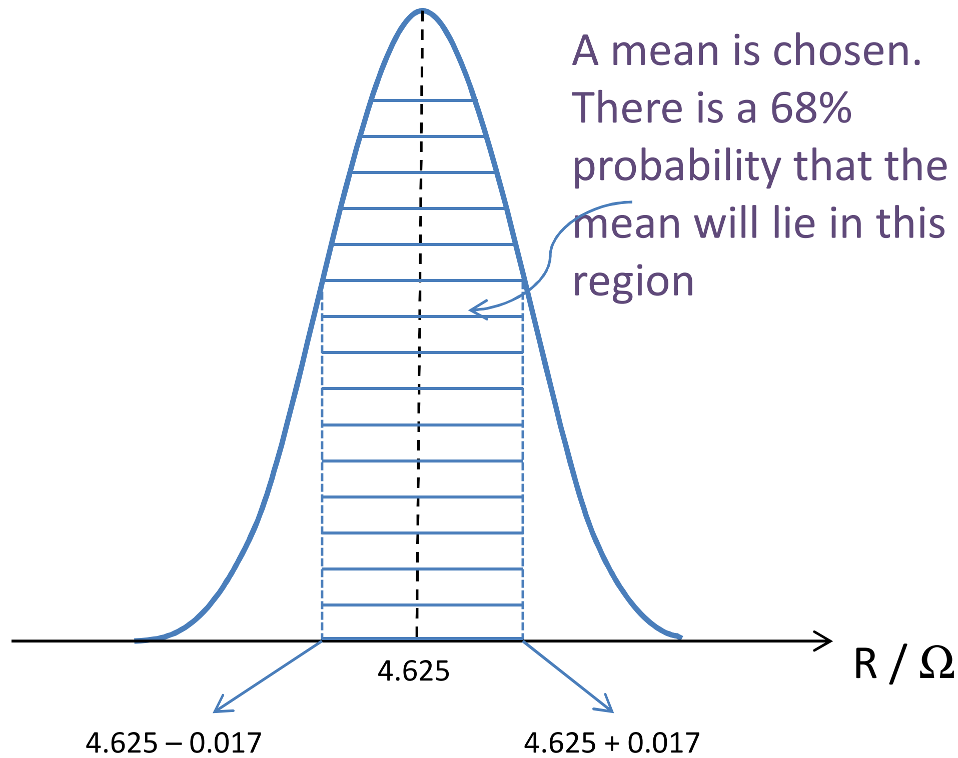
Probabilistic interpretation

$(4.625 \pm 0.017)\ \Omega$

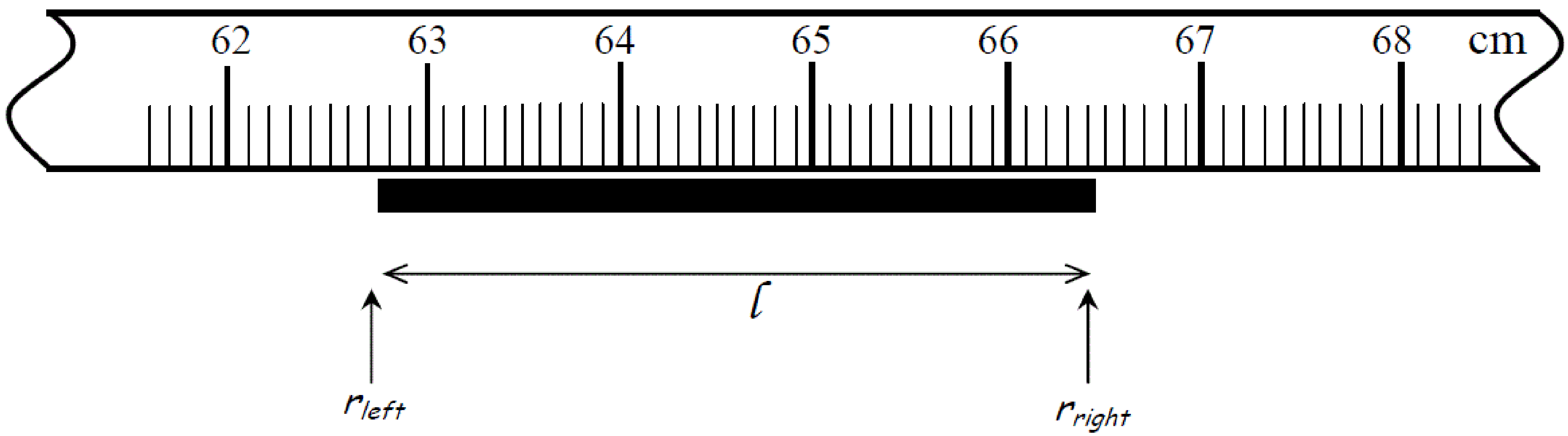
$(4.62 \pm 0.02)\ \Omega$

Gaussian pdf

68% coverage



Propagating Uncertainties

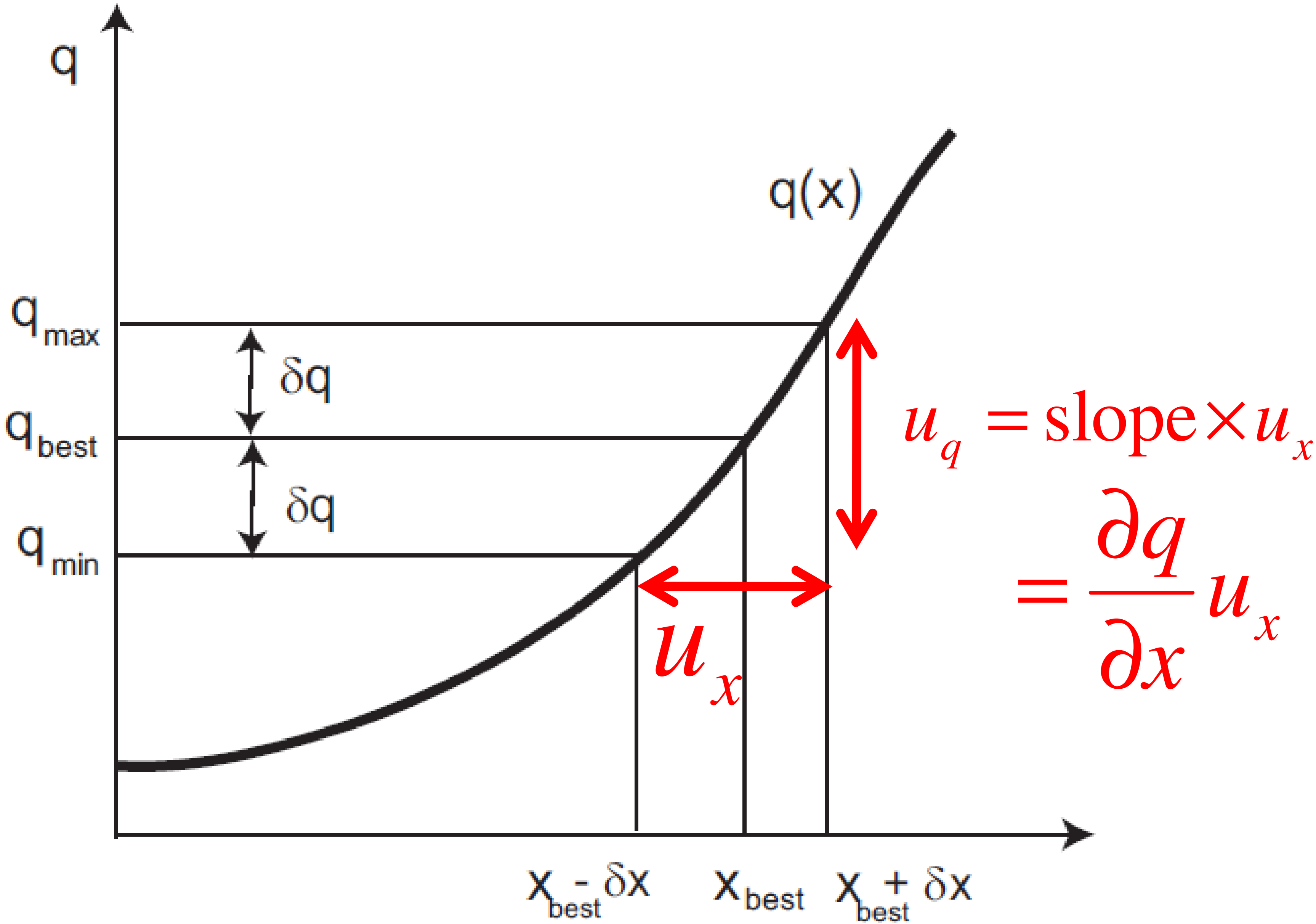


What is the length of the rod?

$$y = y(x_1, x_2, x_3, \ldots, x_n)$$

$$u_y{}^2 = \left(\frac{\partial y}{\partial x_1} u_{x_1}\right)^2 + \left(\frac{\partial y}{\partial x_2} u_{x_2}\right)^2 + \left(\frac{\partial y}{\partial x_3} u_{x_3}\right)^2 + \ldots + \left(\frac{\partial y}{\partial x_n} u_{x_n}\right)^2$$

Rationale


q
q(x)
q_{max}
q_{best}
q_{min}
δq
δq
u_x
$u_q = \text{slope} \times u_x$
$= \frac{\partial q}{\partial x} u_x$
$x_{best} - \delta x$
x_{best}
$x_{best} + \delta x$

Implementing the propagation recipe

$$z = x + y \qquad u_z^2 = u_x^2 + u_y^2$$

$$z = x - y \qquad u_z^2 = u_x^2 + u_y^2$$

$$z = xy \qquad u_z^2 = \left(x u_y\right)^2 + \left(y u_x\right)^2$$

$$z = x / y \qquad u_z^2 = \left(\frac{u_x}{y}\right)^2 + \left(\frac{-x}{y^2}\right)^2$$

$$u_z^2 = (nx^{n-1}u_x)^2$$

$$z = x^n \qquad u_z = nx^{n-1}u_x$$

$$z = x^2 \qquad u_z = 2xu_x$$

$$z = \ln(x) \qquad u_z = \frac{u_x}{x}$$