Spinner with a smartphone*

Muhammad Sabieh Anwar
PhysLab at the LUMS Syed Babar Ali School of Science and Engineering

October 27, 2024
Version 2024-v1

We are all familiar with the iconic $F = ma$ which relates acceleration a of objects of mass m to the unbalanced force F applied on them. A similar relation holds when objects rotate. This leads to an analogous equation $\tau = I\alpha$. The correspondence is one-to-one and highlighted in the table below.

force F	torque τ
mass m	moment of inertia I
linear position x	angular position θ
linear speed v	angular speed ω
linear acceleration a	angular acceleration α

Therefore if F accelerates an object in a straight path, τ *rotates* the object, causing angular acceleration α. A torque comprises a force and a moment arm, which specifies how far does the force act from the point of rotation. This is illustrated in the accompanying diagram (Fig. 1) which I suggest to follow closely.

In the current experiment, we will make measurements with a smartphone which truly, and maybe unnoticeably, is a versatile device for understanding and investigating natural phenomena. Our "smart physics laboratory" [`https://physlab.org/smart-physics/`] in fact lists several interesting endeavors with the phone's in-built sensors. Notably the smartphone has a gyroscope that measures angular velocity ω which is a vector comprising three components $(\omega_x, \omega_y, \omega_z)$. The directions x, y and z for a smartphone are defined in a standard fashion and are shown in Figure 2. So, if the smartphone rotates about the z axis, we expect ω_z to capture the non-zero angular speed in $\mathrm{rad\,s^{-1}}$ and the other components ω_x and ω_y to be zero.

For the experiment, we have made a circular platter on which we mount the smartphone in its purpose-built clamp. The diagram of the setup is sketched in Figure 3. The platter

*No part of this document can be re-used without explicit permission from Dr. Muhammad Sabieh Anwar. I like to thank Muhammad Umar Hassan for help in making the diagrams.

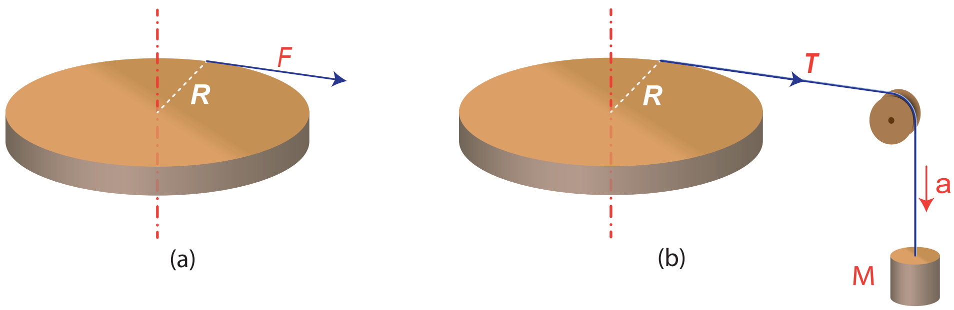

Figure 1: The rotating spinner. (a) A force $\mathbf{F}$ acts at a distance $\mathbf{R}$ from the axis of rotation of a disc. The axis of rotation is shown by a dot-dashed vertical line. The torque in this case would be $\tau = \mathbf{r} \times \mathbf{F}$ and if the radius and force are perpendicular, the scalar form becomes $\tau = r\,F$. (b) The force is the tension $\mathbf{T}$ in the string. The string goes over a pulley and is attached to a mass M at its lowest end. The mass accelerates with a value $\mathbf{a}$ which is equal to the linear acceleration of the string, and the linear acceleration of the spinning disc.

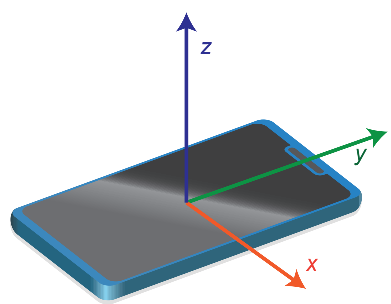

Figure 2: The conventional orientation of the smartphone's axes, used by its internal sensors. For example, ω_z will represent angular speed of the phone as it rotates about an axis perpendicular to its own plane (the z axis), and a_x will be its acceleration parallel to the smaller width, *etc.*

is connected to an axle which runs vertically through the apparatus. Three pulleys co-rotate with the axle and the platter. The torque is applied on the edge of the upper pulley through a string which bends downward over another pulley and is finally connected to a hanging mass M moving under the influence of gravity. The torque τ is made by the application of the tension T in the string which is a distance R away from the axle, $\tau = T\,R$. The axle, pulleys, platter and smartphone all rotate with an angular speed ω_z and angular acceleration α. The smartphone's gyroscope gives us the value of ω_z, whose derivative returns the angular acceleration α. The equation is,

$$\alpha = \frac{d}{dt}\,\omega_z. \tag{1}$$

The linear and angular quantities are also correlated through,

$$
\begin{aligned}
v &= \omega_z R && (2)\\
a &= \alpha R. && (3)
\end{aligned}
$$

In the dynamical equation that has been mentioned above,

$$\tau = I\alpha, \tag{4}$$

the applied torque is simply,

$$\tau = T\,R. \tag{5}$$

The moment of inertia I for our spinner, however, needs to be considered in a bit more detail.

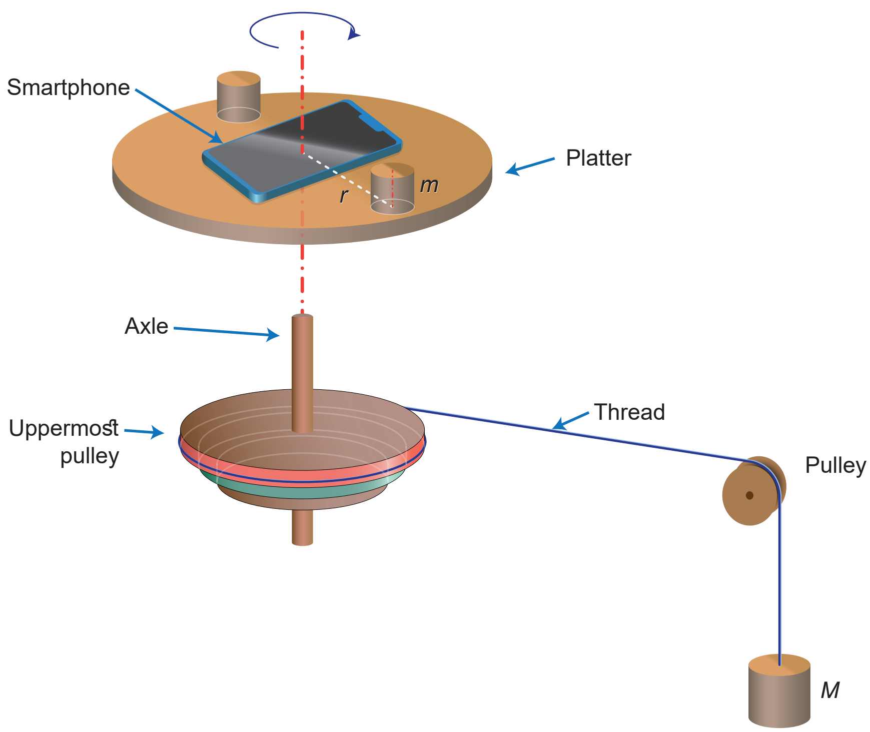

Figure 3: Scheme of the experiment is depicted here. Various parts have been labelled.

The spinning parts of the assembly comprise three horizontally oriented pulleys encircling a vertical shaft which is called the axle (see the diagram), a platter on top of it, and a smartphone positioned inside its clamp affixed on the platter. There are also some screws and minor fasteners. These ingredients remain constant through the experiment and contribute to a moment of inertia I_{fixed} whose total value is (4.4 ± 0.1) kg m^2 for our setup (but can vary from instrument to instrument). Do remember that the I_{fixed} mentioned does include the smartphone as well.

While performing the experiment, we will position two masses, each 50 g symmetrically, a distance r away from either side of the axle. These masses therefore contribute another amount of moment of inertia, $2mr^2$ which can be varied. Therefore, the total moment of inertia in Equation 4 for a certain run of the experiment becomes $I = I_{\text{fixed}} + 2mr^2$.

Now here are a few questions for you. I am sure you will read a bit more about the moment of inertia if you are not already familiar with the concept.

Q 1. What are the dimensions and SI units of all quantities mentioned in the Table on page 1.

Q 2. The gyroscope returns ω_z. How do you aim to obtain the acceleration α from ω_z?

Q 3. Draw the free body diagram for the mass M.

Q 4. Using Equations 2 through 5, derive an expression on how the angular acceleration α depends on the positions r of the masses m placed on the rotating platter. While keeping in mind that friction will reduce the torque from τ to $(\tau - \tau_{\text{friction}})$, come up with an expression for the angular acceleration based on measurable quantities, M, m, R, the acceleration due to gravity g, I_{fixed} and τ_{friction}. This is probably the most creative part in this experiment.

Q 5. Once you have created the expression for α (don't forget to linearize), identify the independent and dependent variables. Perform the experiment. Transfer the data from the smartphone to your PC or laptop. Finally, plot and compare results with theory. What do the plots of ω_z look like? Are they like a sawtooth, going up and down? I am leaving it to your best judgment, on choosing what data to plot and how to interpret. Hence, I am scant on the instructions here!

Q 6. Does your data allow you to estimate the frictional torque τ_{friction})? Does friction depend on the angular speed or any other parameters? See reference [2].

Q 7. The linear acceleration a comes from the angular acceleration α through Equation 3. However, the smartphone is equipped with a three-axis accelerometer as well, that measures the three components (a_x, a_y, a_z). How can you directly measure the a that will be used in your final expression for the α, derived in the preceding questions? This is a bit of an experimental challenge!

References

[1] "The circular Atwood machine", A.C. Marti, M. Monteiro and C. Stari, *The Physics Teacher* **62**, 150–151 (2024).

[2] "Precise measurement of velocity dependent friction in rotational motion", J. Alam, H. Hassan, S. Shamim. W. Mahmood and M.S. Anwar, *European Journal of Physics* **32**, 1367 (2011).