In summary the cosmic-ray muon lifetime was measured with a variety of counters designed to study both the free muon decay and the μ^- capture lifetime. A scintillator box filled with water and various chemical elements dissolved in it was used to investigate the Z dependence of the μ-capture decay. The water detector was found to be as efficient as plastic or liquid scintillators in detecting muons, less expensive, and more versatile, making it an ideal detector material for undergraduate modern physics laboratories.

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Temperature of incandescent lamps

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A method for determining filament temperatures of commercially available incandescent lamps has been presented. The light emitted by a filament at two well-defined wavelengths has been measured by means of two silicon solar cells with interference filters. Since the ratio r of the two signals so determined is a function of the filament temperature, an r against T calibration curve is needed, in order to obtain the filament temperature itself. This calibration curve is determined by means of the well-known dependence of tungsten resistivity on absolute temperature. Considerations about the color index of a star can be raised.

I. INTRODUCTION

In the experiment presented here we want to find the temperature of some commercially available incandescent lamps, analyzing the radiation emitted by their filament at two well-defined wavelengths.

The first step is to determine a calibration curve, using another commercially available incandescent lamp, arbitrarily chosen by us as the standard lamp. By means of a voltmeter and an ammeter, it is possible to find the electrical resistance of its filament, by increasing the voltage until the operating voltage is attained. Then, from the well-known variation law of tungsten resistivity with temperature, the filament temperature can be calculated. A simple relation[1] valid at high temperatures, that permits this calculation, is

$$T = T_0(R/R_0)^{1/1.2}, \tag{1}$$

where R_0 is the resistance of the tungsten filament at ambient temperature T_0.

For each filament temperature, by means of two silicon solar cells with interference filters, the signals $S(\lambda_1)$ and $S(\lambda_2)$ at the two wavelengths allowed by the two filters, can be determined. Then, the ratio $r = S(\lambda_1)/S(\lambda_2)$ of the two signals is plotted against the temperature, to obtain the calibration curve.

The last phase of this experiment consists in the determination of the temperatures of some other incandescent lamps, by measuring the ratio r of the two optical signals, a ratio that allows us to estimate their filament temperature by interpolation on the calibration curve. Then, the reliability of the values so found can be controlled by means of the electrical resistance and Eq. (1), as seen above.

We point out that these considerations are also employed in quite a different field, i.e., astronomy, in order to determine the star's surface temperature. In fact, the *color index of a star* is defined[2] as a value proportional to the logarithm of the ratio seen above.

II. THEORY

Using the Wien approximation to Planck's law for a radiator at temperature T with spectral emission ϵ, the following relation[3] can be derived:

$$S(\lambda) = K\tau\epsilon I(\lambda,T) = K\tau\epsilon c_1\lambda^{-5}\exp(-c_2/\lambda T), \qquad (2)$$

where $S(\lambda)$ is the signal of a proportional detector whose interference filter has a center wavelength λ, $I(\lambda,T)$ is the spectral blackbody radiance, K the detector parameter, c_1 and c_2 the first and the second constants of Planck's law, and τ the transmissivity of the glass bulb of the lamp.

Taking the ratio of the two signals for two wavelengths λ_1 and λ_2, Eq. (2) gives

$$r = \frac{S(\lambda_1)}{S(\lambda_2)} = H\exp\left[\frac{c_2}{T}\left(\frac{1}{\lambda_2} - \frac{1}{\lambda_1}\right)\right], \qquad (3)$$

where H can be considered constant, in first approximation, because it depends very little on the temperature.

Furthermore, taking logarithms,

$$\ln r = -c_2\left(\frac{1}{\lambda_2} - \frac{1}{\lambda_2}\right)\frac{1}{T} + \ln H. \qquad (4)$$

This equation represents a straight line, in a $\ln r$ against $1/T$ diagram, and is fundamental for the determination of high temperatures, in the so-called two-color pyrometry. Advantage can be taken of the fact that this temperature determination depends on the *ratio* of the signals and not on the absolute values of the signals themselves, as in the so-called brightness pyrometry.

Coming back to our experiment, in this case the radiator is the tungsten filament of a lamp, which certainly is not a flat radiator. Thus, the detector does not merely receive radiation emitted in the normal direction, and for this reason the ratio r of the optical signals depends not only on the surface temperature, but also on the geometry of the filament and on its orientation in space. This is because Lambert's cosine law[4] is not valid for tungsten, more precisely because the degree of nonvalidity of this law is also affected by the wavelength. In fact, even if the filament is placed exactly parallel to the receiver, and far away from it, owing to its circular cross section, the receiver sees a high percentage of the filament surface under directions which are quite different from the normal ones to the single portions of areas. In Ref. 3, p. 303, a useful discussion about this topic can be found.

In addition, multiple reflections within the filament coil can cause another serious error. This happens because the radiation coming from inside the coil is more like the blackbody radiation than that which comes directly from the tungsten surface. This causes a drop of the r ratio, as is pointed out in Ref. 3, p. 302. Instead, we note that multiple reflections inside the glass bulb practically do not affect the r ratio.

These considerations lead us to determine the calibration curve we are looking for, on the basis of an *empirical* procedure, as anticipated at the very beginning.

III. EXPERIMENT AND DISCUSSION

The lamp, powered by a stabilized dc power supply, has to be oriented in such a manner that the radiation impinging on the detector does not go through inhomogeneous parts of its glass bulb, for example, where specifications of the lamp are marked. In front of the lamp we put a baffled tube, 1.37 m long, to eliminate stray light and to ensure the perpendicularity of the radiation on the detector's interference filter. Instead of using just one detector with two interchangeable filters, we preferred to use two detectors of a different area with one interference filter each, matching the silicon cell with the larger area with the filter which has a lower value of transmissivity. A dc amplifier is needed to measure the short circuit current of the two detectors.

Obviously, owing to the anisotropy of the filament emission, we had to put the two detectors strictly in the same place, each time we substituted one detector with another. The interference filters used had central wavelengths of $\lambda_1 = 600$ nm and $\lambda_2 = 905$ nm, with a bandwidth each of 15 nm. These wavelengths were chosen, taking the spectral energy distribution of incandescent lamps, the sensitivity of silicon solar cells, and the transmissivity of the glass bulb of the lamps into account.

We used a commercially available 150-W, 220-V, clear glass lamp, with a coil-shaped tungsten filament, in order to obtain the calibration curve of Fig. 1. Then, this "standard" lamp was replaced with another lamp, 100 W, 220 V, with the same kind of filament, and oriented in the same manner to make the results more uniform, bearing in mind

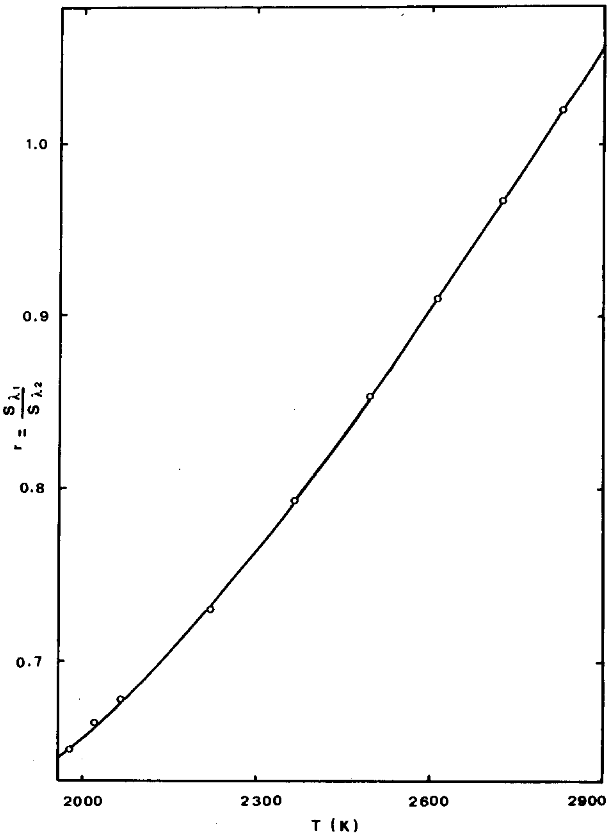

Fig. 1. Calibration curve obtained with a 150-W, 220-V incandescent electric lamp.

Table I. Comparison between the temperatures found by the calibration curve (column 3) and by the electrical resistance (column 4), for three lamps.

1 Lamp rating	2 Voltage (V)	3 T_{cal} (K)	4 T_{el} (K)
100 W, 220 V	140	2330	2320
	220	2775	2760
60 W, 220 V	140	2280	2295
	220	2745	2730
40 W, 220 V	140	2230	2190
	220	2660	2620

the reasons encountered above. This second lamp can be powered for example at an intermediate voltage, and next at the operating voltage, measuring the corresponding r ratios, with the two detectors. Then, on the basis of these values and of the calibration curve, an estimate about the filament temperatures can be made.

Other lamps can be studied by the same method, but they must all have filaments with the same shape, and a power rating lower than the standard. This last requirement is because, in general, only similar lamps have a lower operating temperature of the filament, if their power rating is lower. Thus, interpolation on the calibration curve is made possible.

In Table I we show the temperatures obtained with the optical signal ratio, and in the last column we report for comparison the temperatures obtained with the electrical resistance. It will be noticed that the agreement between the two kinds of temperature is satisfactory in all cases, if one bears in mind the simplicity of the setup. But, on the other hand, we must point out also that the filament temperatures determined by means of the electrical resistance are not the *true* filament temperatures in absolute. There are at least two reasons for this: (i) the formula (1) is not strictly valid; (ii) the temperature is not uniform along the filament axis.

We note also that filament lamps have a cold resistance that varies during their lifetime, especially in the first hours of use.

In order to verify if Eq. (4) applies in our case, we plotted the data of Fig. 1 as illustrated in Fig. 2. In fact, the new diagram shows that the points are not well-aligned, and in our opinion this is due mainly to the causes of error encountered above. This means that we cannot expect to find a good value for Planck's constant h, from the slope of the curve, as we verified taking Eq. (4) and the fact that $c_2 = hc/k$ (c = speed of light; k = Boltzmann's constant) into account.

We also considered projector lamps, which have a quartz bulb instead of a glass bulb, and which have the advantage of allowing measurements to be taken over a wider temperature range, from about 1700 K to about 3400 K. But at the same time these kinds of lamps have a great disadvantage of a very low filament resistance, e.g., 0.120 Ω at 20 °C, for a 250-W, 24-V lamp. This means that errors in the resistance measurements are high, so that the reliability of the filament temperature determination is limited.

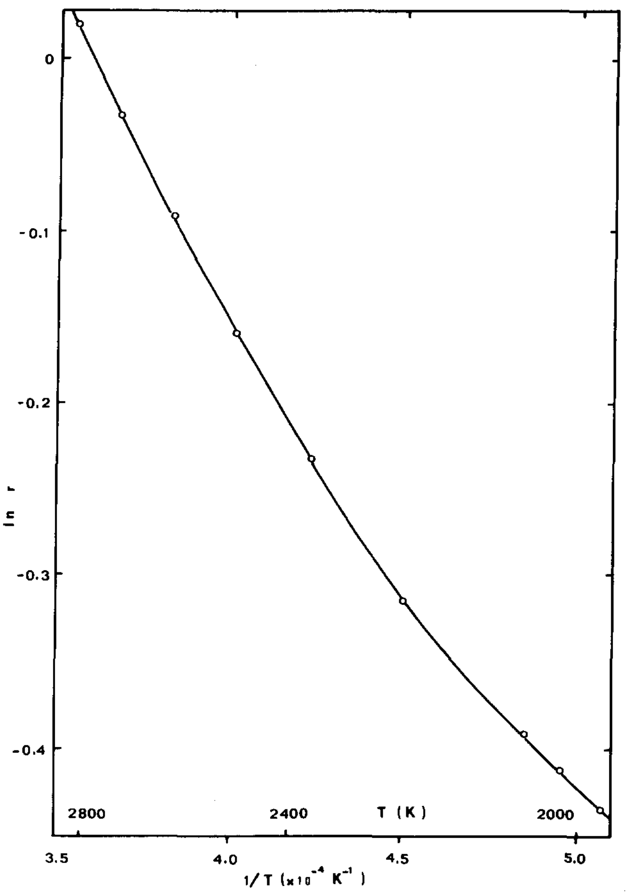

Fig. 2. The same data of Fig. 1, elaborated as suggested by Eq. (4).

As a final comment, let us conclude that incandescent lamps are very interesting thermodynamic systems, whose study is relevant in many respects, as we have seen in this and in other works.[5–7]

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