Magneto-optical Kerr effect in textured magnetic profiles

by

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CERTIFICATE

I hereby recommend that the thesis prepared under my supervision by: —— Muhammad Umer —— on title: —— Magneto-optical Kerr effect in textured magnetic profiles ——— be accepted in partial fulfillment of the requirements for the MS degree.

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ABSTRACT

We have studied the magneto-optical Kerr effect in textured magnetic profiles by using a universal approach. We have employed the thin film approximation in derivation of the analytical expression for the multilayer system. This approximation helps us to ignore the higher order term of layer thickness and magneto-optical coefficient. Different geometries have been studied and simulations have been performed by using MATLAB as a tool. During the simulation we have not employed any approximation and simulate each system as exact. A graphical user interface has also been build using MATLAB which enable ones to simulate different multilayer system.

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Chapter 1

Introduction

1.1 Jones Calculus

This method was introduced by R.C. John in 1941. In this method, the polarized light is represented as a vector which is called Jones vector and an optical component is represented as a matrix which is called Jones matrix. In Jones calculus method, we consider the polarized light in free space or in a medium which is homogeneous and isotropic with zero attenuation. It represent the polarization state and the sum of the squared components of the Jones vector give rise to identity, so it is convenient to keep it normalized at the very beginning.

Jones calculus method is not applicable when the light is partially polarized or is incoherent. Even after passing from an optical component, if the light gets depolarize then the Jones calculus is not able to show that depolarization effect[1].

1.1.1 Jones vector

The elements of the Jones vector are complex in nature, which makes it a complex vector[2]. We can use either component of light, the electric field or the magnetic field to represent the Jones vector of light. Both of them, in general, have two of its components in the Cartesian coordinate system, which both are orthogonal to the direction of prorogation of light simultaneously. By either way we represent the Jones vector, we can get the other representation by just taking the cross product with vector which contains the material properties say $\vec{k}$. Even they can have the three components in the Cartesian coordinate system, while we are discussing the regime of the birefrigerant materials or the material for which $\vec{k}.\vec{E} \neq 0$.

1.1.2 Jones Matrix

Any optical component, i.e; polarizes and retardants, can be represented as a matrix in Jones calculus method[3]. These matrix acts like operators and the vectors are simplify the states. These operators are constructed in the same basis set in which the state is present on which the operator is going to operate.

1.2 Faraday Effect

The effect was first observed by Michael Faraday in 1845. It describes that the plane of polarization of the light get rotated when the light passes through a birefringent material. The materials could be either the linearly birefringent or circularly birefringent in nature. Faraday observes the effect in the isotropic substances which were placed in the magnetic field[2]. Actually the magnetic field applied to the isotropic medium, helps to split the

energy spectrum of the atom and hence give rise to the birefringence properties in that medium. When the light passes through it, either the left and right or horizontal and vertical components of light sees different material properties, depending upon either the material is showing circularly birefringence or linearly birifringence respectively. Due to different material properties for both components of light, they move with different speed. Phase got added to component which is moving with higher speed and the plane of polarization will get rotated. The extent of rotation is linearly proportional to the magnitude and direction of the applied magnetic field[2, 4], and the distance traveled by the light[4].

$$\tau \propto \mathbf{BL},$$
$$\tau = V\mathbf{BL}.$$

Where V is the Verdet constant, τ is the rotation in radians and $\mathbf{B}$ is the component of magnetic field which is in the direction of propagation of light[2]. The Verdet constant depend upon the temperature and the wavelength of the light. It has the units of radians per unit magnetic field strength and per unit length[4].

1.3 Magneto-optical Kerr Effect

This effect was observed by J. Kerr in 1877. When the light reflected from the magnetized material, the plane of polarization gets rotated in the reflected beam[4, 5, 6] and the intensity of the beam also changes[4, 5, 6], this effect is Magneto-optical Kerr effect. Kerr effect is categorized in four ways.

1. Polar Kerr effect

2. Transverse Kerr effect

3. Longitudinal Kerr effect

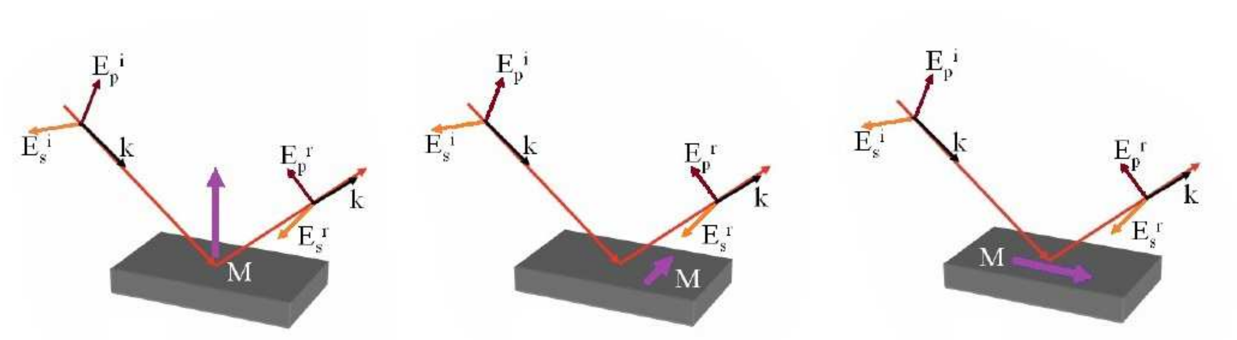
4. Quadratic Kerr effect

Polar Kerr effect rises when the applied magnetic field on the sample has a direction perpendicular to the plane of incidence. After reflection two changes occur in the properties of light. One is the change in the polarization and second is the change in the ellipticity of the light[4, 5].

Equatorial Kerr effect arises when we have the magnetization in the material which is in the plane of sample but perpendicular to the plane of incident light. Equatorial MOKE is directly proportional to the magnitude of the component of applied magnetic filed which is in the direction of the incident light. It has a particular appearance in the absorbing mediums[4, 5].

Meridional Kerr effect arises when the applied magnetization on the sample has a direction in the plane of the incident light and also in the plane of the sample. This has the effects like, change in the polarization of light and change in the ellipticity of light[4, 5].

Despite of these linear effects, there exist a quadratic effect, which depends on the 2*nd* order of magnetization and is called Voigt effect or Quadratic MOKE[4, 6].

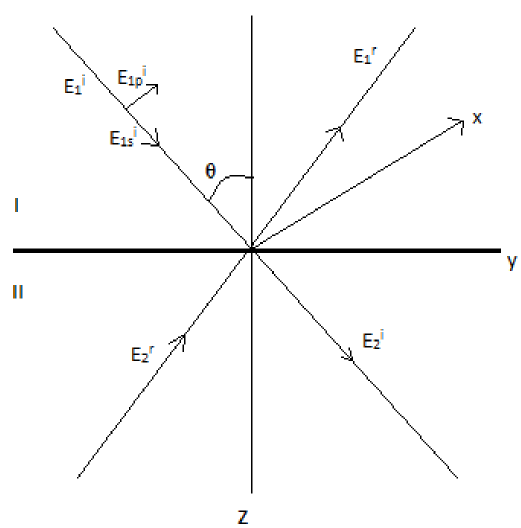


(a). Polar MOKE. (b). Equatorial MOKE (c). Meridional MOKE

Chapter 2

Theoretical background

A universal approach has been adopted to study the Faraday Kerr effect in thin magnetic films. The light has been treated as ray and all the laws of geometrical optics are valid in this case. The universality of this method is, that the incident ray can have arbitrary direction. Either the incident ray is I or II, all the calculation will be valid in both of the cases.



The light ray goes straight while it is traveling within the same medium. At the boundary of two mediums it bends, and the bending is govern by the Snell's law.

$$\sin\theta_1 n_1 = \sin\theta_2\ n_2$$

Where, n_1, n_2 are refractive indexes of first and second layer and θ_1, θ_2 are the incident and refracted angles.

2.1 Single boundary system

In this case, there are two mediums and single boundary. Still we keep the method universal which means that the direction of incident light ray is arbitrary. We resolve the incident and reflected light ray in its s-polarized and p-polarized components. For the incident ray, the components are $E_s^{(i)}$ and $E_p^{(i)}$. For the reflected components, they are $E_s^{(r)}$ and $E_p^{(r)}$. Now as the light passes through the boundary, it undergo transformation. We can write the transformed polarization vector of light in term of polarization vector of incident light.

$$\mathbf{F} = \mathbf{AP}, \tag{2.1}$$

Where,

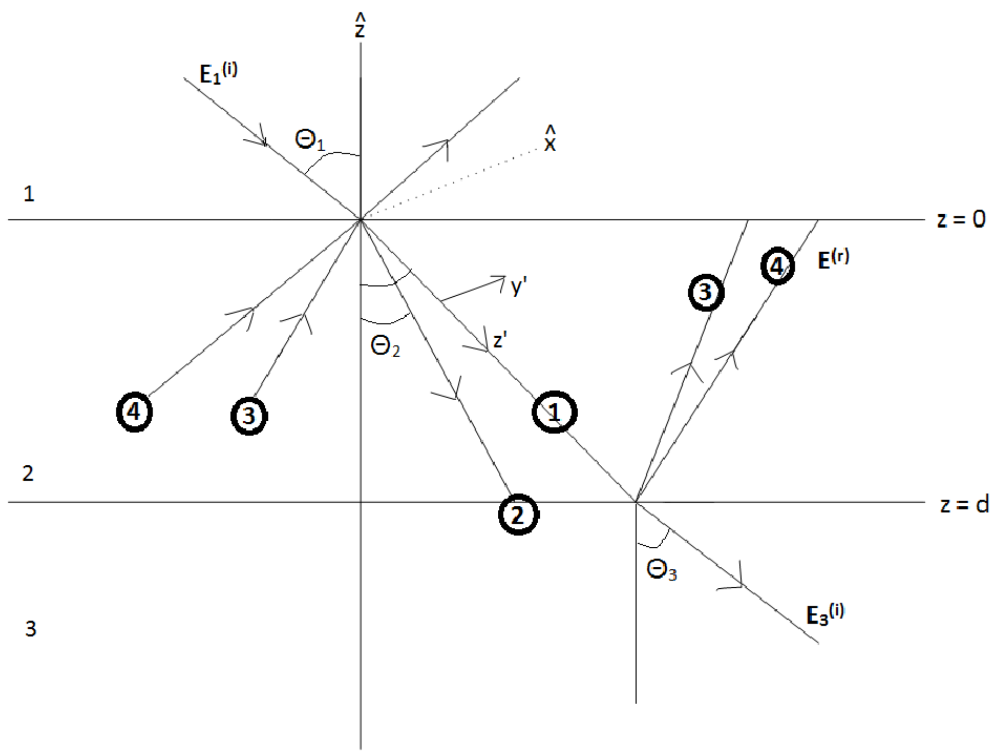
$$\mathbf{P} = \begin{pmatrix} E_s^i \\ E_p^i \\ E_s^r \\ E_p^r \end{pmatrix}, \qquad \mathbf{F} = \begin{pmatrix} E_x \\ E_y \\ H_x \\ H_y \end{pmatrix}. \tag{2.2}$$

So the medium boundary matrix A is a 4×4 matrix with sixteen elements, which is a transforming matrix.It governs the transformation of light components, when it passes through a boundary of two mediums.

2.2 Two boundary system

2.2.1 Medium boundary matrix

Let's take a system with two boundary, three medium.



We have the incident electric field $E^{(i)}$ and reflected electric field $E^{(r)}$. In the medium 2, we have two incident rays, shown as $E^{(1)}$ and $E^{(2)}$ and two reflected electric component denoted as $E^{(3)}$ and $E^{(4)}$. We have the relation for the electric field and D filed which is,

$$D_{(j)} = \sum_{j'} \varepsilon_{jj'} E_{j'}, \tag{2.3}$$

where $j = x, y, z$ and $j' = x, y, z$. The permittivity tensor ε_p for the polar configuration is,

$$\varepsilon_p = N^2 \begin{pmatrix} 1 & iQ & 0 \\ -iQ & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

Where ε_p is for polar permittivity tensor and Q is the magneto-optical constant. It depends on the magnitude and direction of the applied magnetic field and is given by $Q = i\frac{\varepsilon_{xy}}{\varepsilon_{xx}}$ with an assumption that $\varepsilon_{xx} = \varepsilon_{zz}$. This permittivity tensor correspond to only polar configuration when magnetization is applied in z direction according to figure
For the case of arbitrary direction of magnetization, we can have the permittivity tensor ε by applying euler transformation on the polar permittivity tensor ε_p.

$$\varepsilon = R_r \varepsilon_p R_r^{-1} [7]$$

where R_r is given by,

$$R_r = R_z(-\gamma) R_y(-\beta) R_z(\gamma)$$

and

$$R_y(\beta) = \begin{pmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{pmatrix}, \quad R_z(\gamma) = \begin{pmatrix} \cos\gamma & -\sin\gamma & 0 \\ \sin\gamma & \cos\gamma & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

So we get rotation matrix,

$$R_r = \begin{pmatrix} \cos\gamma^2 \cos\beta + \sin\gamma^2 & -\cos\gamma\sin\gamma + \cos\gamma\cos\beta\sin\gamma & -\cos\gamma\sin\beta \\ -\cos\gamma\sin\gamma + \cos\gamma\sin\beta\sin\gamma & \cos\gamma^2 + \cos\beta\sin\gamma^2 & \sin\gamma\sin\beta \\ \cos\gamma\sin\beta & \sin\gamma\sin\beta & \cos\beta \end{pmatrix},$$

and the inverse of rotation matrix is,

$$R_r^{-1} = \begin{pmatrix} \cos\gamma^2 \cos\beta + \sin\gamma^2 & -\cos\gamma\sin\gamma + \cos\gamma\cos\beta\sin\gamma & \cos\gamma\sin\beta \\ -\cos\gamma\sin\gamma + \cos\gamma\sin\beta\sin\gamma & \cos\gamma^2 + \cos\beta\sin\gamma^2 & \sin\gamma\sin\beta \\ -\cos\gamma\sin\beta & -\sin\gamma\sin\beta & \cos\beta \end{pmatrix},$$

and these rotation matrix satisfies $R_r R_r^{-1} = \mathbf{1}$.

So we get the permittivity tensor for arbitrary direction of magnetization by $R_r \varepsilon_p R_r^{-1}$,

$$\varepsilon = R_r \varepsilon_p R_r^{-1} = N^2 \begin{pmatrix} 1 & iQ\cos\beta & -iQ\sin\beta\sin\gamma \\ -iQ\cos\beta & 1 & iQ\cos\gamma\sin\beta \\ iQ\sin\beta\sin\gamma & -iQ\cos\gamma\sin\beta & 1 \end{pmatrix}.$$

So now from (2.3), we have $D = \varepsilon E$, so we get,

$$\begin{pmatrix} D_x^{(j)} \\ D_y^{(j)} \\ D_z^{(j)} \end{pmatrix} = N^2 \begin{pmatrix} 1 & iQ\cos\beta & -iQ\sin\beta\sin\gamma \\ -iQ\cos\beta & 1 & iQ\cos\gamma\sin\beta \\ iQ\sin\beta\sin\gamma & -iQ\cos\gamma\sin\beta & 1 \end{pmatrix} \begin{pmatrix} E_x^{(j)} \\ E_y^{(j)} \\ E_z^{(j)} \end{pmatrix},$$

and we get three equations from here which are,

$$D_x^{(j)} = E_x^{(j)} + iQE_y^{(j)}\cos\beta - iQE_z^{(j)}\sin\beta\sin\gamma, \tag{2.4}$$

$$D_y^{(j)} = -iQE_x^{(j)}\cos\beta + E_y^{(j)} + iQE_z^{(j)}\cos\gamma\sin\beta, \tag{2.5}$$

$$D_z^{(j)} = iQE_x^{(j)}\sin\beta\sin\gamma - iQE_y^{(j)}\cos\gamma\sin\beta + E_z^{(j)}, \tag{2.6}$$

Now we have from the figure that $\left(\frac{D_y}{D_x}\right)^{(j)} = \pm i\cos\theta^{(j)} = \pm i\alpha_z^{(j)}$. and $\left(\frac{D_z}{D_x}\right)^{(j)} = \pm i\sin\theta^{(j)} = \pm i\alpha_y^{(j)}$. Where $j = 1, 2, 3, 4$ and it represents the ray $1, 2, 3$ and 4 respectively.

Now we take the ratio of D_x to D_z,

$$\begin{aligned} \left(\frac{D_y}{D_x}\right)^{(j)} &= \frac{-iQE_x^{(j)}\cos\beta + E_y^{(j)} + iQE_z^{(j)}\cos\gamma\sin\beta}{E_x^{(j)} + iQE_y^{(j)}\cos\beta - iQE_z^{(j)}\sin\beta\sin\gamma}, \\ \mp i\alpha_z^{(j)} &= \frac{-iQE_x^{(j)}\cos\beta + E_y^{(j)} + iQE_z^{(j)}\cos\gamma\sin\beta}{E_x^{(j)} + iQE_y^{(j)}\cos\beta - iQE_z^{(j)}\sin\beta\sin\gamma}, \end{aligned}$$

$$E_y^{(j)} = \frac{iE_x^{(j)}\left(\alpha_z^{(j)} \pm Q\cos\beta\right) + E_z^{(j)}\left(Q\alpha_z^{(j)}\sin\gamma\sin\beta \pm Q\cos\gamma\sin\beta\right)}{\left(Q\alpha_z^{(j)}\cos\beta \pm 1\right)}, \tag{2.7}$$

and

$$\left(\frac{D_y}{D_x}\right)^{(j)} = \frac{iQE_x^{(j)}\sin\beta\sin\gamma - iQE_y^{(j)}\cos\gamma\sin\beta + E_z^{(j)}}{E_x^{(j)} + iQE_y^{(j)}\cos\beta - iQE_z^{(j)}\sin\beta\sin\gamma},$$

$$\pm i\alpha_y^{(j)}\left(E_x^{(j)} + iQE_y^{(j)}\cos\beta - iQE_z^{(j)}\sin\beta\sin\gamma\right) = \left(iQE_x^{(j)}\sin\beta\sin\gamma - iQE_y^{(j)}\cos\gamma\sin\beta + E_z^{(j)}\right),$$

Putting $E_y^{(j)}$ from equtaion (2.7) and putting the value of $E_z^{(j)}$ obtained in above equation, it will give,

$$E_y^{(j)} = E_x^{(j)}\left(\alpha_z^{(j)} + i(\alpha_z^{(j)})^2 Q\cos\beta - i\alpha_y^{(j)}\alpha_z^{(j)} Q\sin\beta\sin\gamma + iQ\cos\beta - iQ\alpha_y^{(j)}\cos\gamma\sin\beta\right),$$

Where

$$\begin{aligned}
\alpha_y^{(1,2)} &= \alpha_y\left(1 \mp \frac{1}{2}g_i Q\right). \\
\alpha_y^{(3,4)} &= \alpha_y\left(1 \mp \frac{1}{2}g_r Q\right). \\
\alpha_z^{(1,2)} &= \alpha_z\left(1 \pm \frac{\alpha_y}{2\alpha_z}g_i Q\right). \\
\alpha_z^{(3,4)} &= -\alpha_z\left(1 \pm \frac{\alpha_y^2}{2\alpha_z^2}g_r Q\right).
\end{aligned}$$

and

$$\begin{aligned}
g_i &= \alpha_z\cos\beta + \alpha_y\sin\beta\sin\gamma. \\
g_r &= -\alpha_z\cos\beta + \alpha_y\sin\beta\sin\gamma.
\end{aligned}$$

With all these substitutions we get,

$$\begin{aligned}
E_y^{(1)} &= E_x^{(1)}\left(-i\alpha_z - \frac{i\alpha_y^2}{2\alpha_z}Qg_i + i\alpha_y^2 Q\cos\beta - i\alpha_y\alpha_z Q\sin\beta\sin\gamma + \alpha_y Q\cos\gamma\sin\beta\right). \\
E_y^{(2)} &= E_x^{(2)}\left(i\alpha_z - \frac{i\alpha_y^2}{2\alpha_z}Qg_i + i\alpha_y^2 Q\cos\beta - i\alpha_y\alpha_z Q\sin\beta\sin\gamma - \alpha_y Q\cos\gamma\sin\beta\right). \\
E_y^{(3)} &= E_x^{(3)}\left(i\alpha_z - \frac{i\alpha_y^2}{2\alpha_z}Qg_r + i\alpha_y^2 Q\cos\beta + i\alpha_y\alpha_z Q\sin\beta\sin\gamma + \alpha_y Q\cos\gamma\sin\beta\right). \\
E_y^{(4)} &= E_x^{(4)}\left(-i\alpha_z - \frac{i\alpha_y^2}{2\alpha_z}Qg_r + i\alpha_y^2 Q\cos\beta + i\alpha_y\alpha_z Q\sin\beta\sin\gamma - \alpha_y Q\cos\gamma\sin\beta\right).
\end{aligned}$$

The x-components of the electric field of different incident and reflected light ray in term of s-polarized and p-polarized components are given as,

$$\begin{aligned} E_x^{(1,2)} &= \frac{1}{2}\left(E_s^{(i)} \pm iE_p^{(i)}\right). \\ E_x^{(3,4)} &= \frac{1}{2}\left(E_s^{(r)} \pm iE_p^{(r)}\right). \end{aligned}$$

So we get,

$$\begin{aligned} E_x &= E_x^{(1)} + E_x^{(2)} + E_x^{(3)} + E_x^{(4)}. \\ &= E_s^{(i)} + E_s^{(r)}. \end{aligned}$$

For for the y-component,

$$\begin{aligned} E_y &= E_y^{(1)} + E_y^{(2)} + E_y^{(3)} + E_y^{(4)}, \\ &= E_s^{(i)}\left(\frac{i\alpha_y}{2\alpha_z}Q\left(\alpha_y g_i - 2\sin\beta\cos\gamma\right)\right) + E_p^{(i)}\left(\alpha_z + i\alpha_y Q\sin\beta\cos\gamma\right) \\ &\quad -E_s^{(r)}\left(\frac{i\alpha_y}{2\alpha_z}Q\left(\alpha_y g_r - 2\sin\beta\cos\gamma\right)\right) + E_p^{(r)}\left(-\alpha_z + i\alpha_y\sin\beta\cos\gamma\right). \end{aligned}$$

The magnetic field components in term of Electric field components is given,

$$H_y^{(j)} = n^{(j)}\alpha_z^{(j)}E_x^{(j)}.$$

The x-component of the electric field contributes toward y-component of magnetic because they are orthonormal. So we get,

$$\begin{aligned} H_y^{(1)} &= N\alpha_z\left(1 + \frac{1}{2}Qg_i + \frac{1\alpha_y^2}{2\alpha_z^2}Qg_i\right)E_x^{(1)}. \\ H_y^{(2)} &= N\alpha_z\left(1 - \frac{1}{2}Qg_i - \frac{1\alpha_y^2}{2\alpha_z^2}Qg_i\right)E_x^{(2)}. \\ H_y^{(3)} &= -N\alpha_z\left(1 - \frac{1}{2}Qg_r - \frac{1\alpha_y^2}{2\alpha_z^2}Qg_r\right)E_x^{(3)}. \\ H_y^{(4)} &= -N\alpha_z\left(1 + \frac{1}{2}Qg_r + \frac{1\alpha_y^2}{2\alpha_z^2}Qg_r\right)E_x^{(4)}. \end{aligned}$$

So the complete y-component of the magnetic field will be ,

$$\begin{aligned} H_y &= H_y^{(1)} + H_y^{(2)} + H_y^{(3)} + H_y^{(4)}, \\ &= N\alpha_z E_s^{(i)} + \frac{iNQg_i}{2\alpha_z} E_p^{(i)} - N\alpha_z E_s^{(r)} - \frac{iNQg_r}{2\alpha_z} E_p^{(r)}. \end{aligned}$$

Similarly the x-component of the magnetic field is given by,

$$H_x^{(j)} = n^{(j)}\alpha_y^{(j)} E_z^{(j)} - n^{(j)}\alpha_z^{(j)} E_y^{(j)}.$$

So, to find $H_x^{(j)}$, First we need to find $E_z^{(j)}$. From equations (2.4),(2.5) and (2.6), we get

$$E_z^{(j)} = E_x^{(j)} \left(\frac{i\alpha_y^{(j)} + Q\sin\beta\sin\gamma - \alpha_z^{(j)} Q\cos\gamma\sin\beta}{i + \alpha_z^{(j)} Q\cos\beta - \alpha_y^{(j)} Q\sin\beta\sin\gamma} \right),$$

Hence,

$$\begin{aligned} E_z^{(1)} &= E_x^{(1)} \left(i\alpha_y - i\frac{\alpha_y}{2} Qg_i + i\alpha_z\alpha_y Q\cos\beta - i\alpha_z^2 Q\sin\beta\sin\gamma + \alpha_z Q\cos\gamma\sin\beta \right). \\ E_z^{(2)} &= E_x^{(2)} \left(-i\alpha_y - i\frac{\alpha_y}{2} Qg_i + i\alpha_z\alpha_y Q\cos\beta - i\alpha_z^2 Q\sin\beta\sin\gamma - \alpha_z Q\cos\gamma\sin\beta \right). \\ E_z^{(3)} &= E_x^{(3)} \left(i\alpha_y + i\frac{\alpha_y}{2} Qg_r - i\alpha_z\alpha_y Q\cos\beta - i\alpha_z^2 Q\sin\beta\sin\gamma - \alpha_z Q\cos\gamma\sin\beta \right). \\ E_z^{(4)} &= E_x^{(4)} \left(-i\alpha_y + i\frac{\alpha_y}{2} Qg_r - i\alpha_z\alpha_y Q\cos\beta - i\alpha_z^2 Q\sin\beta\sin\gamma + \alpha_z Q\cos\gamma\sin\beta \right). \end{aligned}$$

So now, Putting these to get $H_x^{(j)}$ will give,

$$\begin{aligned} H_x^{(1)} &= NE_x^{(1)} \left(+i + \frac{iQg_i}{2} \right). \\ H_x^{(2)} &= NE_x^{(2)} \left(-i + \frac{iQg_i}{2} \right). \\ H_x^{(3)} &= NE_x^{(3)} \left(+i - \frac{iQg_r}{2} \right). \\ H_x^{(4)} &= NE_x^{(4)} \left(-i - \frac{iQg_r}{2} \right). \end{aligned}$$

So from all these, we get,

$$\begin{aligned} H_x &= H_x^{(1)} + H_x^{(2)} + H_x^{(3)} + H_x^{(4)}. \\ &= \frac{iNQg_i}{2} E_s^{(i)} - NE_p^{(i)} + \frac{iNQg_r}{2} E_s^{(r)} - NE_p^{(r)}. \end{aligned}$$

So now from equation (2.1) and (2.2), we can write,

$$A = \begin{pmatrix} 1 & 0 & 1 & 0 \\ \frac{i\alpha_y}{2\alpha_z}Q\left(\alpha_y g_i - 2\zeta\right) & alpha_z + i\alpha_y Q\zeta & \frac{i\alpha_y}{2\alpha_z}Q\left(\alpha_y g_r - 2\zeta\right) & -\alpha_z + i\alpha_y\zeta \\ \frac{iNQg_i}{2} & -N & \frac{iNQg_r}{2} & -N \\ N\alpha_z & \frac{iNQg_i}{2\alpha_z} & -N\alpha_z & -\frac{iNQg_r}{2\alpha_z} \end{pmatrix}. \quad (2.8)$$

Where $\zeta = \sin\beta\cos\gamma$.

<u>For the Polar case:</u>

For the polar case, we have $\beta = 0$, So

$$A = \begin{pmatrix} 1 & 0 & 1 & 0 \\ \frac{i\alpha_y^2}{2}Q & \alpha_z & \frac{i\alpha_y^2}{2}Q & -\alpha_z \\ \frac{iNQ\alpha_z}{2} & -N & -\frac{iNQ\alpha_z}{2} & -N \\ N\alpha_z & \frac{iNQ}{2} & -N\alpha_z & \frac{iNQ}{2} \end{pmatrix}.$$

<u>For the Meridional case:</u>

For the longitudinal case, we have $\beta = \pi/2$ and $\gamma = \pi/2$, So

$$A = \begin{pmatrix} 1 & 0 & 1 & 0 \\ -\frac{i\alpha_y}{2\alpha_z}Q\left(1+\alpha_z^2\right) & \alpha_z & \frac{i\alpha_y}{2\alpha_z}Q\left(1+\alpha_z^2\right) & -\alpha_z \\ \frac{iNQ\alpha_y}{2} & -N & \frac{iNQ\alpha_y}{2} & -N \\ N\alpha_z & \frac{iNQ\alpha_y}{2\alpha_z} & -N\alpha_z & -\frac{iNQ\alpha_y}{2\alpha_z} \end{pmatrix}.$$

<u>For the Equatorial case:</u>

For the transverse case, we have $\beta = \pi/2$ and $\gamma = 0$. so

$$A = \begin{pmatrix} 1 & 0 & 1 & 0 \\ -\frac{i\alpha_y}{2\alpha_z}Q & \alpha_z + iQ\alpha_y & \frac{iQ\alpha_y}{\alpha_z} & -\alpha_z + iQ\alpha_y \\ 0 & -N & 0 & -N \\ N\alpha_z & 0 & -N\alpha_z & 0 \end{pmatrix}.$$

2.2.2 Medium prorogation matrix

The electric field propagate within the medium of same refractive index as,

$$E_x^{(j)}(0) = E_x^{(j)}(z)e^{-i\frac{2\pi}{\lambda}n^{(j)}\alpha_z^{(j)}z},$$

So putting the values for $n^{(j)}$ and $\alpha_z^{(j)}$ and solving it for each of the ray. we get,

$$\begin{aligned}
E_x^{(1)}(0) &= E_x^{(1)}(z)Ue^{-i\frac{\pi NQg_i}{\lambda\alpha_z}z}. \\
E_x^{(2)}(0) &= E_x^{(2)}(z)Ue^{i\frac{\pi NQg_i}{\lambda\alpha_z}z}. \\
E_x^{(3)}(0) &= E_x^{(3)}(z)U^{-1}e^{-i\frac{\pi NQg_r}{\lambda\alpha_z}z}. \\
E_x^{(4)}(0) &= E_x^{(4)}(z)U^{-1}e^{i\frac{\pi NQg_r}{\lambda\alpha_z}z}.
\end{aligned}$$

and we get,

$$\begin{aligned}
\begin{pmatrix} E_x^{(1)}(0) \\ E_x^{(2)}(0) \\ E_x^{(3)}(0) \\ E_x^{(4)}(0) \end{pmatrix} &= \begin{pmatrix} Ue^{-i\frac{\pi NQg_i}{\lambda\alpha_z}z} & 0 & 0 & 0 \\ 0 & Ue^{i\frac{\pi NQg_i}{\lambda\alpha_z}z} & 0 & 0 \\ 0 & 0 & U^{-1}e^{-i\frac{\pi NQg_r}{\lambda\alpha_z}z} & 0 \\ 0 & 0 & 0 & U^{-1}e^{i\frac{\pi NQg_r}{\lambda\alpha_z}z} \end{pmatrix} \begin{pmatrix} E_x^{(1)}(z) \\ E_x^{(2)}(z) \\ E_x^{(3)}(z) \\ E_x^{(4)}(z) \end{pmatrix}, \\
D &= \begin{pmatrix} Ue^{-i\frac{\pi NQg_i}{\lambda\alpha_z}z} & 0 & 0 & 0 \\ 0 & Ue^{i\frac{\pi NQg_i}{\lambda\alpha_z}z} & 0 & 0 \\ 0 & 0 & U^{-1}e^{-i\frac{\pi NQg_r}{\lambda\alpha_z}z} & 0 \\ 0 & 0 & 0 & U^{-1}e^{i\frac{\pi NQg_r}{\lambda\alpha_z}z} \end{pmatrix}.
\end{aligned}$$

And we have,

$$\begin{pmatrix} E_x^{(1)}(0) \\ E_x^{(2)}(0) \\ E_x^{(3)}(0) \\ E_x^{(4)}(0) \end{pmatrix} = \begin{pmatrix} 1 & i & 0 & 0 \\ 1 & -i & 0 & 0 \\ 0 & 0 & 1 & i \\ 0 & 0 & 1 & -i \end{pmatrix} \begin{pmatrix} E_s^{(i)} \\ E_p^{(i)} \\ E_s^{(r)} \\ E_p^{(r)} \end{pmatrix},$$

Similarly,

$$\begin{pmatrix} E_x^{(1)}(z) \\ E_x^{(2)}(z) \\ E_x^{(3)}(z) \\ E_x^{(4)}(z) \end{pmatrix} = \begin{pmatrix} 1 & i & 0 & 0 \\ 1 & -i & 0 & 0 \\ 0 & 0 & 1 & i \\ 0 & 0 & 1 & -i \end{pmatrix} \begin{pmatrix} E_s^{(i)} \\ E_p^{(i)} \\ E_s^{(r)} \\ E_p^{(r)} \end{pmatrix},$$

We take,

$$S = \begin{pmatrix} 1 & i & 0 & 0 \\ 1 & -i & 0 & 0 \\ 0 & 0 & 1 & i \\ 0 & 0 & 1 & -i \end{pmatrix}.$$

So we get the medium propagation matrix $\bar{D}$,

$$\begin{aligned} \bar{D} &= S^{-1}DS, \\ &= \begin{pmatrix} U\cos\frac{\pi NQg_iz}{\lambda\alpha_z} & U\sin\frac{\pi NQg_iz}{\lambda\alpha_z} & 0 & 0 \\ -U\sin\frac{\pi NQg_iz}{\lambda\alpha_z} & U\cos\frac{\pi NQg_iz}{\lambda\alpha_z} & 0 & 0 \\ 0 & 0 & U^{-1}\cos\frac{\pi NQg_rz}{\lambda\alpha_z} & -U^{-1}\sin\frac{\pi NQg_rz}{\lambda\alpha_z} \\ 0 & 0 & U^{-1}\sin\frac{\pi NQg_rz}{\lambda\alpha_z} & U^{-1}\cos\frac{\pi NQg_rz}{\lambda\alpha_z} \end{pmatrix}. \end{aligned}$$

Now if we apply thin film approximation that z is very small. then $\cos\frac{\pi NQg_jz}{\lambda\alpha_z} = 1$ and $\sin\frac{\pi NQg_jz}{\lambda\alpha_z} = \frac{\pi NQg_jz}{\lambda\alpha_z}$. So we get,

$$\bar{D} = \begin{pmatrix} U & U\delta_i & 0 & 0 \\ -U\delta_i & U & 0 & 0 \\ 0 & 0 & U^{-1} & -U^{-1}\delta_r \\ 0 & 0 & U^{-1}\delta_r & U^{-1} \end{pmatrix}.$$

Where

$$\delta_i = \frac{\pi NQg_iz}{\lambda\alpha_z}.$$

$$\delta_r = \frac{\pi NQg_rz}{\lambda\alpha_z}.$$

2.3 Multilayer thin film system

For the complete thin film matrix it would be,

$$\begin{aligned} A_c &= A\bar{D}A^{-1} \\ &= \begin{pmatrix} 1 & 0 & 0 & -\frac{i2d\pi}{\lambda} \\ -\frac{2N\pi Qd\sigma\sin\theta}{\lambda} & \frac{\lambda+2N\pi Qd\sigma\sin\theta}{\lambda} & -\frac{i2d\pi\cos\theta^2}{\lambda} & 0 \\ \frac{2dN^2Q\pi\cos\beta}{\lambda} & \frac{i2dN^2\pi}{\lambda} & \frac{\lambda-2N\pi Qd\sigma\sin\theta}{\lambda} & \frac{2N\pi Qd\sec\theta(-\sigma+\sin\beta\sin\gamma\tan\theta)}{\lambda} \\ -\frac{2idN^2\pi(\cos\theta)^2}{\lambda} & \frac{2dQN^2\pi\cos\beta}{\lambda} & -\frac{2NQd\pi\sin\beta\sin\gamma\sin\theta}{\lambda} & 1 \end{pmatrix}. \end{aligned}$$

where $\sigma = \cos\gamma\sin\beta$ and θ is the angle of light with the horizontal in that thin film.
If we have "m" number of thin films, then the complete matrix for all the films, say M can be given by,

$$M = A_i^{-1}\prod_m A_m\bar{D_m}A_m^{-1}A_f.$$

where i is for incident medium and f for the transmitting medium or substrate medium. And we have,

$$M = \begin{pmatrix} G & H \\ I & J \end{pmatrix}.$$

Where G, H, I and J are 2×2 matrixs and it gives,

$$G^{-1} = \begin{pmatrix} t_{ss} & t_{sp} \\ t_{ps} & t_{pp} \end{pmatrix}, \qquad IG^{-1} = \begin{pmatrix} r_{ss} & r_{sp} \\ r_{ps} & r_{pp} \end{pmatrix}.$$

where t stands for the transmission and r stands for the reflection. Now we can simply find the Kerr rotation and Kerr ellipticity for both s and p polarized light.

For s-polarized light:

$$\text{Kerr rotation} = Re\left[\frac{r_{ps}}{r_{ss}}\right], \quad \text{Kerr ellipticity} = Im\left[\frac{r_{ps}}{r_{ss}}\right].$$

For p-polarized light:

$$\text{Kerr rotation} = -Re\left[\frac{r_{sp}}{r_{pp}}\right], \quad \text{Kerr ellipticity} = Im\left[\frac{r_{sp}}{r_{pp}}\right].$$

2.4 Mueller Matrix Determination Methods

Mueller matrix of an system, gives complete information about the effect of system on light, that has passed through it. The Mueller matrix also gives us the information about the depolarization of light on passing through the system.
For the determination of Mueller matrix, different method have been adopted. Here we will explained the 16 intensity method and the dual compansators rotator method.

2.4.1 16-intensity method

For this method, we have a simple of arrangment of two compansators, polarizer, an analyzer along with source and detector as shown in the figure below.

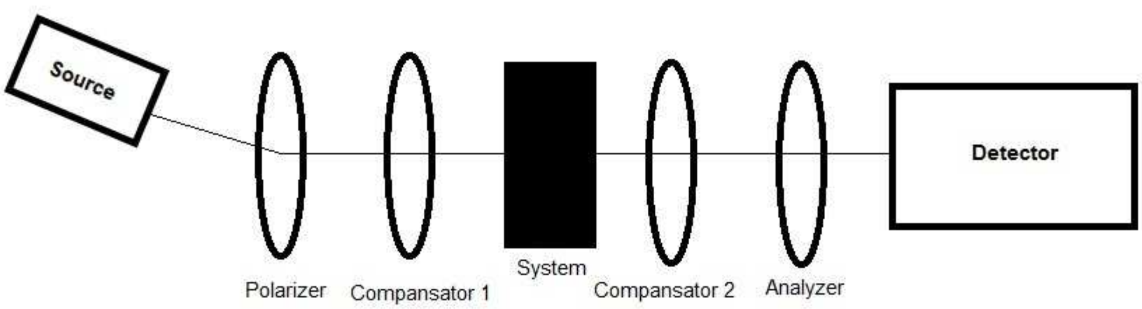


Figure 2.1: 16-Intensity Method for the determination of the Mueller matrix for a system.

Let we take any arbitrary polarization of the light which is emitted from the source. After

passing through the polarizer, which is placed with an arbitrary angle of P with the horizontal. At the output of the polarizer we will get the polarized light of the form, let we call it $P^{'}$.

$$P^{'} = \begin{pmatrix} 1 \\ \cos 2P \\ \sin 2P \\ 0 \end{pmatrix}.$$

General matrix for the compansator is given as,

$$C = \begin{pmatrix} 1 & c & d & 0 \\ c & h & i & -e \\ d & i & j & g \\ 0 & e & -g & p \end{pmatrix}.$$

Where

$$\begin{aligned} c &= s\cos 2C, & h = k\cos 4C + (1+k), \\ d &= s\sin 2C, & i = k\sin 4C, \\ e &= r\sin 2C, & j = -k\cos 4C + (1-k), \\ g &= r\cos 2C, & k = (1-p)/2, \end{aligned}$$

And we have $s = \cos 2\psi_c, r = \sin 2\psi_c \sin\delta_c$ and $p = \sin 2\psi_c \cos\delta_c$ are the stokes parameters[10]. We get the output light just after the first compansator and we denote it like $S^{'}$ and it is given as,

$$
\begin{aligned}
S^{'} &= \begin{pmatrix} S_1^{'} \\ S_2^{'} \\ S_3^{'} \\ S_4^{'} \end{pmatrix} = C.P^{'}, \\
&= I_p \begin{pmatrix} 1 + s\cos 2(C-P) \\ k\cos 4C - 2P + s\cos 2C + (1-k)\cos 2P \\ k\sin 4C - 2P + s\sin 2C + (1-k)\sin 2P \\ r\sin 2C - 2P \end{pmatrix}.
\end{aligned}
$$

Where I_p is the intensity emerging from the polarizer.

The Mueller matrix of the system is define in a general way, which has to be derived by calculations, and denoted by M.

$$
M = \begin{pmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \\ m_{41} & m_{42} & m_{43} & m_{44} \end{pmatrix}.
$$

Second compansator is also defined in the similar way, with $C^{'}$ be the angle with the horizontal and $\delta_c^{'}$ be the retarding angle. And the analyzer matrix is denoted by A with an angle A with horizontal and is given as,

$$
A = \begin{pmatrix} 1 & \cos 2A & \sin 2A & 0 \\ 1 & \cos 2A & \sin 2A & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}.
$$

So we get a general output with arbitrray direction of all the components, and is given as,

$$\begin{pmatrix} I_g \\ I_g \\ 0 \\ 0 \end{pmatrix} = A.C^{'}.M.S^{'}$$

And from here, we get a general form of the output light just after the analyzer. Now from here, we can apply different configurations of the polarizer, analyzer and compansators to get a number of equation, which then on solving simultaneously, will give the Mueller matrix of the system. We used (P,C) settings to be $(0,0),(\pi/4,\pi/4),(\pi/2,\pi/2)$ and $(0,\pi/4)$ with all the $(C^{'},A)$ settings of $(0,0),(\pi/4,\pi/4),(\pi/2,\pi/2)$ and $(\pi/4,0)$, we get sixteen equations[10]. These equations, when solved simultaneously give rise to the elements of Mueller matrix, which are,

$$\begin{aligned}
m_{11} &= (B_1 + B_{11} + B_3 + B_9)/(2Ip(1+s)(1+s^{'})), \\
m_{12} &= (B_1 - B_{11} + B_3 - B_9)/(2Ip(1+s)(1+s^{'})), \\
m_{13} &= -((B_1 + B_{11} + B_3 - 2(B_5 + B_7) + B_9)/(2Ip(1+s)(1+s^{'}))), \\
m_{14} &= -2B_1 + 2B_{13} + 2B_{15} - 2B_3 + 2B_1k - 2B_{11}k + 2B_3k - 2B_9k + B_1s + B_{11}s \\
&\quad +2B_{13}s + 2B_{15}s + B_3s - 2B_5s - 2B_7s + B_9s/(2Ipr(1+s)(1+s^{'})),
\end{aligned}$$

$$\begin{aligned}
m_{21} &= (B_1 - B_{11} - B_3 + B_9)/(2Ip(1+s)(1+s^{'})), \\
m_{22} &= (B_1 + B_{11} - B_3 - B_9)/(2Ip(1+s)(1+s^{'})), \\
m_{23} &= (-B_1 + B_{11} + B_3 + 2B_5 - 2B_7 - B_9)/(2Ip(1+s)(1+s^{'})), \\
m_{24} &= -2B_1 + 2B_{13} - 2B_{15} + 2B_3 + 2B_1k + 2B_{11}k - 2B_3k - 2B_9k + B_1s - B_{11}s \\
&\quad +2B_{13}s - 2B_{15}s - B_3s - 2B_5s + 2B_7s + B_9s/(2Ipr(1+s)(1+s^{'})),
\end{aligned}$$

$$
\begin{aligned}
m_{31} &= -((B_1 - 2B_{10} + B_{11} - 2B_2 + B_3 + B_9)/(2Ip(1+s)(1+s^{'}))),\\
m_{32} &= (-B_1 - 2B_{10} + B_{11} + 2B_2 - B_3 + B_9)/(2Ip(1+s)(1+s^{'})),\\
m_{33} &= (B_1 - 2B_{10} + B_{11} - 2B_2 + B_3 - 2(B_5 - 2B_6 + B_7) + B_9)/(2Ip(1+s)(1+s^{'})),\\
m_{34} &= 2B_1 - 2B_{13} + 4B_{14} - 2B_{15} - 4B_2 + 2B_3 - 2B_1k - 4B_{10}k + 2B_{11}k + 4B_2k - 2B_3k\\
&\quad +2B_9k - B_1s + 2B_{10}s - B_{11}s - 2B_{13}s + 4B_{14}s - 2B_{15}s + 2B_2s - B_3s + 2B_5s - 4B_6s\\
&\quad +2B_7s - B_9s/(2Ipr(1+s)(1+s^{'}))
\end{aligned}
$$

$$
\begin{aligned}
m_{41} &= 2B_1 - 2B_{12} - 2B_4 + 2B_9 - 2B_1k^{'} + 2B_{11}k^{'} + 2B_3k^{'} - 2B_9k^{'} - B_1s^{'} + 2B_{10}s^{'}\\
&\quad -B_{11}s^{'} - 2B_{12}s^{'} + 2B_2s^{'} - B_3s^{'} - 2B_4s^{'} - B_9s^{'}/(2Ipr^{'}(1+s)(1+s^{'}))\\
m_{42} &= 2B_1 + 2B_{12} - 2B_4 - 2B_9 - 2B_1k^{'} - 2B_{11}k^{'} + 2B_3k^{'} + 2B_9k^{'} - B_1s^{'} - 2B_{10}s^{'}\\
&\quad +B_{11}s^{'} + 2B_{12}s^{'} + 2B_2s^{'} - B_3s^{'} - 2B_4s^{'} + B_9s^{'}/(2Ipr^{'}(1+s)(1+s^{'})),\\
m_{43} &= -2B_1 + 2B_{12} + 2B_4 + 4B_5 - 4B_8 - 2B_9 + 2B_1k^{'} - 2B_{11}k^{'} - 2B_3k^{'} - 4B_5k^{'} + 4B_7k^{'}\\
&\quad +2B_9k^{'} + B_1s^{'} - 2B_{10}s^{'} + B_{11}s^{'} + 2B_{12}s^{'} - 2B_2s^{'} + B_3s^{'} + 2B_4s^{'} - 2B_5s^{'} + 4B_6s^{'} - 2B_7s^{'}\\
&\quad -4B_8s^{'} + B_9s^{'}/(2Ipr^{'}(1+s)(1+s^{'})),\\
m_{44} &= -4B_1 + 4B_{13} - 4B_{16} + 4B_4 + 4B_1k + 4B_{12}k - 4B_4k - 4B_9k + 4B_1k^{'} - 4B_{13}k^{'} + 4B_{15}k^{'}\\
&\quad -4B_3k^{'} - 4B_1kk^{'} - 4B_{11}kk^{'} + 4B_3kk^{'} + 4B_9kk^{'} + 2B_1s - 2B_{12}s + 4B_{13}s - 4B_{16}s - 2B_4s\\
&\quad -4B_5s + 4B_8s + 2B_9s - 2B_1k^{'}s + 2B_{11}k^{'}s - 4B_{13}k^{'}s + 4B_{15}k^{'}s + 2B_3k^{'}s + 4B_5k^{'}s - 4B_7k^{'}s\\
&\quad -2B_9k^{'}s + 2B_1s^{'} - 2B_{13}s^{'} + 4B_{14}s^{'} - 2B_{15}s^{'} - 4B_{16}s^{'} - 4B_2s^{'} + 2B_3s^{'} + 4B_4s^{'} - 2B_1ks^{'}\\
&\quad -4B_{10}ks^{'} + 2B_{11}ks^{'} + 4B_{12}ks^{'} + 4B_2ks^{'} - 2B_3ks^{'} - 4B_4ks^{'} + 2B_9ks^{'} - B_1ss^{'} + 2B_{10}ss^{'}\\
&\quad -B_{11}ss^{'} - 2B_{12}ss^{'} - 2B_{13}ss^{'} + 4B_{14}ss^{'} - 2B_{15}ss^{'} - 4B_{16}ss^{'} + 2B_2ss^{'} - B_3ss^{'} - 2B_4ss^{'}\\
&\quad +2B_5ss^{'} - 4B_6ss^{'} + 2B_7ss^{'} + 4B_8ss^{'} - B_9ss^{'}/2Iprr^{'}(1+s)(1+s^{'}),
\end{aligned}
$$

Where B_j are the outputs of the 16 experiments respectively.

2.5 Simulation results

Some of the simulation were performed in MATLAB using this theory of thin film systems. the code can be found in Appendix A. Here I would like to show that simulated results along with the geometries.

For the Bulk iron (Fe)[7]:

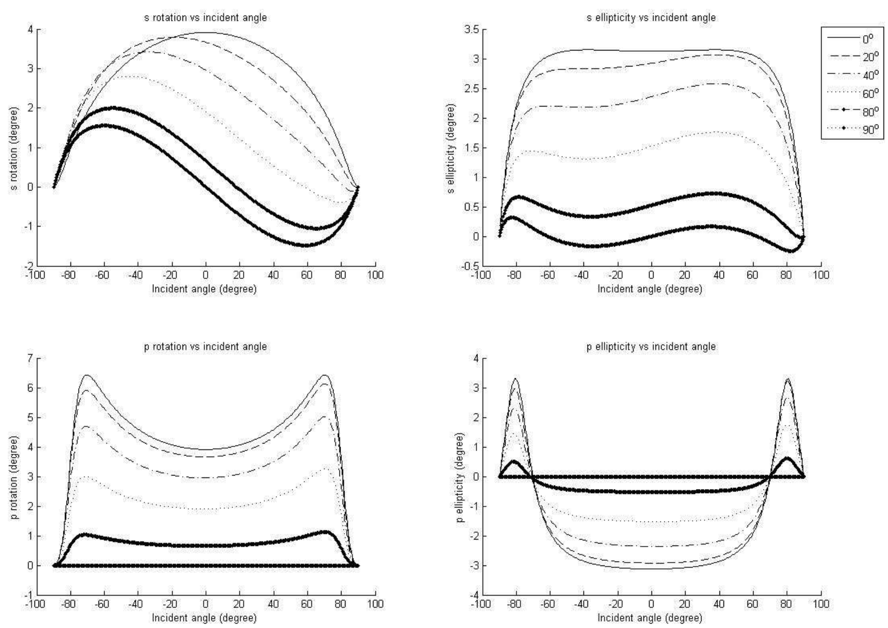


Figure 2.2: The refractive index for iron is $2.87 + i3.36$ and the magneto-optical constant Q is $.376 + i0.0066$, with a wavelength of $6328A^o$.

The zero angle for the direction of magnetization represents the polar case and an angle of 90^oC represents the longitudinal case of geometry. Polar case is quite symmetric with the center of symmetry at zero incident angle in case of $s-$rotation and this symmetry got lost when move toward the longitude case. Longitude case is completely asymmetric, and same case is true for $s-$ellipticity. We can see from the figure that the $p-$rotation

and $p-$ellipticity are quit symmetric with the center of symmetry at zero incident angle.

For the $50A^o$ Fe on the Au substrate[7]:

This the case in which we have a thin film of $50A^o$ of ferromagnetic material, Fe on non-magnetic gold substrate. Where the incident medium is free space.

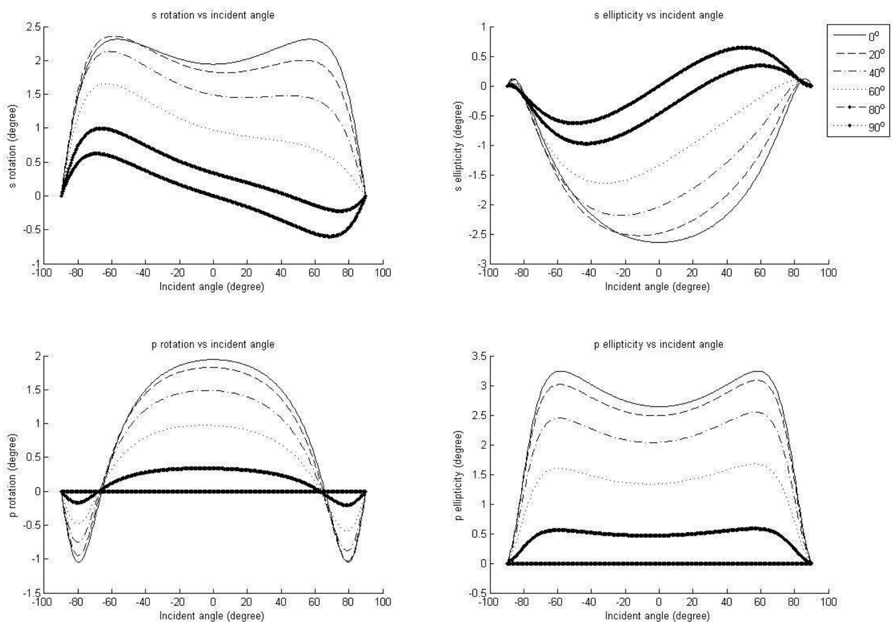


Figure 2.3: Kerr rotation and ellipticity for the $50A^o$ iron film on gold substrate, with an refractive index of $2.87 + i3.36$ and $.12 + i3.29$ respectively, the magneto-optical constant Q for iron is $.376 + i0.0066$, with a wavelength of $6328A^o$.

This also shows full symmetry in the $p-$rotation and $p-$ellipticity with the zero incident angle as the center of symmetry. Where the $s-$rotation and $s-$ellipticity are symmetric for the case of polar magnetization and completely asymmetric in the case of longitudinal magnetization geometry.

For the 50/50 periods Fe $10A^o$ on the $10A^o$ Au substrate[7]:

Here we have 50 periods of back to back $10A^o$ Fe and $10A^o$ gold thin films. The incident

medium is free space and the substrate is $10A^o$ gold film.

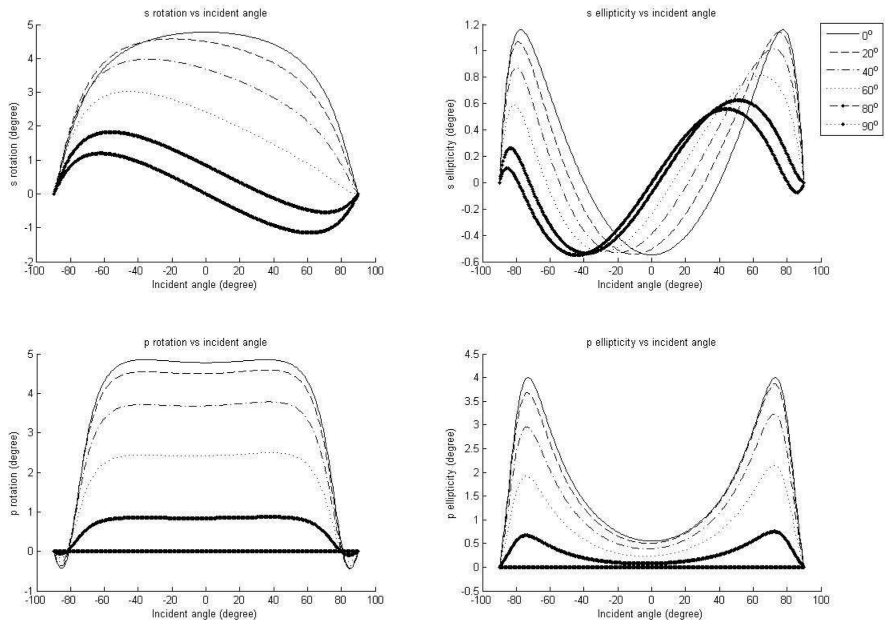


Figure 2.4: Kerr rotation and ellipticity for the 50 periods of $10A^o$ iron film and $10A^o$ gold film substrate, with an refractive index of $2.87 + i3.36$ and $.12 + i3.29$ respectively, the magneto-optical constant Q for iron is $.376 + i0.0066$, with a wavelength of $6328A^o$.

Same is the case here, for the $s-$rotation, $s-$ellipticity, $p-$rotation and $p-$ellipticity respectively. These configurations are from the Badar et. al[7].

For 10 periods of $50A^o$ Cu/ $55.8A^o$ Co[8]:

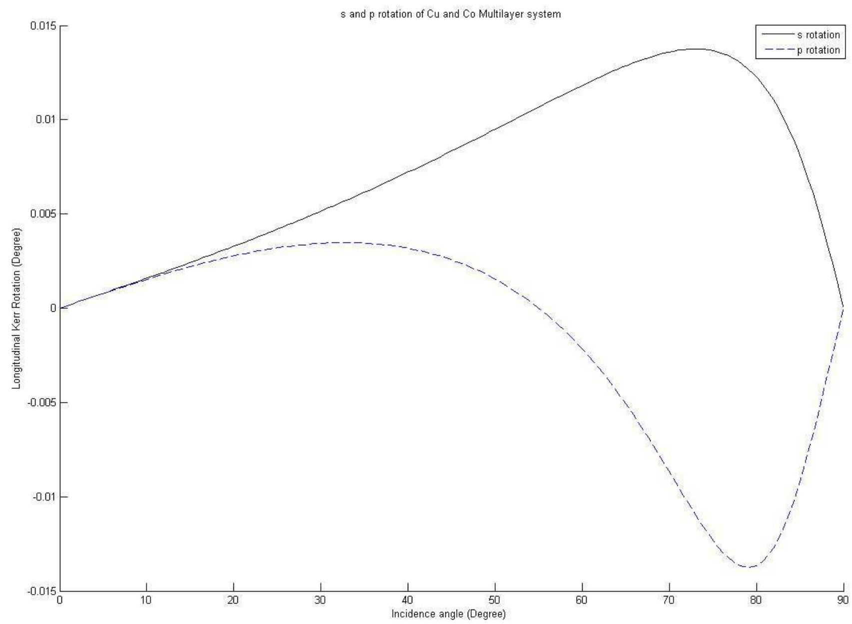


Figure 2.5: Kerr rotations for the 10 layer system of $50A^o$ Cu with refractive index of $1.58+i3.58$ and $55.8A^o$ Co with refractive index of $2.212+i4.170$, magneto-optical constant Q is $.00038+i.00314$. Wavelength of the incident light is kept at $6328A^o$.

The direction of magnetization in the Co layer, which is a ferromagnetic medium, is longitudinal. The incident angle varies from $0-\pi/2$ in radians.The $s-$rotation is positive and the $p-$rotation is negative. We can notice a dip at about 79^oC of incident angle which a grazing angle, and similarly $p-$rotation is also maximum around the grazing angle.

For 200 periods of $1.8A^o$ Co/ $9A^o$ Pd[8]:

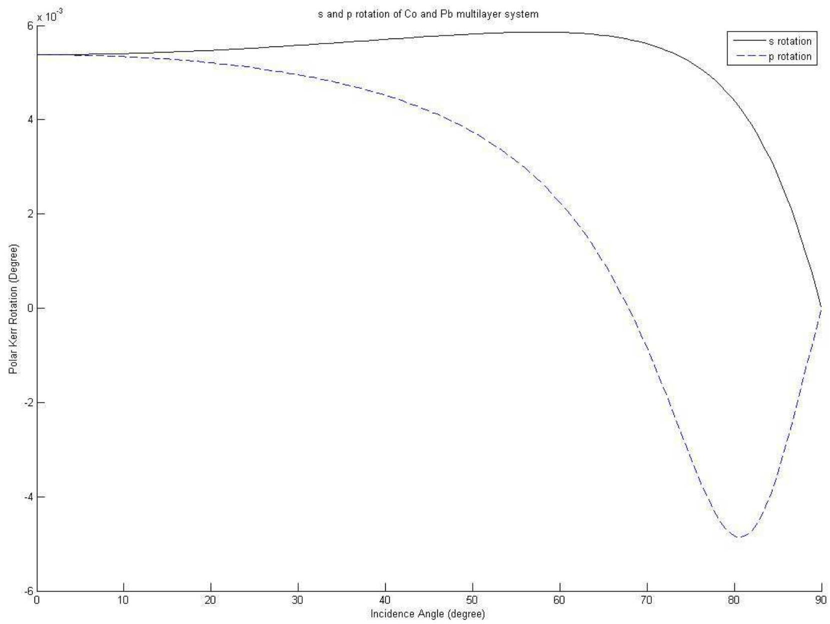


Figure 2.6: Kerr rotations for 200 layer system of $1.8A^o$ Co with refractive index of $2.212+i4.170$, magneto-optical constant Q is $.00038+i.00314$ and $9A^o$ Pd with refractive of $1.768+i4.289$. Wavelength of the incident light is kept at $6328A^o$.

The direction of magnetization in Co layer is fixed and is perpendicular to the plane of incident and parallel to the direction of incidence. Again here, the $p-$rotation at grazing angle is large then the other angles of incident.

Two layer systems with arbitrary direction of magnetization in one of them.

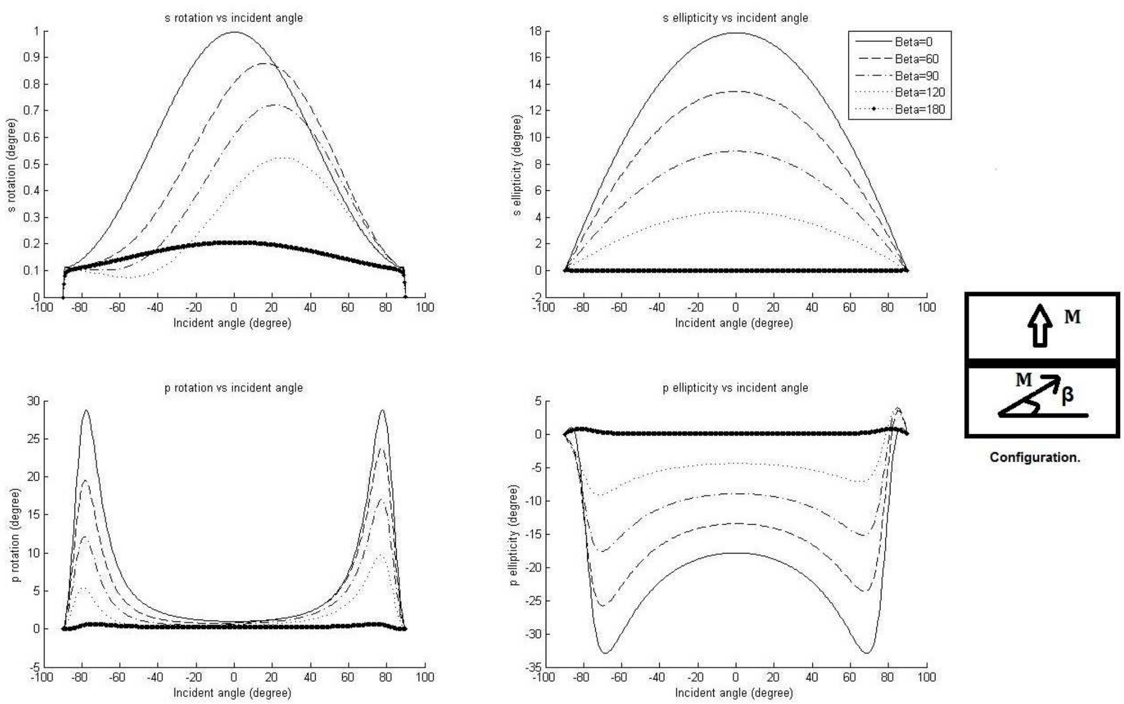


Figure 2.7: Both layers are of iron with refractive index of $2.87 + i3.36$ and magneto-optical coefficient $.376 + i0.0066$. Thickness of both layers is $10A^o$.

The magnetization direction in the first layer is fixed and has polar geometry while in the second layer it is arbitrary, it varies from $0 - \pi$ radians. When we have the same direction of magnetization in both layers, the rotations and ellipticities are very large, and they are suppressed as we change the direction of the magnetization in the second layer. When the direction of magnetization got reversed as compared to that of first one, the rotations and ellipticities are very small.

Now if we keep fix the first layer in longitudinal geometry and change the magnetization direction in the second layer from $0 - \pi$ radians.

We can see the behavior of the rotations and ellipticities in both s and p polarized cases, and we can see from the figure 2.5 that with the reversal of the direction of

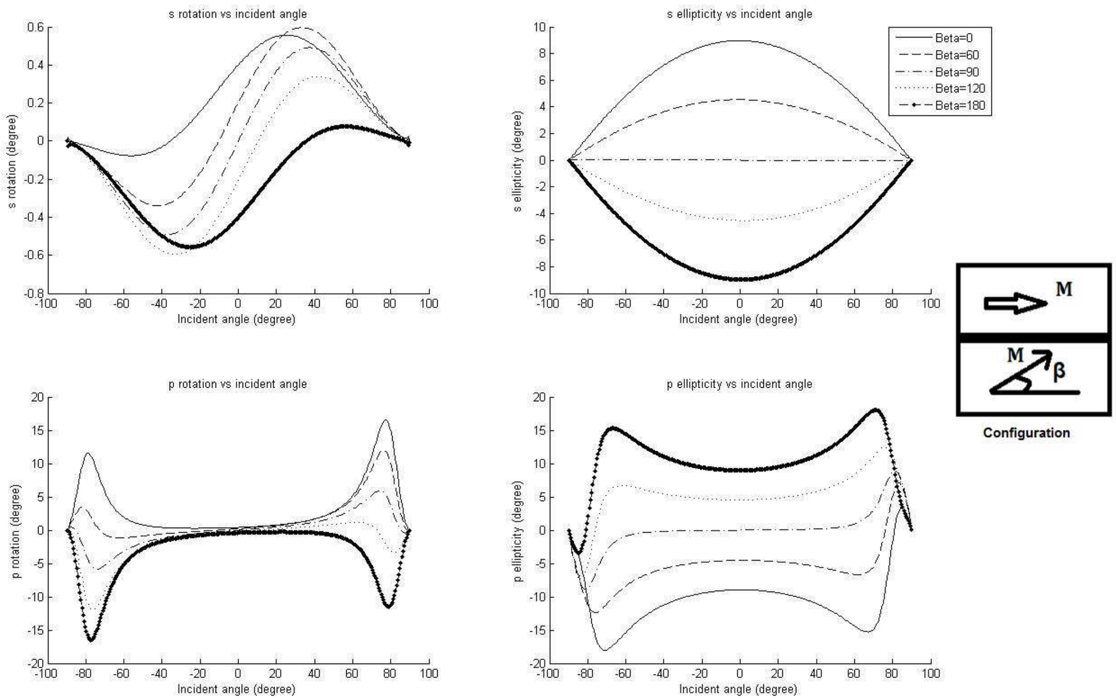


Figure 2.8: Both layers are of iron with refractive index of $2.87 + i3.36$ and magneto-optical coefficient $.376 + i0.0066$. Thickness of both layers is $10A^o$.

magnetization, the rotation also change its sign, let say from positive to negative.
We have also seen the behavior of Kerr rotation and ellipticity with respect to the changes in the direction of magnetization at specific angles of incident. At an incident angle of $\pi/2$, their is no change in the polarization and ellipticity of the light irrespective to the direction of applied magnetization. Highest changes in the polarization and ellipticity of $s-$polarized light is maximum at an angle of zero degree, and it decreases as we deviate with the polar geometry of magnetization direction in the second layer of iron. Which implies that the similar directions of magnetization in both layers reinforce each other and if we deviate in either direction, it will suppress the outcome.

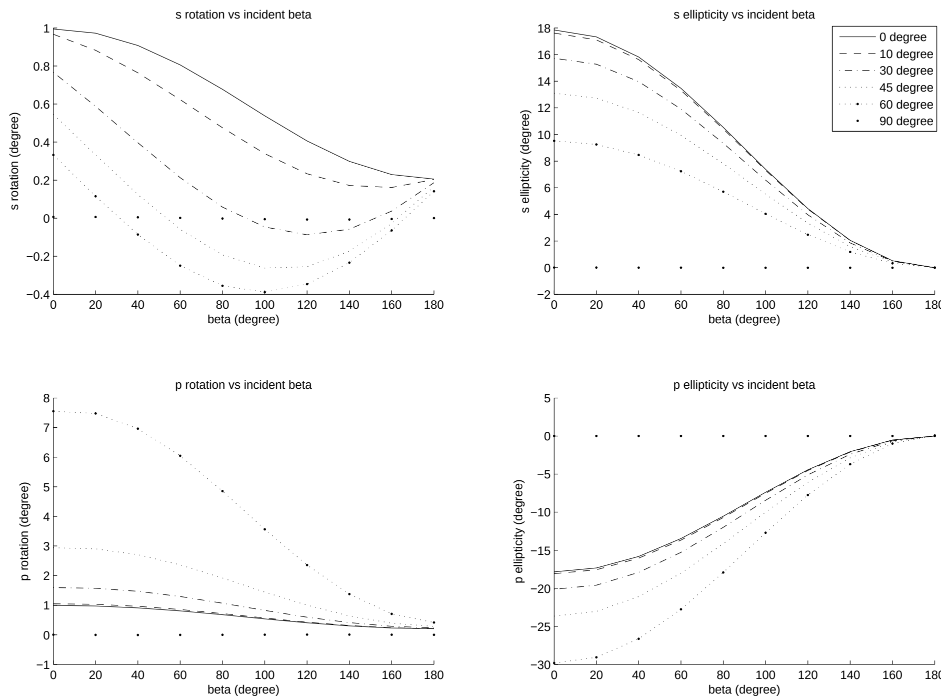


Figure 2.9: Both layers are of iron with refractive index of $2.87 + i3.36$ and magneto-optical coefficient $.376 + i0.0066$. Thickness of both layers is $10A^o$.

<u>Single layer system with different direction of magnetization:</u>

We have studied the effect of magnetization direction, in a single layer system and noticed that with the flip in the direction in the magnetization the rotations and ellipticities changes their sign, let say from positive to negative or vice versa.

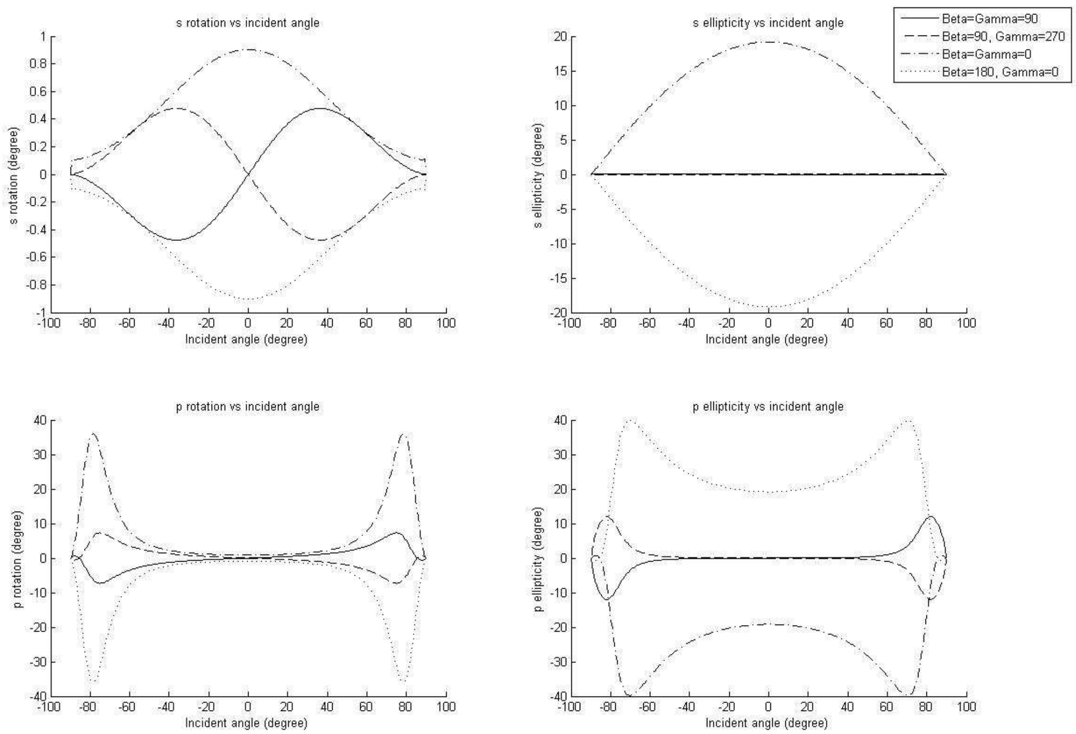


Figure 2.10: The layer is of iron with refractive index of $2.87 + i3.36$ and magneto-optical coefficient $.376 + i0.0066$. Thickness of the layers is $10A^o$ in each case.

Where, $Beta = Gamma = 90$ represent the longitudinal geometry of magnetization, the $Beta = 90, Gamma = 270$ represented the flipped longitudinal case. Similarly $Beta = Gamma = 0$ represent the polar geometry of magnetization and $Beta = 180, Gamma = 0$ represent the flipped polar geometry in the single layer of iron with a thickness of $10A^o$.

Textured magnetic profile in iron:

$10nm$ film of iron is divided in 100 intervals to form a stack of 100 layers, and the magnetization direction is changing continuously and form a spiral structure from $1-100$ layers. We have change the direction of magnetization in a plane, say we have only change the parameter $beta$ from $0-\pi$ which form a half spiral shape, and we have compared the result with a single layer of $10nm$ in which the magnetization direction has a polar geometry.

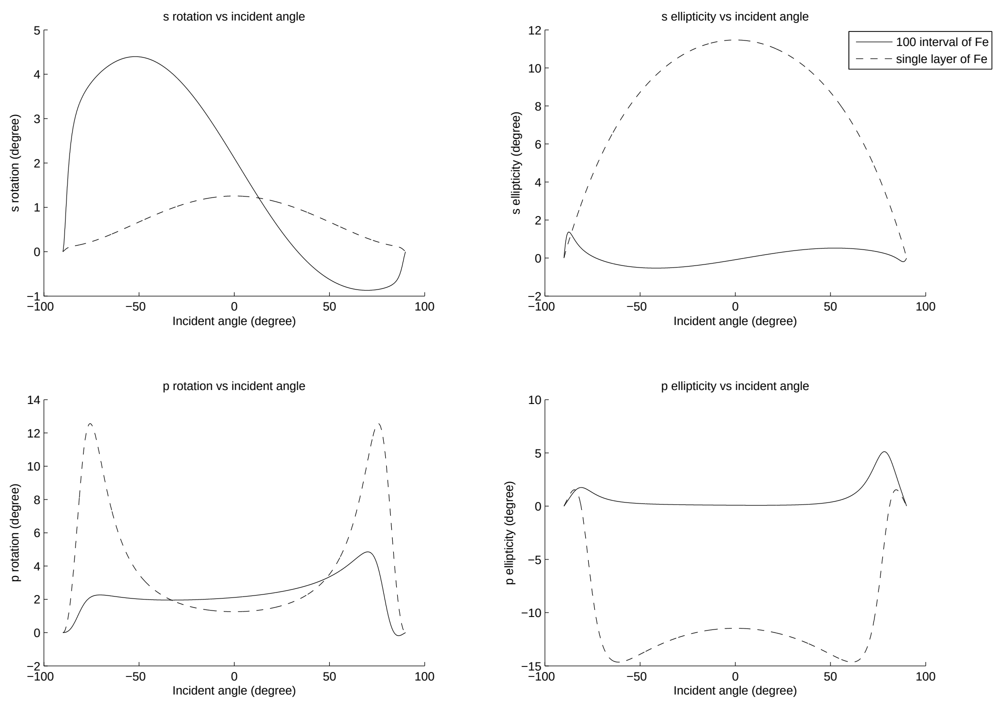


Figure 2.11: The layer is of iron with refractive index of $2.87 + i3.36$ and magneto-optical coefficient $.376 + i0.0066$. Thickness of the layers is $100A^o$ in each case.

Appendix A

Numerical Code

First of all, we will define all the needed variables, which are, β, γ, λ, *pol* the type of polarization of light, we have labeled s-polarized light as 0 and p-polarized light as 1, the type of species and the number of repeating units.

```
spec= ;      % Enter the Number of different type of layers excluding the
             % incident and refracted medium
mot= ;       % Enter the number of repeating units present in the thin film
             % system.
N=   ;       % Define the array of the refractive indeces, containing the index
             % for each film.
Q=   ;       % Define the array of the magneto-optical coefficient, containing
             % the coefficient for each film.
z=   ;       % Define the array of the thickness, containing the thickness for
             % each film.
Beta=;       % Define the array of the angle beta, containing the angle for
```

```
            % each film.
Gamma=;     % Define the array of the angle beta, containing the angle for
            % each film.
lambda= ;   % Enter Lambda here, the wavelength of the incident light.
pol= ;      % Enter the polarization, 0 for s-polarized and 1 for p-polarized;
theta1=-90*pi/180:0.001:90*pi/180;
theta2=theta1.*(180/pi);
if spec==0
    for k=1:length(theta1)
    for count=1:1:2
    theta(count)=asin((N(1)*sin(theta1(k)))/N(count));
    gi(count)=cos(beta)*cos(theta(count))+
                sin(theta(count))*sin(beta)*sin(gamma);
    gr(count)=-cos(beta)*cos(theta(count))+
                sin(theta(count))*sin(beta)*sin(gamma);
    A(:,:,count)=[1,0,1,0;
    (i*sin(theta(count))*Q(count)*(sin(theta(count))*gi(count)
    -2*sin(beta)*cos(gamma)))/(2*cos(theta(count))),
    (cos(theta(count))+i*sin(theta(count))*sin(beta)*cos(gamma)*Q(count)),
    (-i*sin(theta(count))*Q(count)*(sin(theta(count))*gi(count)
    -2*sin(beta)*cos(gamma)))/(2*cos(theta(count))),
    (-cos(theta(count))+i*sin(theta(count))*sin(beta)*cos(gamma)*Q(count));
    i*N(count)*gi(count)*Q(count)/2,-N(count),
    i*N(count)*gr(count)*Q(count)/2,-N(count); N(count)*cos(theta(count)),
    i*N(count)*gi(count)*Q(count)/(2*cos(theta(count))),
    -N(count)*cos(theta(count)),-i*N(count)*gr(count)*Q(count)/(2*cos(theta(count)))];
    Ai(:,:,count)=inv(A(:,:,count));
```

```
    end
    M=Ai(:,:,1)*A(:,:,2);
    G=M(1:2,1:2);
    Gi=inv(G);
    II=M(3:4,1:2);
    RR=II*Gi;
    rss(k)=RR(1,1); % rss
    rsp(k)=RR(1,2); % rsp
    rps(k)=RR(2,1); % rps
    rpp(k)=RR(2,2); % rpp
    kerrs(k)=rps(k)./rss(k);  % kerr in case of s polarization
    kerrp(k)=rsp(k)./rpp(k);  % kerr in case of p polarization
    end
else
num=(mot*spec)+2;
if spec>1
 for i=2:num-(spec+1)
N(i+(spec))=N(i);
Q(i+(spec))=Q(i);
z(i+(spec))=z(i);
Beta(i+(spec))=Beta(i);
Gamma(i+(spec))=Gamma(i);
 end
end
for k=1:length(theta1)
for count=1:1:num
    theta(count)=asin((N(1)*sin(theta1(k)))/N(count));
```

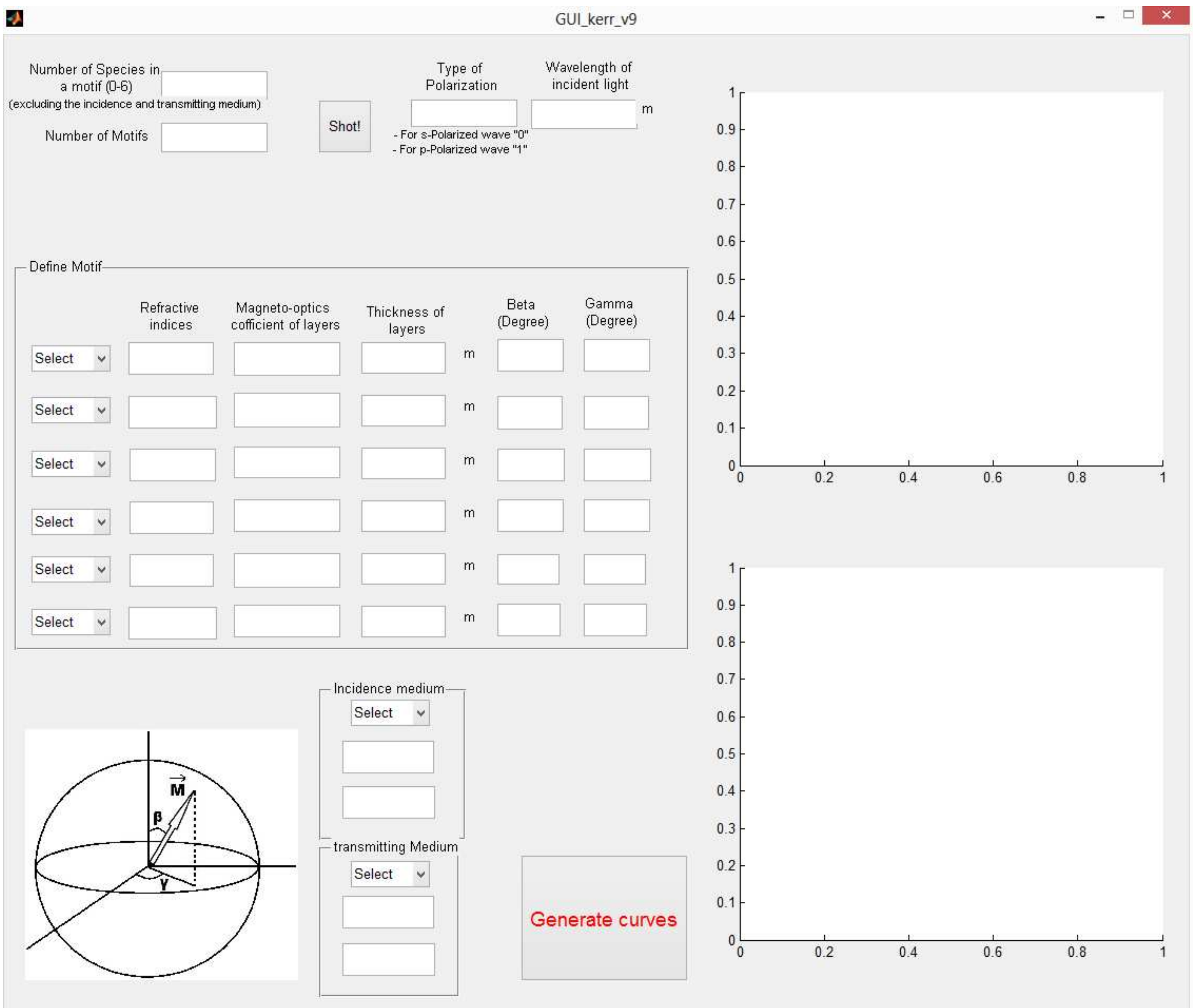
```
gi(count)=cos(Beta(count))*cos(theta(count))
+sin(theta(count))*sin(Beta(count))*sin(Gamma(count));
gr(count)=-cos(Beta(count))*cos(theta(count))
+sin(theta(count))*sin(Beta(count))*sin(Gamma(count));
A(:,:,count)=[1,0,1,0;
(i*sin(theta(count))*Q(count)*(sin(theta(count))*gi(count)
-2*sin(beta)*cos(gamma)))/(2*cos(theta(count))),
(cos(theta(count))+i*sin(theta(count))*sin(beta)*cos(gamma)*Q(count)),
(-i*sin(theta(count))*Q(count)*(sin(theta(count))*gi(count)
-2*sin(beta)*cos(gamma)))/(2*cos(theta(count))),
(-cos(theta(count))+i*sin(theta(count))*sin(beta)*cos(gamma)*Q(count));
i*N(count)*gi(count)*Q(count)/2,-N(count),
i*N(count)*gr(count)*Q(count)/2,-N(count); N(count)*cos(theta(count)),
i*N(count)*gi(count)*Q(count)/(2*cos(theta(count))),
-N(count)*cos(theta(count)),-i*N(count)*gr(count)*Q(count)/(2*cos(theta(count)))];
Ai(:,:,count)=inv(A(:,:,count));
D(:,:,count)=[exp(-1i*2*pi*N(count)*z(count)*cos(theta(count))/
lambda),exp(-1i*2*pi*N(count)*z(count)*cos(theta(count))/
lambda)*pi*N(count)*z(count)*Q(count)*gi(count)/(lambda*cos(theta(count))),0,0;
-exp(-1i*2*pi*N(count)*z(count)*cos(theta(count))/
lambda)*pi*N(count)*z(count)*Q(count)*gi(count)/(lambda*cos(theta(count))),
exp(-1i*2*pi*N(count)*z(count)*cos(theta(count))/lambda),0,0;
0,0,exp(1i*2*pi*N(count)*z(count)*cos(theta(count))/lambda),
-exp(1i*2*pi*N(count)*z(count)*cos(theta(count))/
lambda)*pi*N(count)*z(count)*Q(count)*gr(count)/(lambda*cos(theta(count)));
0,0,exp(1i*2*pi*N(count)*z(count)*cos(theta(count))/
lambda)*pi*N(count)*z(count)*Q(count)*gr(count)/(lambda*cos(theta(count))),
```

```
    exp(1i*2*pi*N(count)*z(count)*cos(theta(count))/lambda)];
end
% here we will get all the A D and inverse A matrices saved in 3-Dimesional
% arrays
M=Ai(:,:,1);


for n=2:count-1;
M=M*A(:,:,n)*D(:,:,n)*Ai(:,:,n);
end
M=M*A(:,:,count);
% here we will get final M matrix which has the form Ai(1)*(ADAi)^m * A(f).
G=M(1:2,1:2);
Gi=inv(G);
II=M(3:4,1:2);
RR=II*Gi;
rss(k)=RR(1,1); % rss
rsp(k)=RR(1,2); % rsp
rps(k)=RR(2,1); % rps
rpp(k)=RR(2,2); % rpp
kerrs(k)=rps(k)./rss(k);  % kerr in case of s polarization
kerrp(k)=rsp(k)./rpp(k);  % kerr in case of p polarization
end
end
if pol==0
    krsr=(real(kerrs))*180/pi;
    krsi=(imag(kerrs))*180/pi;
    plot(handles.axes1,theta2,krsr);
```

```
    xlabel(handles.axes1,'angle of incidence (Degree)');
    ylabel(handles.axes1,'s rotation (degree)');
    title(handles.axes1,'s rotation vs angle of incidence')
    plot(handles.axes2,theta2,krsi);
    xlabel(handles.axes2,'angle of incidence (Degree)');
    ylabel(handles.axes2,'s ellipticity (degree)');
    title(handles.axes2,'s ellipticity vs angle of incidence')
    uisave({'theta2','krsr','krsi'},'filename')
else
    if pol==1
       krpr=-(real(kerrp))*180/pi;
       krpi=(imag(kerrp))*180/pi;
       plot(handles.axes1,theta2,krpr);
       xlabel(handles.axes1,'angle of incidence (Degree)');
       ylabel(handles.axes1,'p rotation (degree)');
       title(handles.axes1,'p rotation vs angle of incidence')
       plot(handles.axes2,theta2,krpi);
       xlabel(handles.axes2,'angle of incidence (Degree)');
       ylabel(handles.axes2,'p ellipticity (degree)');
       title(handles.axes2,'p ellipticity vs angle of incidence')
       uisave({'theta2','krpr','krpi'},'filename')
    end
end
```

A.1 GUI Panel


GUI_kerr_v9
Number of Species in a motif (0-6)
(excluding the incidence and transmitting medium)
Number of Motifs
Shot!
Type of Polarization
- For s-Polarized wave "0"
- For p-Polarized wave "1"
Wavelength of incident light
m
Define Motif
Refractive indices
Magneto-optics cofficient of layers
Thickness of layers
Beta (Degree)
Gamma (Degree)
Select
m
Incidence medium
transmitting Medium
Generate curves

Bibliography

[1] E. Hecht, "*Optics*" Addison Wesley Publishers . 4, (2002).

[2] A. Yariv, P. Yeh, "*Photonics, Optical electronics in modern communications*" Oxfored university press . 6, (2007).

[3] G. R. Fowles, "*Introduction to modern optics*" Dover Publications . 2, (1975).

[4] A. K. Zevzdin and V. A. Kotov, "*Modern magnetooptics and magnetooptical materials*" IOP publishing Ltd. (1977).

[5] J. M. D. Coey, "*Magnetism and magnetic materials*" Cambridge university press. 1, (2010).

[6] J. Zak, E. R. Moog, C. Liu and S. D. Bader, "*Universal approach to magneto-optics,*" Journal of magnetism and magnetic materials. 89, 107-123 (1990).

[7] J. Zak, E. R. Moog, C. Liu and S. D. Bader, "*Magneto-optics of multilayers with arbitrary magnetization directions,*" Physical review B. 43(8), 6423-6429 (1991).

[8] C. Y. You and S. C. Shin, "*Generalized analytical formulae for magneto-optical Kerr effects,*" Journal of applied physics. 84(1), 541-546 (1998).

[9] C. Y. You and S. C. Shin, "*Derivation of simplified analytical formulae for magneto-optical Kerr effects,*" Applied physical letter. 69(9), 1315-1317 (1996).

[10] P. S. Hauge, "*Mueller matrix ellipsometry with imperfect compensators*" Optical society of America. 68(11), 1519-1528 (1978).