Measurement of Planck's constant using a light bulb*

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1 Introduction

The purpose of this experiment is to measure Planck's constant through the observation of black body radiation, which all matter emits by virtue of its temperature [1, 2, 3]. A simple setup involving a tungsten-filament light bulb as a source of blackbody radiation, an optical filter as a wavelength selector and a silicon photodetector is used in this experiment. Planck's radiation law is then used to derive an expression for h in terms of the filament temperature, wavelength and photo-intensity. Last, students are expected to keep track of the uncertainties associated with measurements, specifically their propagation to the experimentally determined value of Planck's constant and comment on the validity of the experimental method.

KEYWORDS Planck's radiation law · Blackbody · Intensity · Stefan's law · Wein's displacement law · Tungsten light bulb ·

2 Conceptual Objectives

In this experiment, we will,

1. understand the concept of black body radiation and its relation to temperature T and wavelength λ,
2. understand Planck's radiation law and Stefan's law,
3. learn how to infer meaningful data from experimental graphs,
4. learn how error propagates from measured to inferred quantities,
5. practice manipulating error bars and weighted fit of a straight line,
6. determine numerical value of Planck's constant h.

3 Experimental objectives

This experiment is designed to determine the value of Planck's constant using a commercially available incandescent light bulb (60 W) as a source of black-body radiation. A single color optical filter is also introduced for selecting a wavelength in the visible range. An electrical setup is used to measure resistance and power dissipation. AC voltage is used to illuminate the incandescent bulb at different voltages and the corresponding photointensity is measured on a photodetector. The experiment is divided into two sections. In the first part students will measure the temperature of the incandescent bulb filament. In the second part students will determine the value of Planck's constant by taking into account the measured intensities and bulb temperatures.

4 Theoretical background

4.1 Thermal or blackbody radiation

We see objects when light is reflected from them. At high temperatures, bodies become self luminous and start glowing even in the dark. Radiation emitted by a body due to its temperature is called thermal radiation. All bodies not only emit but also absorb such radiations from their surroundings and finally come into thermal equilibrium. If we steadily increase the temperature we notice that the predominant color shifts from dull red through bright yellow-orange to bluish white heat. This change in color shows that the frequency distribution of the emitted radiation changes with temperature. Since the thermal radiation spectrum strongly depends on temperature, we can easily estimate the temperature of a hot body through the emitted radiation. This is the basis of color thermometry [1].

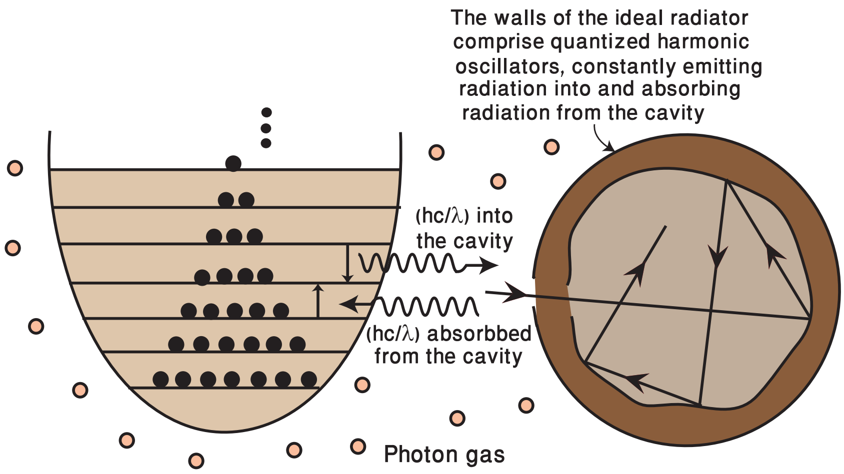

Figure 1: A physical model of a blackbody.

In real practice, the radiation emitted by a body not only depends on the temperature but also depends on the material, shape and nature of its surface. Such factors make it difficult to understand thermal radiation in terms of simple physical models, just like difficulties arise in understanding properties of real gases in term of simple atomic model. The 'gas problem' was resolved by introducing an 'ideal gas'. Likewise the radiation problem is solved by introducing the concept of an 'ideal radiator' for which the spectral distribution depends only on the temperature and on nothing else.

An ideal radiator can be made by forming a cavity within a body, the walls being held at uniform temperature. A small hole is also pierced inside the cavity to examine the nature of the radiations inside the cavity. This ideal radiator is a called black-body, it is a perfect emitter and perfect absorber of all the electromagnetic radiation that falls on it [1, 2, 3]. A physical model is shown in Figure (1).

Some properties of the cavity radiation are given below,

Stefan-Boltzmann law The total emitted power per unit area, over all wavelengths is called radiant intensity $I(T)$ and is given by,

$$I(T) = \frac{P}{A} = \int_0^\infty R(\lambda) d\lambda = \sigma T^4, \tag{1}$$

and

$$P = \sigma A T^4, \tag{2}$$

where, σ ($5.670 \times 10^{-8}\,\mathrm{W/m^2\text{-}K^4}$) is a constant called the Stefan-Boltzmann constant and A is the surface area of the radiating object. $R(\lambda)$ is called the spectral radiancy, tells about how the intensity of the radiation varies with wavelength for a particular temperature. A body which is not an ideal radiator will obey the same rule but with a coefficient,

$$I(T) = \varepsilon \sigma T^4, \tag{3}$$

where ε is a dimensionless quantity, called the emissivity of the material. For a black body, $\varepsilon = 1$, but for all other objects the emissivity is always less than one.

Wein's displacement law Wilhelm Wein (1864-1928) deduced that λ_{max} at which the spectral radiancy is maximum varies as $1/T$ and the product is a universal constant,

$$\lambda_{max} T = 2.898 \times 10^{-3}\,\mathrm{m\,K}. \tag{4}$$

The spectrum of intensity as a function of wavelength for cavity radiation for selected temperatures is sown in Figure (2). The diagram indicates a wavelength shift of the most intense radiation (indicated by the peak) towards lower wavelength as the temperature of the black body increases.

Q 1. For sun, the wavelength at which the spectral radiance is maximum is 500 nm. The radius of the sun is 7.0×10^8 m [1].

(a) What is the surface temperature of the sun?

(b) What is the radiant intensity of the sun?

(c) What is the total radiated power output of the sun?

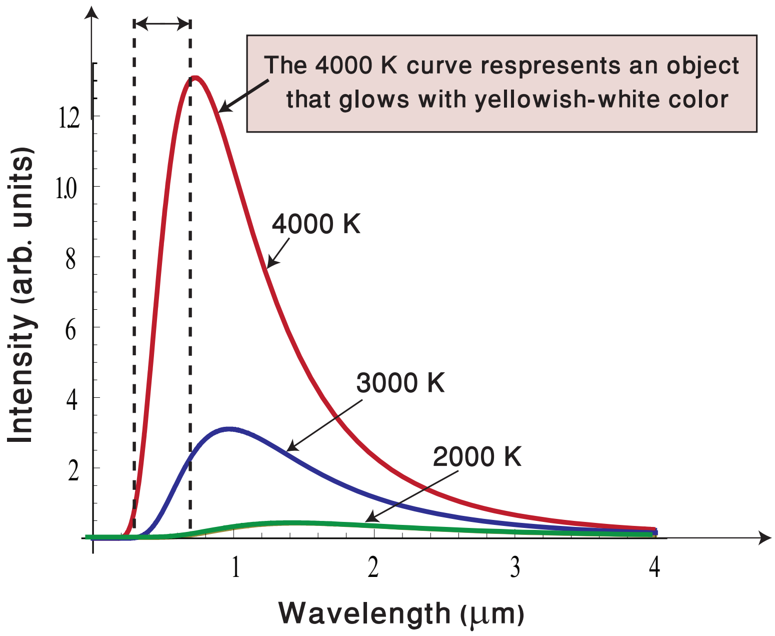

Figure 2: The intensity distribution at various temperatures as a function of wavelength for an ideal blackbody. The arrow shows the region that is approximately equal to the visible range.

Q 2. A surface that is 'white hot' emits about 10 times more power than a 'red hot' surface of the same area. What does this tells us, quantitatively about the relative temperatures?

Q 3. The star Betelgeuse appears to glow red, whereas Rigel appears blue in color. Which star has a higher surface temperature [2]?

Q 4. Estimate the surface temperature of the sun from the following information. The sun's radius is given by $R_s = 7.0 \times 10^8$ m. The average earth-sun distance is $R = 1.5 \times 10^{11}$ m. The power per unit area (at all frequencies) from the sun is measured at the earth to be $1400\,\mathrm{W/m^2}$. Assume that the sun is a blackbody [3].

5 Planck's radiation law

Around the year 1900, attempts were made to find a simple formula that can fit the experimental curves similar to the ones shown in Figure (2). For example, Rayleigh and Jeans derived a relationship based on classical physics and his formula fit the curves in the limit of very long wavelengths (low frequencies) much larger than $50\,\mu$m. Wein's theoretical expression, though a 'guess' fit the experimental curves at short wavelengths but departed at longer wavelengths. A comparison is displayed in Figure (3).

Max Planck tried to reconcile the two radiation laws. He made an interpolation that remarkably fit the experimental data at all wavelengths. Planck's formula related the intensity of the emitted radiation at a particular wavelength λ to the temperature T by,

$$I(\lambda, T) = \frac{2\pi hc^2}{\lambda^5}\left(\frac{1}{e^{hc/\lambda k_B T} - 1}\right). \tag{5}$$

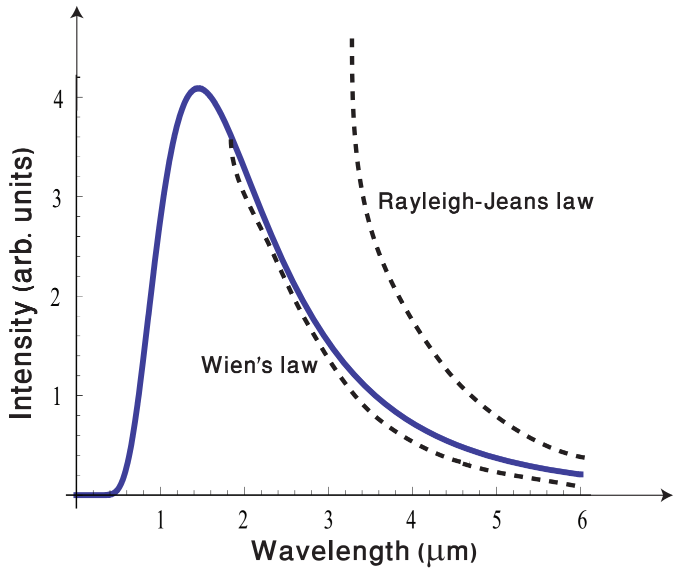

Figure 3: The solid line shows the experimental data at 2000 K, while the dashed lines represent the predictions of the Rayleigh-Jeans's and Wien's law.

Here T is the temperature (in Kelvin), h is Planck's constant, c is the speed of light and k_B is Boltzmann's constant (1.38×10^{-23} JK^{-1}).

Planck's radiation law (5) cab be used to find the intensity ratio being measured at the same wavelength λ but at two different temperatures T_1 and T_2 [4],

$$\frac{I_1(T_1)}{I_2(T_2)} = \frac{e^{(hc/\lambda k_B T_2)} - 1}{e^{(hc/\lambda k_B T_1)} - 1} . \tag{6}$$

In the limit of usable temperatures $(hc/\lambda) \gg k_B T$, the above expression becomes,

$$\frac{I_1(T_1)}{I_2(T_2)} \approx \frac{e^{(hc/\lambda k_B T_2)}}{e^{(hc/\lambda k_B T_1)}} . \tag{7}$$

Thus, Equation (7) holds good for the visible range and typical filament temperatures (upto 2500 K) in incandescent light bulbs. Planck's constant can be determined for any pair of measured intensities and their respective temperatures.

Q 5. Use Equation (7) to express h in terms of $I_1(T_1)$, $I_2(T_2)$, λ, T_1, and T_2.

Q 6. If we fix $I_1(T_1)$ and the corresponding temperature T_1 as reference intensity I_o at temperature T_o, and vary T_2, what will be the expression for h?

Q 7. By integrating the Planck's radiation law (5) over all wavelengths, show that the power dissipated per square meter of a cavity surface is given by [1],

$$I(T) \quad = \quad \left(\frac{2\pi^5 k_B^4}{15 h^3 c^2}\right) T^4 \quad = \sigma T^4 .$$

Calculate the value of Stefan-Boltzmann's constant σ using the above expression. What are its units.

6 Primer on light bulbs and their temperature

Incandescent light bulbs are supposed to be the hottest things in our homes. Some typical temperatures in a common household are given below,

Object Name	Temperature (K)
Cooking oven	600
Candle flame	1700
Sui gas flame	2300

Table 1: Approximate steady state temperatures of some household items.

A tungsten light bulb is shown in Figure (4).

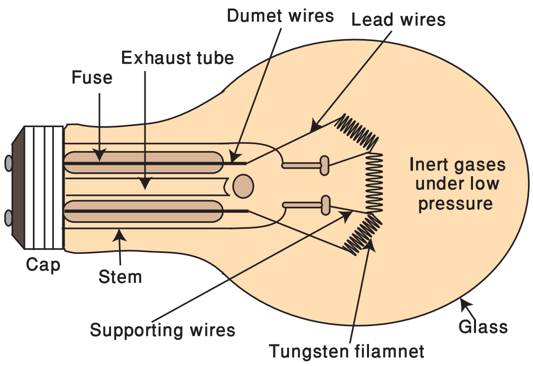

Figure 4: The inner construction of an incandescent bulb.

The measurement of the temperature of the filament without making direct contact with bulb is somewhat tricky task [6, 7, 8, 9]. We will use the following technique to get reasonably accurate results.

For an ideal blackbody, the emitted power can be found out using Stefan-Boltzmann law,

$$P_{rad} = \sigma A T^4.$$

Suppose we supply electrical energy to the incandescent bulb. The electric power that goes into the bulb is

$$P_{elec} = I^2 R., \tag{8}$$

where I is the current and R is the resistance. Assume that the total electrical power going into the filament is entirely emitted by radiative processes, i.e.

$$\begin{aligned} P_{elec} &= P_{rad}, \\ I^2 R &= \sigma A T^4. \end{aligned} \tag{9}$$

For temperatures ranging from room to about 2500 K, a tungsten filament supposedly obeys a linear relation between its resistance and temperature,

$$R(T) = R_o[1 + \alpha(T - T_o)] \tag{10}$$

where R_o represents ambient resistance measured at ambient temperature T_o and α is temperature coefficient of resistivity. However, α may itself depend on temperature. (Relying on published temperature coefficients for commercial tungsten may also be problematic since the filament contain unknown amounts of impurities). Instead we assume a non-linear relation between R and T given by the power law,

$$T = KR^{\gamma}, \tag{11}$$

where K is an unknown constant. Under this assumption, Equation (9) becomes,

$$P_{elec} = VI = I^2R = \sigma AT^4 = \sigma AK^4R^{4\gamma}. \tag{12}$$

Later in the experiment we will use this equation to determine the value of γ for our light bulb. This value of γ will later be used to estimate the temperature of the filament using the following equation,

$$T = \left(\frac{R}{R_o}\right)^{\gamma} T_o. \tag{13}$$

Here R_o is the resistance of the unenergized bulb at temperature T_o.

Q 8. Give one reason for assuming a non linear relation between R and T as opposed to Equation (10)?

Q 9. Convert the proportionality given in Equation (12) into a logarithmic equation relating P_{elec}, R, and γ. State the advantage of working with logs.

7 The experiment

7.1 Apparatus

1. A commercially available tungsten filament incandescent light bulb rated at 60 W, 220-240 V.
2. Variac (0-220 V),
3. Connecting wires,
4. Digital multimeters to measure current I and voltage V,
5. Bandpass filter of red wavelength (650 nm, Thorlabs FB 650-10),
6. Silicon photo-detector (Newport 818-SL),
7. Aperture to modulate power density incident on detector,
8. Digital oscilloscope (BK Precision 2534),
9. Enclosing black box and cylindrical tube (inner diameter= 87.42 mm, outer diameter= 89.39 and length= 41 cm), homemade,
10. Optical rail, homemade.

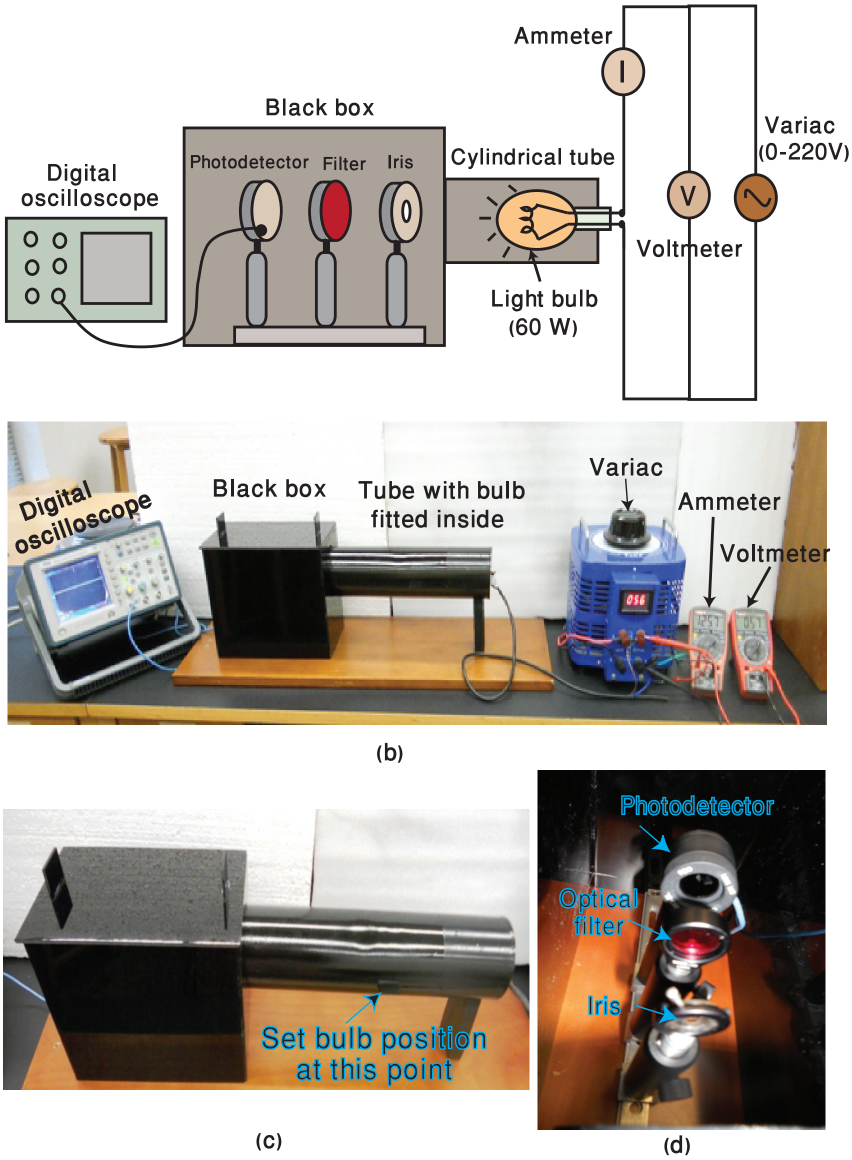

Figure 5: Experimental setup. (a) Schematic diagram, (b) assembled view, (c) adjusting the position of the bulb using a sliding lever, and (d) assembly of optical components inside the black box.

7.2 Experimental procedure

The experimental setup is shown in Figure (5). Complete the assembly as sketched in figure (5a). Connect a 60 W incandescent light bulb to the variac through an ammeter (digital mulimetrer) in series. A digital voltmeter is also connected in parallel to measure voltage values. The variac is an AC transformer that gives variable voltages.
Note: Make sure the variac is unplugged when you are making electrical connections. Wear safety gloves. Remember, never touch bare electric wires.

You are provided with a cylindrical tube with bulb fitted inside it. Inert this cylindrical tube into the black box. Fix the bulb position as shown in Figure (5c). Now assemble the optical components in a way shown in Figure (5d) and connect the silicon photodiode to digital

V (V)	ΔV (V)	I (mA)	ΔI (mA)	R (Ω)	ΔR (Ω)	P (W)	ΔP (W)	T (K)	ΔT (K)	I_{rad} (mV)	ΔI_{rad} (1%) (mV)
34	0.4	100	1	329	6	3.31	0.09	976	12	0.4	1% of I value
57											
.											
.											
210											

Table 2: Model table for experimental results.

oscilloscope.

Q 10. You are provided with digital multimeter and a thermometer. Measure the room temperature resistance R_0 of light bulb. Note down the laboratory temperature T_0. Note down the uncertainties in the measured quantities.

Q 11. Now we would like to measure $P_{elec} = VI$ for different settings of the variac. We will also note the corresponding intensity recorded from the oscilloscope. Take measurements between 30 to 220 V. Note down I, V, I_{rad} and calculate R and P_{elec}. Table (2) provides you with a sample template.

Q 12. Plot a graph of $\log(P)$ versus $\log(R)$ and determine the value of the slope and the intercept using weighted fit of a straight line.

Q 13. Calculate the value of γ using equation (12). You will require this parameter to calculate bulb's temperature.

Q 14. What is the uncertainty in your calculated value of γ. (Hint: Use mathematical relationship for uncertainty in slope).

Q 15. Use the value of γ obtained in Q 18 and Equation (13) to determine the filament temperature T. Calculate the corresponding uncertainties ΔT for the complete range of voltage.

Q 16. Verify that the expression you derived for Planck's constant h has the form:

$$\left(\frac{\lambda k_B}{c}\right)\left(\ln\left(\frac{I}{I_o}\right)\right) = h\left(\frac{1}{T_o} - \frac{1}{T}\right). \tag{14}$$

Q 17. From the table of results chose appropriate values (e.g., the first value) of intensity I and temperature T to be used as reference parameters for I_o and T_o in Equation (14).

Q 18. Use Equation (14) to plot $[(\frac{\lambda k_B}{c})(\ln \frac{I}{I_o})]$ versus the $[\frac{1}{T_o} - \frac{1}{T}]$ data points.

Q 19. Perform weighted fit of the straight line to determine the value of the slope and the intercept.

Q 20. Calculate the value of h using Equation (14).

Q 21. What is the uncertainty in your calculated value of h.

Q 22. The published value of Planck's constant has a value of 6.626176x10^{-34} Js^{-1} [10].

Calculate the percentage accuracy of your determination of h.

Q 23. What are the major sources of uncertainty in this experiment? Suggest two improvements to the experimental procedure that will account for a higher percentage accuracy with respect to the published values.

References

[1] David Halliday, Robert Resnick and Kenneth S. Krane, "*Physics*", John Willey & Sons, Inc, pp. 1021-1025, (1992).

[2] Raymond A. Serway, John W. Jewett, Jr. "*Physics for Scientists and Engineers with modern Physics*", pp. 1186-1191, (2010).

[3] Raymond A. Serway, Clement J. Moses, and Curt A. Moyer, "*Modern Physics*, Thomson learning, pp. 68-74 (2005).

[4] Brizuela, Graciela and Alfredo Juan, "*Planck's constant determination using a light bulb*", Am. J. Phys. 64(6), pp. 819-821 (1996).

[5] Dryzek, J. and Ruebenbauer, "Planck's constant determination from black-body radiation." Am. J. Phys, 60(3), pp. 251-253 (1992).

[6] Izarra, Charles and Jean-Michel Gitton, "*Calibration and temperature profile of a tungsten filament lamp*" Eur. J. Phys, 31, pp. 933-942 (2010).

[7] Koninklijke Philips Electronics N.V, "*Standard A-shape clear*" 11 April 2012. Philips/lighting. 25 June 2012 www.philips.com/lighting.

[8] MacIsaac, Dan, Gary Kanner and Anderson Graydon, "*Basic Physics of the Incandescent Lamp (Lightbulb)*" The Physics Teacher, 37, pp. 520-525 (1999).

[9] Vittorio Zanetti, "*Temperature of incandescent lamps*" Am. J. Phys, 53(6), pp. 546-548 (1985).

[10] Handbook of Chemistry and Physics, 58th ed. (Chemical Rubber Company, 2255 Palm Beach Lakes Blvd, West Palm beach, Florida), pp. E218, E223-E224 (1978).