(b) Here $N = 9$, $n = 6$, $k = 5$, $2 \le x \le 4$. Using the hypergeometric formula, we get

$$P(2 \le X \le 4) = P(X = 2) + P(X = 3) + P(X = 4)$$

$$= \frac{\binom{5}{2}\binom{4}{4}}{\binom{9}{6}} + \frac{\binom{5}{3}\binom{4}{3}}{\binom{9}{6}} + \frac{\binom{5}{4}\binom{4}{2}}{\binom{9}{6}}$$

$$= \frac{(10)(1)}{84} + \frac{(10)(4)}{84} + \frac{(5)(6)}{84} = 0.9524$$

(c) Here $N = 9$, $n = 6$, $k = 5$, $x \le 3$. Using hypergemetric formula, we get

$$P(X \le 3) = P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3)$$

$$= 0 + 0 + \frac{\binom{5}{2}\binom{4}{4}}{\binom{9}{6}} + \frac{\binom{5}{3}\binom{4}{3}}{\binom{9}{6}} = \frac{(10)(1)}{84} + \frac{(10)(4)}{84} = 0.5952.$$

In a sample of size $n = 6$, we need to select at least two master's degree holders.

Example 7.24 A company employs six operators of whom three are women. Four operators are chosen at random from six operators. What is the probability that of those chosen, the number of women will be (a) exactly two (b) two or more (c) at least two?

Solution (a) Here $N = 6$, $n = 4$, $k = 3$, $x = 2$. Using the hypergeometric formula, we get

$$P(X = 2) = \binom{3}{2}\binom{3}{2}\Big/\binom{6}{4} = \frac{9}{15}$$

(b) Here $N = 6$, $n = 4$, $k = 3$, $x \ge 2$, Using the formula, we get

$$P(X \ge 2) = P(X = 2) + P(X = 3) + P(X = 4)$$

$$= \frac{\binom{3}{2}\binom{3}{2}}{\binom{6}{4}} + \frac{\binom{3}{3}\binom{3}{1}}{\binom{6}{4}} + 0 = \frac{9}{15} + \frac{3}{15} + 0 = \frac{4}{5}$$

In a sample of size $n = 4$, we can choose upto 3 women only

(c) $P(\text{at least two}) = P(X \ge 2) = 4/5$

7.6 The Poisson Distribution Another important probability distribution that is also discrete in nature is the Poisson distribution named after S.D. Poisson (1781 – 1840) who first discovered it in 1837.

Experiments yielding observations of *rare events* (i.e. events having a very small chance of occurrence when the experiment is repeated a very large number of times) during a given *unit of time* or *region of space* are called *Poisson experiments.* The unit of time may be a minute, an hour, a day, a month, etc. while the region of space may be a length, area or volume. Following examples are excellent models of Poisson experiments: (i) the number of calls received at a switchboard during a fixed time interval, (ii) the number of typing errors on a page, (iii) the number of defects in a manufactured article, (iv) the number of traffic accidents at a busy crossing per month, (v) the number of arrivals at a service counter during a specified period of time, (vi) the number of breakdowns or failures of a piece of equipment during a specified time interval, (vii) the number of diseased trees per acre of a certain woodland, (viii) the number of death claims per month received by an insurance company.

Like a binomial experiment, Poisson experiment possesses the following properties:

(i) A trial of the experiment results in one of the two possible outcomes; an event occurs or it does not occur.

(ii) The probability that an event occurs in a given unit of time or region of space is the same for all the units.

(iii) The number of events that occur in one unit of time or region of space is independent of the number that occurs in another unit or region.

(iv) The probability that more than one outcome will occur in a short interval of time or fall in a region of space is negligible.

The random variable X, the number of outcomes occurring in a Poisson experiment, is called a *Poisson random variable* and its probability distribution is called the *Poisson distribution.*

7.6.1 Poisson Probability Function The probability function of the Poisson random variable, which represents the number of occurrences of an event during a specified interval, is defined as

$$f(x; \lambda) = P(X = x) = \frac{e^{-\lambda}\lambda^x}{x!}, \quad x = 0, 1, 2, \ldots \tag{7.8}$$

where λ represents the average number of occurrences of the event within the specified interval and $e = 2.71828\ldots$ The value of $f(x; \lambda)$ in (7.8) can be obtained by using the following table (Table A) which gives values of $e^{-\lambda}$ for various values of λ or by using logarithms.

7.6.2 Mean and Variance of the Poisson Distribution The mean (μ) and variance (σ^2) of the Poisson distribution are

$$\mu = E(X) = \lambda \quad \text{and} \quad \sigma^2 = Var(X) = \lambda, \quad \sigma = \sqrt{\lambda}$$

7.6.3 Poisson Process Experience shows that the Poisson probability function may be used in a number of applications with quite satisfactory results. The number of automobile accidents in some unit of time or the number of claims in some unit of time is often assumed to be a random variable which has a Poisson distribution. Each of these instances can be thought of as a process that generates a number of changes (accidents, claims, etc.) in a fixed interval of time or space, etc. If a process leads to a Poisson distribution, that process is called a *Poisson Process*, whose characteristics are as follows:

(i) The occurrences of the event are independent. That is, occurrence of an event in an interval of time or space has no effect on the probability of a second occurrence of the event in the same or any other interval.

(ii) Theoretically, an infinite number of occurrences of the event must be possible in the interval.

(iii) The probability of a single occurrence of the event in a given interval is proportional to the length of the interval.

(iv) In any infinitesimally small portion of the interval, the probability of more than one occurrence of the event is negligible.

Values of $e^{-\lambda}$

$(0 < \lambda < 1)$

λ	0	1	2	3	4	5	6	7	8	9
0.0	1.000	.9900	.9802	.9704	.9608	.9512	.9418	.9324	.9231	.9139
0.1	.9048	.8958	.8869	.8781	.8694	.8607	.8521	.8437	.8353	.8270
0.2	.8187	.8106	.8025	.7945	.7866	.7788	.7711	.7634	.7558	.7483
0.3	.7408	.7334	.7261	.7189	.7118	.7047	.6977	.6907	.6839	.6771
0.4	.6703	.6636	.6570	.6505	.6440	.6376	.6313	.6250	.6188	.6126
0.5	.6065	.6005	.5945	.5886	.5827	.5770	.5712	.5655	.5599	.5543
0.6	.5488	.5434	.5379	.5326	.5273	.5220	.5169	.5117	.5066	.5016
0.7	.4966	.4916	.4868	.4819	.4771	.4724	.4677	.4630	.4584	.4538
0.8	.4493	.4449	.4404	.4360	.4317	.4274	.4232	.4190	.4148	.4107
0.9	.4066	.4025	.3985	.3946	.3906	.3867	.3829	.3791	.3753	.3716

$(\lambda = 1, 2, 3, \ldots, 10)$

λ	1	2	3	4	5	6	7	8	9	10
$e^{-\lambda}$	.36788	.13534	.04979	.01832	.006738	.002479	.000912	.000335	.000123	.000045

Note: To obtain values of $e^{-\lambda}$ for other values of λ, use the laws of exponents.
Example: $e^{-3.48} = (e^{-3.00})(e^{-.48}) = (0.04979)(0.6188) = 0.03081$.