The Moment of Inertia of a Tennis Ball

Howard Brody, University of Pennsylvania, Philadelphia, PA

The moment of inertia of a tennis ball about its center of mass is one of the physical properties that determine how the ball reacts in play, yet there is no measurement of this parameter found in the literature nor is it mentioned in the Rules of Tennis. The moment of inertia determines how much spin the ball acquires for a given angular impulse applied by the racket's strings and also how the ball behaves when it bounces. When a ball bounces, the friction between the ball and the court surface produces a substantial torque. For a given torque, the magnitude of the moment of inertia determines whether the ball slides throughout the bounce or goes into the rolling mode, and if it does roll, the moment of inertia determines the ratio of final horizontal velocity to initial horizontal velocity.[1]

There are several types of tennis balls available in tennis shops that meet the requirements of the International Tennis Federation (ITF) and therefore can be used in tournament tennis. There is the standard ball, which acquires its rebounding ability due to an internal pressure of an atmosphere or so above the ambient, outside pressure. These are called "pressurized" tennis balls and they are used at Wimbledon, the U.S. Open, etc. Because the internal pressurized gas slowly leaks out through the ball cover, these balls go "dead" in a few weeks. That is why they are sold in pressurized containers. To get around this problem, there is the nonpressurized or pressureless tennis ball, which acquires its rebounding characteristics from the resilience of its walls. There is the Tretorn Micro X ball, which is filled with a very light elastic powder to provide the required rebounding characteristics. The powder does not leak out through the walls of the ball, so unlike an air pressurized ball, it retains its rebound characteristics. There is a fourth type of ITF-approved ball, the oversize ball, which is pressurized and conforms to all the ITF standards for mass, compression, bounce, and so on, except that it is about 6% larger in diameter. This type of ball was designed (by the author of this article) to slow down the game of tennis due to its greater air resistance.

The moment of inertia (I) of these four types of tennis balls about an axis through their centers of mass has been measured using a torsion pendulum. For a torsion pendulum, torque = $k\theta = I\, d^2\theta/dt^2$, where k is the torsion constant of the torsion wire and θ is the angular displacement. The solution of this equation is $\omega = 2\pi/T = \sqrt{k/I}$, where ω is the angular velocity of the pendulum.

Using an object of known moment of inertia such as a uniform disk ($I_0 = 0.5mR^2$) on the end of the torsion wire and measuring T_0, the period of the oscillation, the torsion constant k for the pendulum can be determined:

$$k = 4\pi^2 I_0/T_0^2 . \tag{1}$$

The moment of inertia of any object (I_b) can now be determined by attaching the object to the disk on the end of the pendulum and measuring the new period of the oscillation. The new moment of inertia will be

$$I_t = I_0 + I_b . \tag{2}$$

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With the objects of total moment I_t on the pendulum, the new period of oscillation is T_t, and the value of $I_t = k\, T_t^2/4\pi^2$. Substituting Eqs. (1) and (2) into this equation yields for the unknown moment of inertia

$$I_b = I_0\,(T_t^2/T_0^2 - 1). \quad (3)$$

This allows the moment of inertia I_b of an object to be determined if I_0 and T_0 are known and T_t is measured.

A metal disk of mass 0.080 kg and diameter 0.0965 m (with calculated moment of inertia, $I_0 = 9.31 \times 10^{-5}\ \text{kg}\cdot\text{m}^2$) was fixed to the end of the torsion wire using a micro chuck (Wire Gauge Drill Chuck, MSC Industrial Supply Co., part #08601007 @ $7.50) that was threaded and bolted to the center of the disk. The other end of the wire was fixed to the laboratory bench also using a micro chuck. The wire was a standard piece of piano wire (available on eBay and many music stores) of diameter 0.30 mm and length 1.25 m. The period T_0 of the disk-wire system was determined to be 8.140 s.

As a test of the system, a steel disk of mass 0.0638 kg, outer diameter 0.0599 m, with a hole 0.0064 m in diameter in it [$I_d = 0.5m\,(R_o^2 + R_i^2) = 2.89 \times 10^{-5}\ \text{kg}\cdot\text{m}^2$] was attached to the disk already in place on the bottom of the pendulum. The period of oscillation was now 9.325 s and this gives, using Eq. (3), $I_b = 2.91 \times 10^{-5}\ \text{kg}\cdot\text{m}^2$, an agreement to better than 1%.

Tennis Balls

If the moment of inertia of a ball is parameterized by $I_b = \alpha m R^2$, then for a uniformly dense ball $\alpha = 0.40$ and for a hollow shell $\alpha = 0.67$. The results for several different types of tennis ball are

for a pressurized tennis ball:
$T_t = 9.435$ s $m = 0.0549$ kg $R = 0.033$ m
$I_b = 3.20 \times 10^{-5}\ \text{kg}\cdot\text{m}^2$ $\alpha = 0.535$

for a pressureless tennis ball:
$T_t = 9.455$ s $m = 0.0586$ kg $R = 0.033$ m
$I_b = 3.25 \times 10^{-5}\ \text{kg}\cdot\text{m}^2$ $\alpha = 0.509$

for an oversize ball:
$T_t = 9.460$ s $m = 0.0556$ kg $R = 0.0345$ m
$I_b = 3.75 \times 10^{-5}\ \text{kg}\cdot\text{m}^2$ $\alpha = 0.567$

for a Tretorn Micro X ball:
$T_t = 9.441$ s $m = 0.0574$ kg $R = 0.033$ m
$I_b = 3.21 \times 10^{-5}\ \text{kg}\cdot\text{m}^2$ $\alpha = 0.514$

These data would indicate that the pressureless ball had a thicker wall than the standard, pressurized ball and that the oversize ball had a thinner wall than the standard ball. It would also indicate that the powder inside the Micro X ball has low density compared to the density of the outside shell of the ball.

The tennis balls measured in this experiment were used balls, not new balls. They were attached to the pendulum disk by double-sided masking tape. There is an uncertainty of at least a millimeter in the measurement of a fuzzy ball's diameter, which introduces about a 5% uncertainty in the absolute value of α. However, since all the tennis balls were measured in the same manner, their relative values of α are considerably better.

This problem has been investigated theoretically by Rod Cross[2] and he predicted $\alpha = 0.55$ on the basis of the construction of a standard tennis ball. Subsequently, Cross has refined his calculation by taking into account that the ball is made of two concentric shells (the fabric and the rubber) of different density and he now predicts $\alpha = 0.534$.[3]

Other Sport Balls

The α of a wood croquet ball was measured and its $\alpha = 0.402$, indicating it is, as expected, a uniformly dense sphere. The α of a baseball was measured and its $\alpha = 0.378$, indicating it is more dense at its center. The α of a golf ball was measured and its $\alpha = 0.400$. These balls were attached to the torsion pendulum using a small Nd alloy disk magnet (12.5 mm x 5 mm EMSCO, Sergeant Welch part #43516 @ $0.79) and a steel screw embedded in each ball, because the double-sided tape was not strong enough to hold the balls in place. The combined moments of inertia of the magnet, screw, nut, and the micro chuck was estimated to be $1.5 \times 10^{-7}\ \text{kg}\cdot\text{m}^2$, which is much less than 1% of the moment of inertia of the standard

disk (I_0 = 9.31 *10^{-5} kg·m^2) used to calibrate the torsion wire.

Introductory Laboratory

This experiment has been adopted to be a laboratory exercise associated with the introductory physics course. In the lab experiment, there is more emphasis on the other types of sport balls rather than the various types of tennis ball. The baseball reported on above (α = 0.378) was used in collegiate competition. The "baseballs" measured in the undergraduate lab were inexpensive balls (made in China) purchased at a discount store (Target) and had an α close to 0.40, which we did not discover until the student laboratory write-ups were handed in.

The experiment was performed by groups of students (four students per group), and the final write-up was turned in to the lab TA within the two-hour lab period. The instructions for this experiment were available on the course web page a few days before the lab was run.

Appendix

For a sphere with thick walls (outer wall = R_o, inner wall = R_i) and mass = m, the moment of inertia about a diameter is $I = (2/5)m\,(R_o^5 - R_i^5)/(R_o^3 - R_i^3)$. This can be approximated by $I = (2/3)m\,R_o R_i$. For a wall thickness of $0.1R$, this approximation is good to better than 1%.

References

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Howard Brody *is an emeritus professor of physics at the University of Pennsylvania, where he was interim varsity tennis coach for part of the 1991 season. He earned his bachelor's degree at MIT and his master's and doctoral degrees at Cal Tech. He is a member of the International Tennis Federation Technical Commission and author of the book* Tennis Science for Tennis Players *and co-author of the book* The Physics and Technology of Tennis.

Department of Physics and Astronomy, University of Pennsylvania, Philadelphia, PA 19104