Moment of Inertia of a Ping-Pong Ball

Xian-Sheng Cao, Changzhou University, Changzhou, People's Republic of China

This note describes how to theoretically calculate and experimentally measure the moment of inertia of a Ping-Pong® ball. The theoretical calculation results are in good agreement with the experimental measurements that can be reproduced in an introductory physics laboratory.

A number of recent papers dealing with properties of Ping-Pong balls have appeared in the literature.[1,2] However, while the moment of inertia of the ball about an axis through its center of mass is one of the important physical properties that determines how it behaves during play, there evidently have been no published measurements of this parameter. The experimental method described here was developed previously for studying tennis balls.[3]

The moment of inertia of a hollow spherical shell of mass m about an axis through its center of mass is given by[4]

$$I_S = \frac{2m}{5}\frac{r_E^5 - r_I^5}{r_E^3 - r_I^3}, \qquad (1)$$

where r_E and r_I are, respectively, the external radius and internal radius of the spherical shell. If the shell is sufficiently thin, that is to say $r_I \approx r_E = r$, Eq. (1) reduces to the more familiar expression for a very thin spherical shell,

$$I = \tfrac{2}{3}mr^2. \qquad (2)$$

Measurements

Five Ping-Pong balls, from the same manufacturer, were used. The external diameter of each Ping-Pong ball was measured 10 times using Vernier calipers with a precision of 0.005 cm. The internal diameters were measured by simply cutting out some very small pieces of the ball and measuring their thickness (H) with Vernier calipers with a precision of 0.005 cm. The internal diameter of each ball is $R_I = R_E - 2H$. The mass of each Ping-Pong ball was also measured 10 times using an analytical balance with a precision of 5 mg.

The results were as follows:
$r_E = (1.988 \pm 0.002)$ cm, $r_I = (1.947 \pm 0.002)$ cm, $m = (2.456 \pm 0.002)$ g. Substituting these values into Eq. (1) yields $I = (6.338 \pm 0.114)$ g•cm^2. If Eq. (2) is used with a value of r equal to the average of r_E and r_I, the result is $I = (6.207 \pm 0.002)$ g•cm^2.

Experimental verification

Now that the moment of inertia has been calculated theoretically, how can we measure the moment of inertia of a Ping-Pong ball? We do this by a method similar to that originally described by Brody.[3] The ball is suspended at the end of a very thin steel wire (0.15-mm diameter) and allowed to oscillate as a torsional pendulum. The period of the simple harmonic oscillations depends on the moment of inertia of the ball and the torsional restoring constant K of the wire ($\tau = -K\theta$), where τ is the torque and θ is the angular displacement. The expression for the oscillation period T of a torsional pendulum is

$$T = 2\pi\sqrt{IK}, \qquad (3)$$

where I is the moment of inertia of the oscillating mass. The moment of inertia I_B of the ball can be found from the measured value of the period T_B if the value of K is also known. To find K we attach an object having very simple geometry and known moment of inertia to the end of the wire. I chose a thin metal disk (mass $M = 5.521$ g $\pm$ 0.002 g and diameter $2R = 5.103$ cm $\pm$ 0.002 cm). The moment of inertia I_D of this disk about an axis perpendicular to the plane of the disk and passing through its center of mass is $I_D = \tfrac{1}{2}MR^2 = (17.971 \pm 0.001)$ g•cm^2. Substituting this value into Eq. (3) along with the measured period $T_D = (1.63 \pm 1)$ s of the oscillating disk, we obtain a value of $K = (1.21 \pm 0.01)$ s^2•g• cm^2 for the wire. Finally, we use this value along with the measured period $T_B = (2.81 \pm 0.01)$ s of the oscillating ball in Eq. (3) to find the moment of inertia of the ball. The result is $I_B = (6.045 \pm 0.002)$ g•cm^2, which agrees with the theoretical values given above to within 5%.

Acknowledgment

This project is supported by the priming scientific research foundation of Changzhou University (ZMF 1002133).

References

1. L. Pauchart and S. Rica, "Contact and compression of elastic spherical shells: The physics of a 'ping-pong' ball," *Phil. Mag.* **78** 225–233 (1998).
2. Paulo A. de Souza Jr. and Gutemberg Hespanha Brasil, "Assessing uncertainties in a simple and cheap experiment," *Eur. J. Phys.* **30,** 615–622 (2009).
3. Howard Brody, "The moment of inertia of a tennis ball," *Phys. Teach.* **43,** 503–505 (2005).
4. See, for example, physics.info/rotational-inertia/practice.shtml.
5. David Halliday, Robert Resnick, and Jearl Walker, *Fundamentals of Physics,* 6th ed. (Wiley, Hoboken, NJ, 2001), p. 354.

Xian-Sheng Cao *received his BA and MS in physics from Lanzhou University and his PhD in condensed matter physics from Northwestern Polytechnical University. He currently is an assistant professor in the School of Mathematics and Physics at the University of Changzhou in China. He has been teaching physics for nine years in China.*
The University of Changzhou, 507 Yifu Science Building, Changzhou City, Jiangsu Province, 213164, China; phygreen@cczu.edu.cn

 DOI: 10.1119/1.3703546