Natural Radioactivity and Statistics

Amrozia Shaheen, Hafsa Hassan, Muhammad Umar Hassan and Muhammad Sabieh Anwar

LUMS School of Science and Engineering

October 23, 2024
Version 2024-1

In statistics, events can be classified either as certain or random. Events whose outcomes can be predicted definitely, such as the boiling of a substance at its boiling point, are certain events. However, in experimental physics, we often deal with random events, i.e., where we can only predict the outcome probabilities e.g., the tossing of a fair coin or a dice are random events. Poisson statistics arise from the counting of random, rare statistically independent processes, examples of which abound in nature. One of the main tasks of this experiment is to statistically verify that radioactivity its statistics are best described by a Poisson distribution.

KEYWORDS

Radioactivity · Background radiation · Poisson distribution · GM tube and radiation detector.

APPARATUS

Gamma source (activity = 10.1 μCi) · Geiger Counter · Geiger Muller Tube · Safety gloves for handling lead.

1 Conceptual Objectives

In this experiment, we will,

1. learn about the Poisson distribution,
2. learn the safe handling of radioactive material,
3. learn about the random statistical nature of radioactivity, and

4. practice mathematical analysis of data, and perform comparisons of experimental data with theoretical predictions.

2 Theoretical Introduction

2.1 The Poisson distribution

An important model for predicting the outcome of random, independent events is the Poisson distribution, named after S. D. Poisson who first proposed it in 1837. Experiments based on observations of rare events during a given unit of time or region of space are called Poisson experiments. The unit of time may be a minute, an hour, a day or a month, while the region of space might be a length, area or volume. Some examples of Poisson experiments are the number of calls received during a fixed time interval, the number of typing errors on a page, the number of accidents at a busy crossing per month, and the number of breakdowns or failures of a piece of equipment during a specified time interval.

The probability density function of the Poisson random variable X represents the number of occurrences of a particular event in a specified interval, and is given by

$$P(x,\mu) \quad = \quad \frac{\mu^x e^{-\mu}}{x!}, \qquad x = 0, 1, 2 \ldots \tag{1}$$

where μ represents the mean number of occurrences of the event within a specified interval, $e = 2.71828$ and x is the number of outcomes occurring in an experiment. For example, $P(x = 1)$ is the probability that an event occurs once in the specified time interval, $P(x = 2)$ is the probability that two events occur in the same interval and so on.

The Poisson distribution has its origins in the Binomial distribution, which models the success of an event x with a given probability p over n measurements, and is given by the equation:

$$P_n(x) = \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x}, \qquad x = 0, 1, 2 \ldots \tag{2}$$

Fixing the mean rate $\mu = pn$, we can evaluate $P_n(x)$ as n goes to infinity, while p remains very small, which is true for rare events.

$$
\begin{aligned}
\lim_{n\to\infty} P_n(x) &= \lim_{n\to\infty} \frac{n!}{x!(n-x)!}\left(\frac{\mu}{n}\right)^x\left(1-\frac{\mu}{n}\right)^{n-x} \\
&= \lim_{n\to\infty} \frac{n!}{x!(n-x)!}\,\frac{\mu^x}{n}\left(1-\frac{\mu}{n}\right)^n\left(1-\frac{\mu}{n}\right)^{-x} \\
&= \frac{\mu^x}{x!}\left[\lim_{n\to\infty}\frac{n!}{n^x(n-x)!}\right]\left[\lim_{n\to\infty}\left(1-\frac{\mu}{n}\right)^{-x}\right]\left[\lim_{n\to\infty}\left(1-\frac{\mu}{n}\right)^{n}\right]
\end{aligned}
$$

The second term is 1, the third term simplifies to $e^{-\mu}$ while the first term is calculated as,

$$
\begin{aligned}
\lim_{n\to\infty}\left[\frac{n!}{n^x(n-x)!}\right] &= \lim_{n\to\infty}\frac{n(n-1)(n-2)\dots(n-x+1)(n-x)!}{(n-x)!n^x} \\
&= \lim_{n\to\infty}\left[\left(\frac{n}{n}\right)\left(\frac{n-1}{n}\right)\left(\frac{n-2}{n}\right)\dots\left(\frac{n-x+1}{n}\right)\right] \\
&= \lim_{n\to\infty}\left[1\left(1-\frac{1}{n}\right)\left(1-\frac{2}{n}\right)\dots\left(1-\frac{x+1}{n}\right)\right] \\
&= 1
\end{aligned}
$$

Hence the binomial distribution for large n, reduces to the Poisson distribution,

$$
\lim_{0\to\infty} P_r(x) \approx \frac{\mu^x e^{-\mu}}{x!}. \qquad (3)
$$

Q 1. Consider the following series of measurements of the counts per minute from a detector viewing a $^{22}_{11}$Na source, What is the decay rate and its uncertainty?

Number	**Counts**
1	2201
2	2145
3	2222
4	2160
5	2300

2.2 Radioactivity as a Poisson process

An important property of the Poisson distribution is that the standard deviation is equal to the square root of the mean value. The poisson distribution can be approached as a limiting case of the binomial distribution for large number of occurrences. Experience shows that the Poisson probability distribution has numerous applications satisfactorily explaining many natural processes. Radioactivity is a random process in which atomic decays are independent. This means that the probability of decay in a time interval has no affect on the probability of decay in any subsequent time interval.

Q 2. This is a challenging, optional question. Show that the mean of a Poisson distribution, Eq.(1) is μ and the standard deviation is also μ

Let us turn to an experiment in which we record counts registered by a Geiger counter in ten $1-s$ intervals, and the counts are [2 6 2 3 2 6 2 3 2 1] 2 counts in first $1-s$ interval, 6 in the second, 2. The mean value of 2.7, the number of counts is 2.7 and the standard deviation is 1.7.

Counts	**Frequency**
0	0
1	1
2	6
3	2
4	0
5	0
6	2

Table 1: Frequency of counts from background radiation.

The number of successes of an event can be obtained through the frequency distribution. Table (1) shows a relationship between the number of counts detected and respective frequencies. The histogram of the data sowing frequency distribution is depicted in the Figure (1).

Q 3. The number of particles emitted each minute by a radioactive source is recorded for a period of 10 hours and a total of 1800 counts are registered. During how many 1-minute intervals should we expect to observe [3]

(a) no particles,

(b) 10 particles.

3 Experimental Objectives

The Physlab is authorized by the Pakistan Nuclear Regulatory Authority (PNRA) for the permissible use of radioactive sources for this experiment.

We will start the experiment with the use of a Geiger-Muller tube, GM counter and data acquisition script within Matlab, first measuring background radiation. We will then

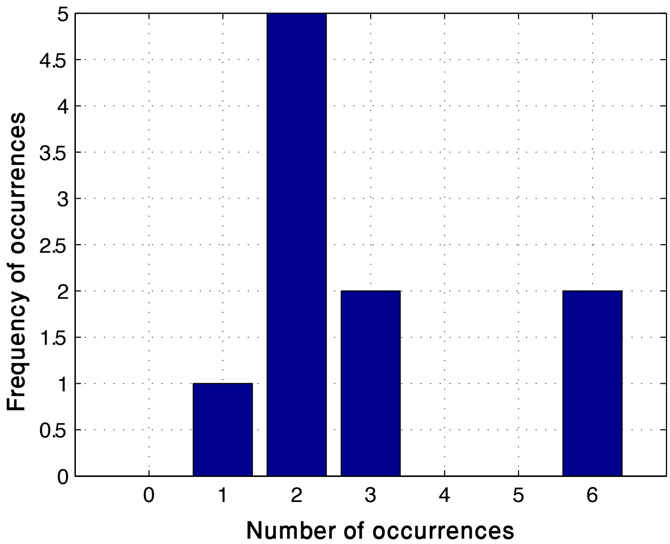

Figure 1: A histogram for background radiation, corresponding to Table 1

demonstrate the random nature of radioactive decay by using the gamma source and will fit our results with the Poisson distribution.

4 Apparatus

The experiment involves the following equipment.

- Geiger Muller tube and counter
- Radioactive source
- Clamp and holders

Photograph for the experimental apparatus is given in Figure (4). Here is a brief description of the important components listed.

4.1 Geiger Muller (GM) tube and counter

The precision Geiger counter manufactured by (Daedalon), takes input from the GM tube (also from Daedalon), detecting the radiation particles and feeding the signal into the computer. The GM counter clicks every time a particle is detected. The GM tube works best when supplied with 900 V. Below this value, its efficiency decreases and we risk losing our data but remember higher voltage levels can also damage the GM tube. **So do not increase the input voltage beyond 900 V.**

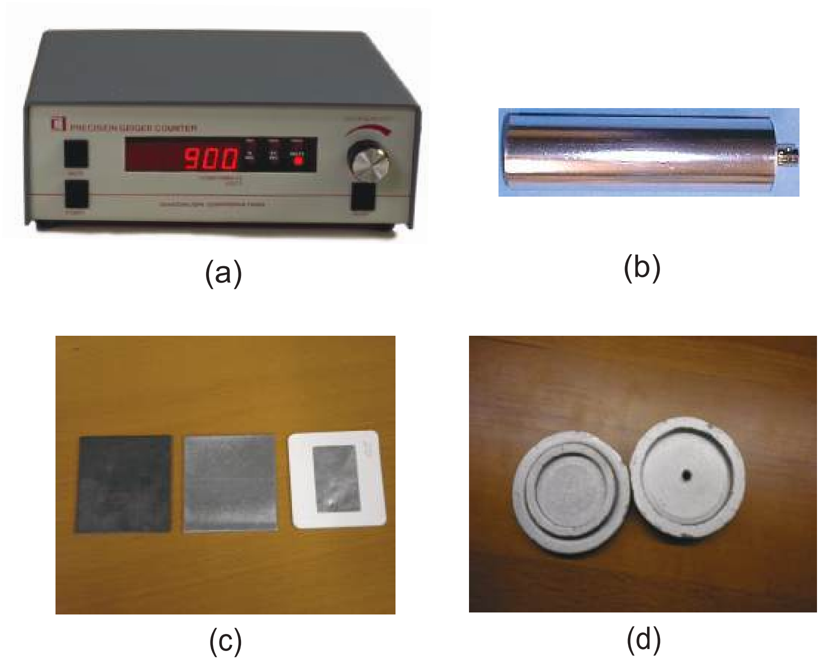

Figure 2: The apparatus provided in the experiment. (a) GM counter, (b) GM tube, (c) Absorbers, Left to Right: Lead sheet, Aluminum sheet and Aluminum foil (d) source holder.

The GM tube is the sensing element of the Geiger counter that detects a single particle of ionizing radiation. It consists of a tube filled with a low pressure (0.1 atm) inert gas (helium, neon or argon). When a radiation beam is incident on it, the gas ionizes creating ions and electrons. The ions move towards the cathode and electron towards the anode due to strong electric field created by the electrodes. The tube is schematically shown in Figure (3). The ions pairs gain sufficient energy to ionize further molecules through collision, and in this way a short, intense current pulse is produced that is detected by the counter.

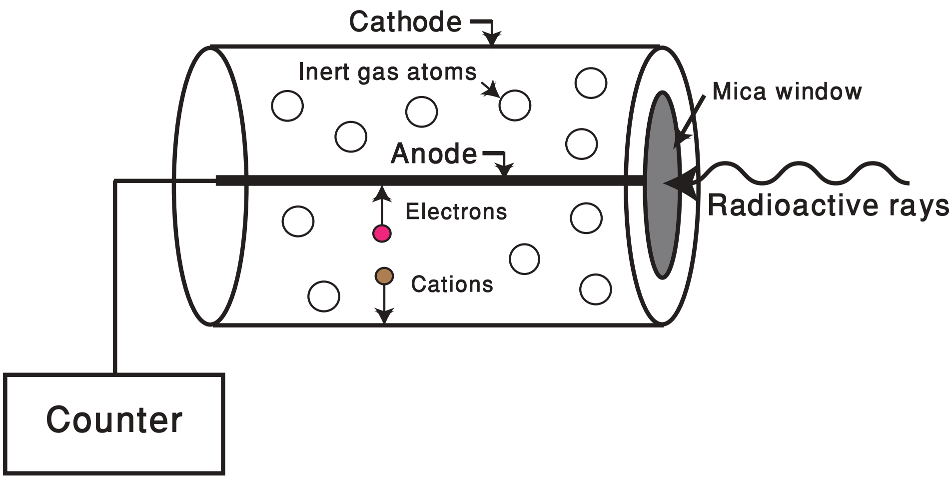

Figure 3: The construction and working model of the Geiger Muller tube.

4.2 Radioactive sources

Some properties and reaction schemes of radioactive sources used in this experiments are listed in Table (2),

Radiation	Source	Activity	Half life	Reaction scheme
Alpha	Polonium, $^{210}_{84}$Po	0.1 μCi	133 days	$^{210}_{84}\text{Po}\rightarrow^{206}_{82}\text{Pb}+^{4}_{2}\text{He}$
Beta	Strontium, $^{90}_{38}$Sr	0.1 μCi	28.6 years	$^{90}_{38}\text{Sr}\rightarrow^{90}_{39}\text{Y}+e^{-}+\bar{\nu}$
Gamma	Cobalt, $^{60}_{27}$Co	10.1 μCi	5.26 years	$^{59}_{27}\text{Co}+^{1}_{0}n\rightarrow^{60}_{27}\text{Co}\rightarrow^{60}_{28}\text{N}+e^{-}+\gamma$

Table 2: Important properties of radioactive sources

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4.3 Lead source holders

The sources will be placed inside the lead source holders and mounted on the metal clamp holder. **Note that gloves should be worn while handling lead, and sources should not be touched with bare hands**. The forceps should be used to hold the sources at all times. The provided Allen key can be used to loosen or tighten the clamps in the setup.

You must ask the demonstrator to issue you the sources, which must be safely returned to him/her after completion of the experiment. Do not leave the experimental arrangement unattended.

5 Experimental Method

5.1 Background radiation measurement

Q 4. Turn the **VOLTAGE ADJUST** knob of EN-03 Geiger counter fully counter clockwise and switch it ON from the **POWER** button. Press **MODE** button 5 times so that LED behind *VOLTS* blinks, and set the voltage to about 900 V. You should hear distinct *clicks* from the counter. First of all you are required to record the background counts.

Q 5. Familiarize yourself with the downloaded Matlab codes.

- Download the software codes for this experiment, extract and copy them to a directory of your choice (preferably in drive `D:` after your registration number i.e. `D:2024001001001`)
- By default, the automation script starts collecting data through the default serial port, with 1 s sample window and 10 minutes session time.

- All the scripts are fairly commented. You can open each script or use the Matlab `help` command to explore a particular script. e.g. `help collect_data`.
- Open Matlab and change the current directory to where you copied the codes. Make sure you have copied all the script files.

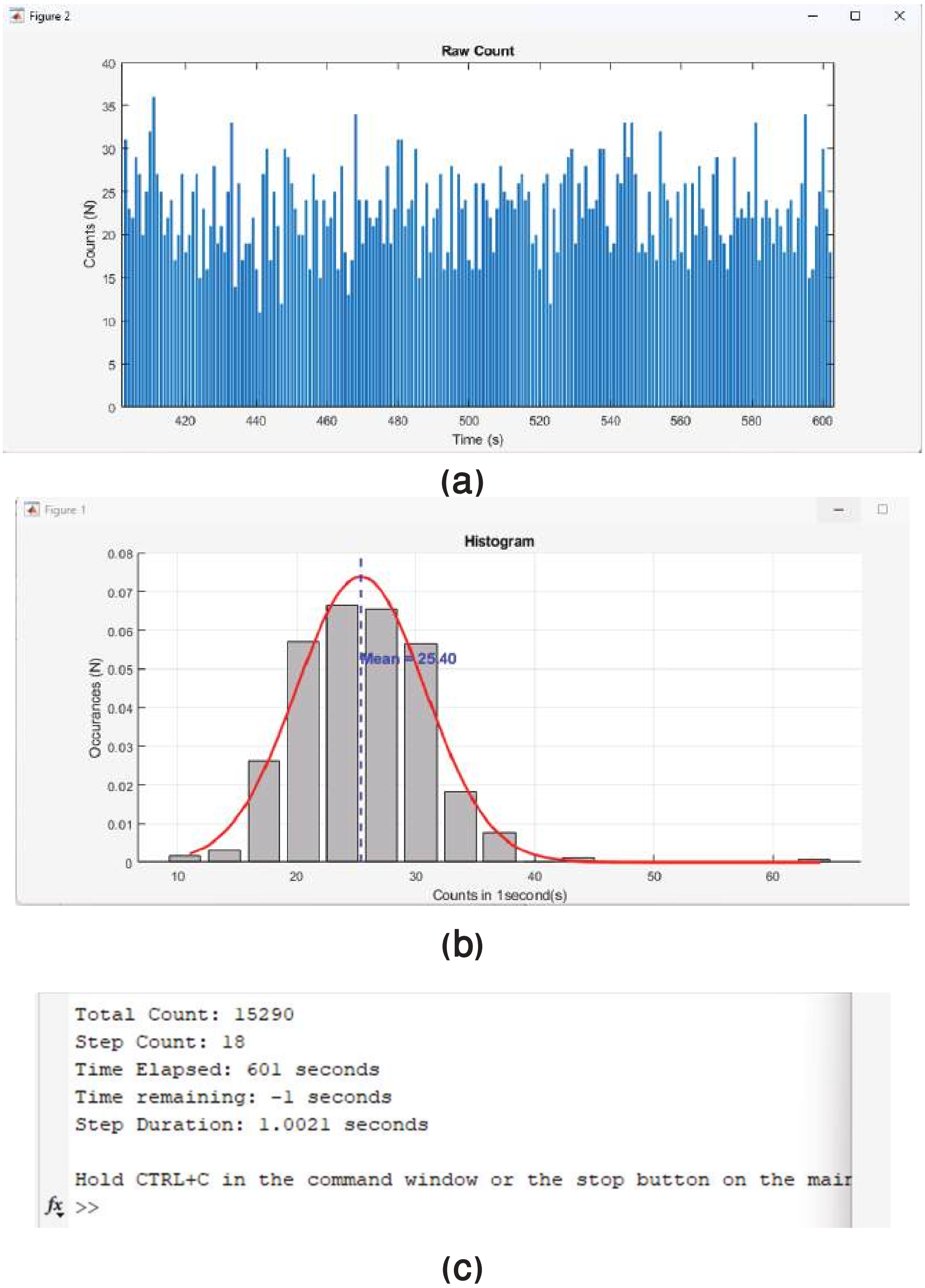

Figure 4: Screenshot of the various components of the PhysLab Geiger Matlab Utility. (a) Bar graph of the raw counts, (b) histogram, (c) console preview

- Although you can learn advance usage of the included scripts from the help command, for basic usage enter `collect_data` in the command window and follow the on-screen instructions to begin collecting the data. The script asks for the duration of session and collection window size, and also displays the names of the variables it uses to save the data. You can use these variables later to plot and analyze the results.

Q 6. Use Matlab to find, from your data, the mean background count per second, and the mean background counts per minute. Record this in your notebook. Repeat this experiment 5-6 times, and take the mean background rate. **You need to subtract this background rate from all the results of the subsequent experiments**.

5.2 Investigating the statistical nature of radiation

In this part you will verify that radioactive decay follow the Poisson distribution. You will use the γ source. You will record counts for a particular time interval. Subsequently, you will process and analyze your data using MATLAB and do a comparison between obtained experimental and theoretical results.

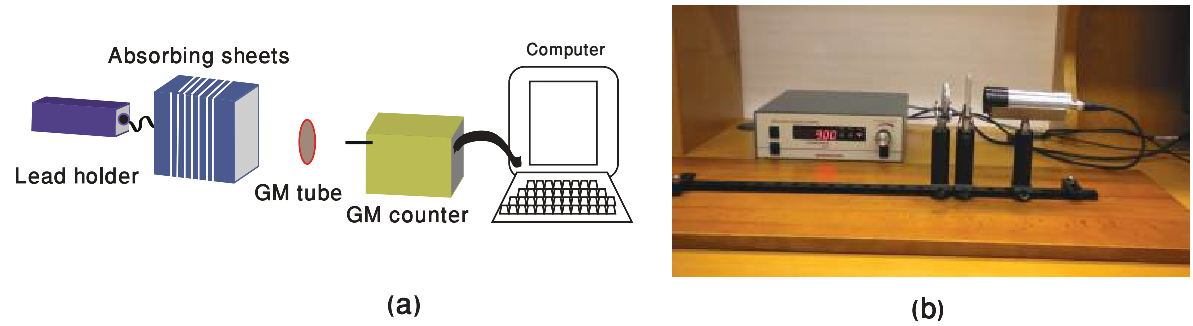

Figure 5: Experimental setup: (a) schematic and, (b) assembled views.

You can now collect the radioactive sources from the instructor.

Q 7. Set up the experiment as shown in Figure (5b), but without absorbing sheets.

Q 8. Place the γ-source in the back lead holder using tongs. Ensure that the printed surface of the γ-source is towards the GM tube. Place the source at a distance of about 2 cm from the GM tube.

Q 9. Enter `collect_data` in the command window and follow the on-screen instructions to begin collecting the data.

Q 10. Once the session has finished, find the mean value of counts per second (call it μ) and the standard deviation of the counts per second (call it 5).

Q 11. For a Poisson distribution, one expects $s=\sqrt{\mu}$. Is this true for your data?

Q 12. Plot a histogram of the counts per second data. Look up the command **hist** in Matlab.

Q 13. Does this look like a Poisson distribution with mean μ. Use the command **poisspdf** in Matlab to verify.

Q 14. Try fitting your histogram from Q14 to a Poisson distribution using the command **histfit** in Matlab.

Q 15. Comment on your data.

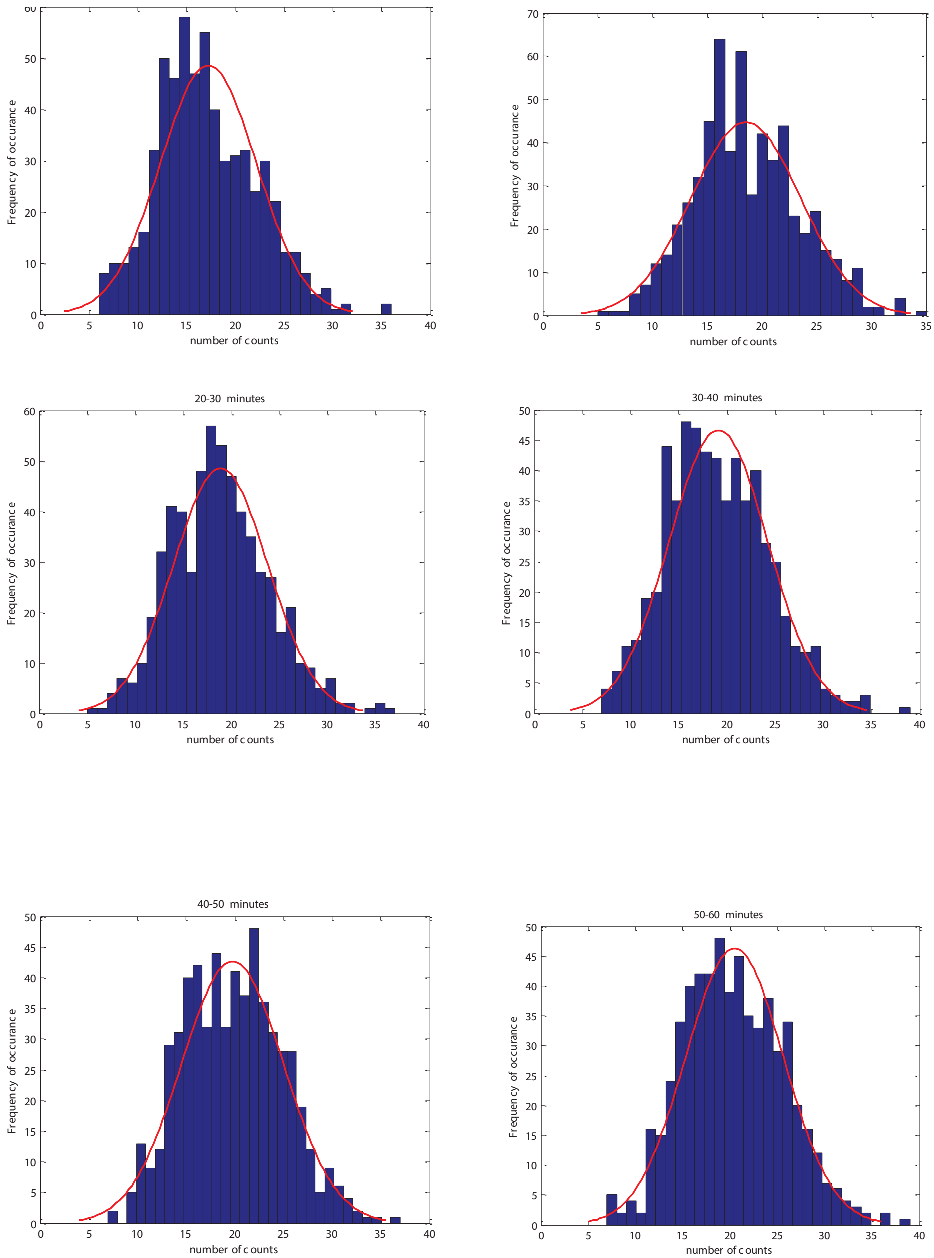

Figure 6: Experimental data distribution with overlaid Poisson distribution curves. Time 600 s, $d = 2$ cm.

References

[1] A. W. Harrison, *Intermediate atomic and nuclear physics*, St. Martin's Press. New York, (1966), pp. 116-122.

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[3] M. L. Boas, *Mathematical methods in the physical sciences*, pp.729.