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A demonstration of dry and viscous damping of an oscillating pendulum

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Abstract. Damped oscillatory motion is one of the most widely studied movements in physics courses. Despite this fact, dry damped oscillatory motion is not commonly discussed in physics textbooks. In this work, we discuss the dry and viscous damped pendulum, in a teaching experiment that can easily be performed by physics or engineering students.

1. Introduction

Oscillatory motion is one of the main topics in physics, from elementary up to advanced courses. A precise understanding of this motion is of vital importance, not only for physicists but also for engineering students. Considering oscillatory motion, two kinds of damping can occur: the first is viscous damping and the second is dry damping. Although viscous damping is a standard topic in all physics textbooks and there are also a number of papers that deal with viscous damping of mechanical oscillations [1], to the best of our knowledge dry damped oscillatory motion is not normally demonstrated. In this work, we describe the dry and viscous damped motion of an oscillating pendulum as a laboratory exercise to be done in a basic course by physics or engineering students. To achieve our goal, we will give a brief review of both motions.

2. A brief review of damped oscillators

2.1. Dry damped oscillator

The basic characteristics of this type of friction are well defined considering that, for low velocities, the friction force is constant in modulus and always acts in the opposite direction to the velocity. Figure 1 shows a simplified view of the system that we are going to study. As can be seen from the figure, the pendulum consists of a rod with a mass at its end, and it is fixed on the axis of a potentiometer R, C is the centre of mass of the pendulum and l is the distance from the centre of mass to the axis of the potentiometer. The friction force occurs inside the potentiometer between its mobile parts. For this situation, the equation of motion (neglecting the static friction force and considering small angles) is as follows:

$$I\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2} + mgl\theta = \pm\tau_\mathrm{f} \tag{1}$$

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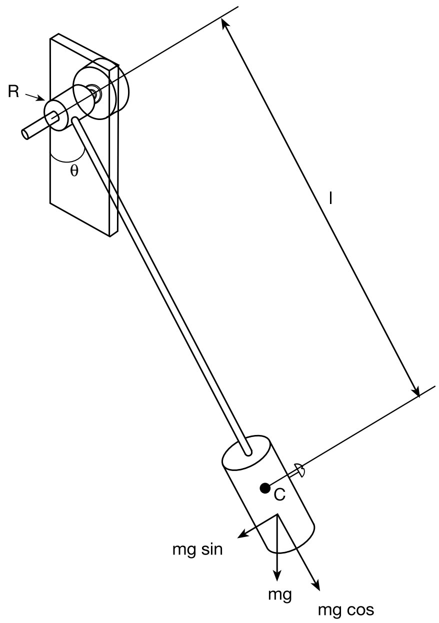

Figure 1. Simplified view of the pendulum showing the forces acting on the system. C is the centre of mass, l the distance from the centre of mass to the fixed point, $mg\sin\theta$ the restoring force and $mg\cos\theta$ the normal force which is the origin of the friction force.

where I is the moment of inertia of the pendulum, the second term on the left-hand side is the restoring torque, l is the distance between the centre of mass and the axis, and the right-hand side is the torque due to friction. The plus or minus sign is chosen so that the torque due to friction acts in the opposite direction to the velocity. Equation (1) can be solved by using a new variable $\Theta = (\theta \pm \tau_f / I\omega_0^2)$ and the solutions for consecutive half-periods put together [2], or in a straightforward manner using Laplace transforms [3]. Due to the nature of this damping, the amplitude of the motion will decrease linearly by a quantity proportional to the friction torque. From the solution, we can find an expression to represent the maxima of $\theta(\cos\omega_0 t = 1)$:

$$\theta(t) = \left(\theta_0 - \frac{4t\tau_f}{\omega_0^2 I T}\right) \tag{2}$$

where T is the period of the oscillation and ω_0^2 is equal to mgl/I. As explained, this damping is linear, and the amplitude decreases by $4\tau_f/mgl$ after each complete period.

2.2. Viscous damped oscillator

The differential equation of the oscillatory motion considering viscous damping is given by

$$I\frac{d^2\theta}{dt^2} + b\frac{d\theta}{dt} + mgl\theta = 0 \tag{3}$$

where $b\,d\theta/dt$ is the damping term given by the friction between the pendulum and the air. The solution of this equation can be found in any basic physics textbook [4] and has the form

$$\theta = \theta_0 e^{-bt/2I}\cos(\omega_0 t + \varphi) \tag{4}$$

where ω_0 is the frequency of motion given by $[mgl/I - (b/2I)^2]^{1/2}$, φ is the initial phase of the motion, θ_0 is the amplitude of the oscillation and $\gamma = -b/2I$ is the damping constant.

3. Experiment

The experiment is to analyse the amplitude of the pendulum as a function of time. Figure 2 shows its arrangement. The pendulum rod is fixed to the axis of a linear potentiometer R (1 kΩ).

The oscillatory motion of the pendulum causes a variation in the resistance of the potentiometer and, consequently, in the potential difference (VR). Using a commercial interface such as PASCO Series 6500 or a homemade one†, VR can be measured, and it is directly proportional to the position of the pendulum, as a function of time.

Dry damping is caused by the friction between the parts of the potentiometer, and viscous damping by the friction between the pendulum and the fluid surrounding it (air or water in this case). When the pendulum is immersed in air, the contribution of dry friction is much larger than viscous friction, and when it is immersed in water, the contribution of viscous friction dominates.

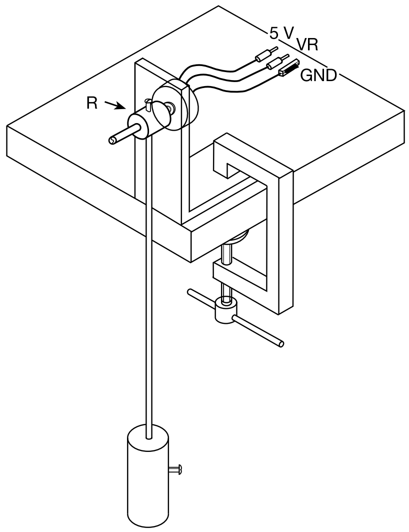

Figure 2. Experimental arrangement of the pendulum to be connected to the computer. The 5 V pin is the supply voltage and VR is the signal that has to be read.

4. Results

The results presented in figure 3 clearly show us the linear and the exponential damping of the two types of friction acting on the pendulum. With the coordinates of the maximum points of the curves, we can construct the slope of the damping for each case, and as can easily be seen in the inset, the two situations are very well separated. When the pendulum was immersed in air, no deviation from linear behaviour was observed, indicating that viscous damping is negligible in this situation; the converse is true when the pendulum is immersed in water. This fact is an experimental verification of the validity of the models used for the two kinds of damping torques. The fits to these constructed curves give us the damping term in each case. For dry damping, the angular coefficient of the straight line observed was 0.014 s^{-1}. In this way, knowing the constant l, the friction torque can be evaluated. Also, the dry damping can be varied using a rubber ring attached to the axis of the potentiometer with the pressure being controlled by a screw. In the case of viscous damping, the constant $b/2I$ determined by an exponential fit to the respective curve was 0.425 s^{-1}. We suggest that this experiment be made with liquids of different values of viscosity in order to compare the damping constants in each case, as a relative measurement of the viscosity of these media.

By changing the dry friction as mentioned and using liquids with different viscosities, the experimenter can achieve a situation where the two dampings are of the same scale. The

† Further information about a quite simple homemade interface can be obtained from the authors via e-mail to: dione@ultra3000.ifqsc.sc.usp.br

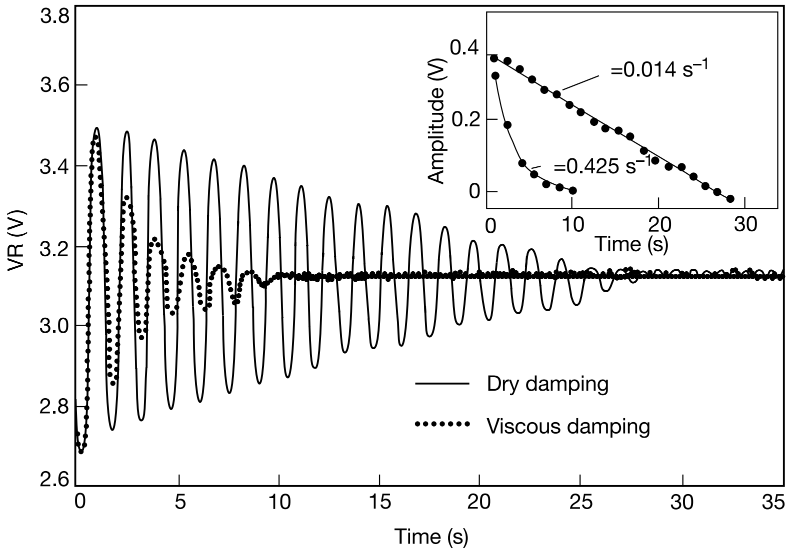

Figure 3. Plots of the pendulum motion as a function of time. The voltage on the abscissa is directly related to the position of the pendulum. The solid line refers to the dry damped pendulum (the pendulum in air) and the dots refer to the viscous damped one (the pendulum immersed in water). The inset in the figure shows the damping slopes of the two curves and the damping constants γ are obtained from fits to these slopes.

equation can be solved for the case where the two types of damping are present, in order to obtain a comparison with experimental results.

5. Conclusions

In this work we have demonstrated a simple way to observe viscous and dry damping effects on an oscillating pendulum. The demonstration of this latter problem is of relevance to physics and engineering undergraduate students, but in spite of this it is not commonly treated in physics textbooks.

Acknowledgments

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