It is a contact force exerted by the starting blocks against the foot of a sprinter that accelerates him forward. An equal and opposite force is exerted on the starting block, and, because the block is attached to Earth, on Earth itself.

Chapter 5

Applications of Newton's Laws

In this chapter we apply Newton's laws to a variety of situations in which forces act. We will consider several types of forces, including gravity, tension, normal forces, friction, and drag forces, all of which act on us and the objects around us. We shall also look at the role of forces in circular motion as well as the features of motion in rotating frames. Finally, we will look at how the forces are ultimately described in terms of more fundamental forces—in particular, ones that act at a microscopic level.

5–1 Common Forces Revisited

The forces that we encountered in Chapter 4 demand further study, as we have not yet explored all of their significant features. These forces include the force of *gravity* $\vec{F}_g$, here treated as a constant force that acts on every object, the tension $\vec{T}$ exerted by a taut rope, and the normal force $\vec{F}_N$ that keeps you from falling through your chair. We'll study friction in Section 5–2.

Gravity

The parabolic form of a projectile's trajectory near Earth (Fig. 5–1) is due to the force of gravity. As we saw in Chapter 3, projectiles in the vicinity of Earth's surface travel with a

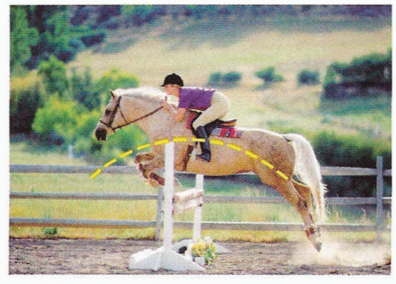


FIGURE 5–1 The horse follows a parabolic path as it jumps the rail.

constant acceleration $\vec{a}$ that points downward and has magnitude g. With Newton's second law, we can say that the force of gravity causes the projectile's constant acceleration and that the force has constant magnitude near Earth's surface and is always directed down toward the center of Earth. More precisely, the acceleration $\vec{a}$ of our projectile is given in terms of the force of gravity $\vec{F}_g$ on the projectile and the inertial mass m of the projectile as

$$\vec{a} = \frac{\vec{F}_g}{m}. \tag{5–1}$$

A very special characteristic of the force of gravity is that, at any given location, *it causes all objects to accelerate in the same way—no matter what their mass.* In other words, the right-hand side of Eq. (5–1) is independent of the mass m. The only way this can happen is for the *force itself to be proportional to the mass*, so that the factor m in the denominator cancels with another such factor in the force itself.

Let's look more carefully at the meaning of this remarkable fact. That an object has constant acceleration under the influence of gravity means only that this force takes the form

$$\vec{F}_g = m_g\vec{g}, \tag{5–2}$$

where $\vec{g}$ is a constant vector with dimensions of acceleration and points to the center of Earth.† The "gravitational mass" m_g is the property of the object that determines the strength of the gravitational force acting on it. However, the acceleration of *all* objects under the influence of gravity is *precisely the same*; mathematically, this statement means

$$m_g = m. \tag{5–3}$$

This relation between gravitational mass and inertial mass is remarkable because, at least until Einstein's theory of general relativity, not developed until about 1915, we had no reason to think that the force of gravity has anything special to do with the inertial mass. (See Chapter 12 for further discussion.) We recall from Chapter 4 that the inertial mass determines the *response* to a force, and tests to determine the inertial mass are possible without using gravity at all—say, by seeing how much the object accelerates when *any* known force, even one that has absolutely nothing to do with gravity, acts on it. Conversely, tests to determine the force of gravity on an object can be performed without using motion at all—say, by observing the equilibrium stretch of a spring while the object hangs from it under the influence of gravity. Nevertheless, the equality of Eq. (5–3) has been experimentally verified to a very high degree of accuracy.‡ Thus the force of gravity has the simple form

$$\vec{F}_g = m_g\vec{g} = m\vec{g}. \tag{5–4}$$

FORCE OF GRAVITY

The acceleration $\vec{a}$ of any object at Earth's surface under the influence of gravity alone is then given by the constant vector $\vec{g}$:

$$\vec{a} = \frac{\vec{F}_g}{m} = \vec{g}. \tag{5–5}$$

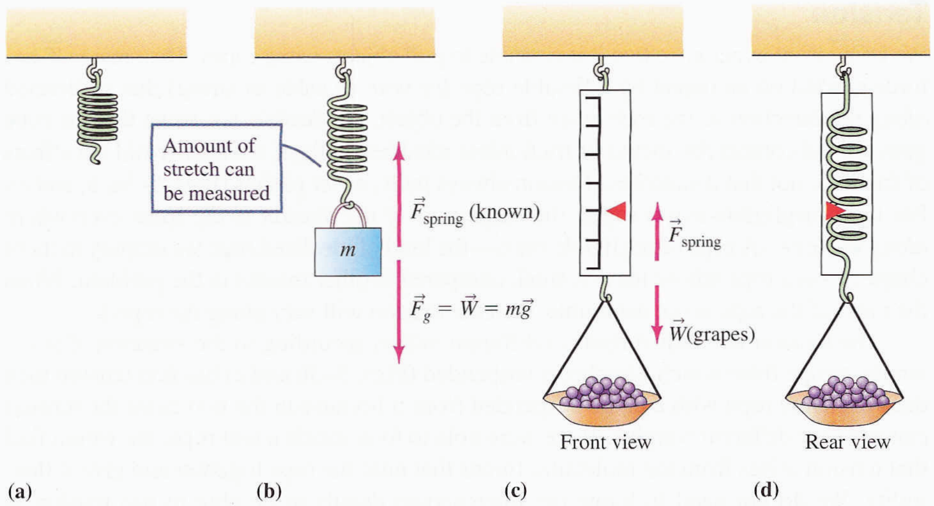
As described in Chapter 3, the magnitude of $\vec{g}$ at Earth's surface is roughly

$$g = 9.80\ \text{m/s}^2. \tag{5–6}$$

Experiments show that this value varies by about 1 percent over Earth's surface, with the higher values occurring at the poles. This variation is due to irregularities in the shape and density of Earth and to Earth's rotation.

† The force near any astronomical object has the same form and direction, but the magnitude of g is different. We'll learn more about this in Chapter 12.

‡ The so-called Eötvös experiments verify that the inertial and gravitational masses are the same to 1 part in 10^{12}.



▲ **FIGURE 5–2** The weight of an object is found by attaching it to a spring, for which the amount of stretch corresponds to a given force. In (a) the spring is unstretched. (b) Calibration (the force corresponding to a given amount of stretch) is mapped out by hanging a series of known masses. (c, d) Once the calibration is done, the weight of any other object can be found.

We noted in Chapter 4 that the force of gravity on an object is commonly called the object's weight, $\vec{W}$,

$$\vec{W} = \vec{F}_g = m\vec{g}. \tag{5–7}$$

WEIGHT

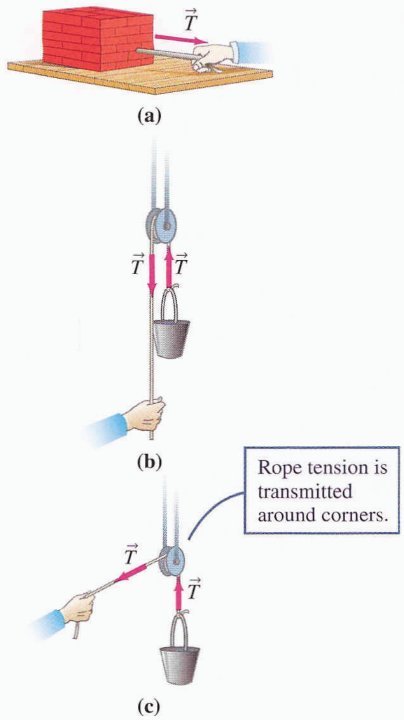
We can experimentally determine the weight of an object by balancing the force of gravity against a second calibrated force such as that exerted by the stretched spring shown in its unstretched state in Fig. 5–2a and stretched, with a mass attached, in Fig. 5–2b. Once we know how much force is exerted for a given stretch, we can find the weight of any object by suspending it from the spring and observing the amount of stretch (Figs. 5–2c and d). In fact, when you step on a bathroom scale, you are indirectly measuring the force of gravity on your body in a similar way; the second force is the calibrated bathroom scale, which is in essence a compressible spring. While we might in everyday conversation sometimes use the words *weight* and *mass* interchangeably, this is not correct. Be careful not to confuse the weight of an object, measured in newtons or pounds, with its mass. We know from Chapter 4 that the mass of an object is the quantity of matter it contains, which is measured by its inertia—its resistance to any change in motion. The mass and weight of an object, though, are numerically proportional to each other through g—the weight of an object equals the object's mass times $\vec{g}$. In other words, the mass is intrinsic to the object, but its weight depends on where it is: If you went to the Moon, where a falling object falls with a different acceleration $\vec{g}_{Moon}$, a 1-kg mass would still be a 1-kg mass, but its weight would have magnitude mg_{Moon} rather than mg_{Earth}.

CONCEPTUAL EXAMPLE 5–1 Let us imagine that the inertial mass of any object is exactly 5 percent larger than its gravitational mass. How would this affect our treatment of the projectile motion?

Answer The force of gravity $\vec{F}_g = m_g\vec{g}$ can be determined by weighing the object. However the two factors cannot be determined without knowing more about gravity (this is the subject of Chapter 12). The acceleration due to the force of gravity is given by

$$\vec{a} = \frac{\vec{F}_g}{m} = \frac{m_g}{m}\vec{g} = \frac{1}{1.05}\vec{g}.$$

It is this quantity that is determined by dropping an object from a given height, and it is this acceleration that has the value 9.81 $\mathrm{m/s^2}$ on Earth. Furthermore it is this quantity that enters into projectile motion. Thus nothing would be changed, *provided* that the ratio of m to m_g (here 1.05) is a universal constant, and does not change from object to object.



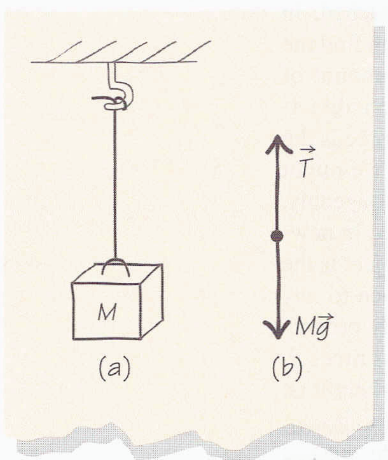
Tension

We know from experience that it is possible to pull objects using ropes. The tension $\vec{T}$ is a force exerted on an object by a flexible rope (or wire or cable or string) that is directed *along* the direction of the rope away from the object. By *flexible*, we mean that the rope goes around corners (by means of frictionless idealized pulleys, a way to avoid the effects of friction), not that it stretches. Tension always pulls, never pushes (Figs. 5–3a, b, and c). For light (negligible-mass) ropes, the *magnitude of the tension is the same everywhere along the rope*. (A rope of negligible mass—the kind of idealized rope we employ in these chapters—is a rope whose mass is small compared to other masses in the problem. When the mass of the rope is not negligible, then the tension will vary along the rope.)

The tension will adjust itself to different values according to the situation. For example, a rope from which a bucket is suspended (Figs. 5–3b and c) has less tension than does the same rope with a piano suspended from it because in the two cases the tension cancels very different weights. If we were able to look inside a taut rope, we would find that tension arises from the molecular forces that hold the rope together and give it flexibility. We do not need to know the microscopic details to be able to use tension in everyday problems.

◀ **FIGURE 5–3** (a) The tension force pulls but cannot push. (b, c) A rope of negligible mass with a given tension can maintain the magnitude of the tension even if the direction of the rope is changed. Tension can be transmitted around corners without change of magnitude because we are assuming an ideal pulley.

EXAMPLE 5–2 Consider Fig. 5–4a, which shows a fishing line hanging from a hook attached to the ceiling. This fishing line is rated as 10-lb-test line, which means that it should hold as long as the tension within it does not exceed 10 lb. A box of mass 2.0 kg is attached to the line. Find the tension in the line. Will the line hold? What will happen if a box of mass 5.0 kg is attached instead?



▲ **FIGURE 5–4** (a) Mass hangs from hook using 10-lb-test line. (b) Free-body diagram for a 2.0-kg package suspended from a line.

Setting It Up We are given the mass of the box, which we label as M. If we can calculate the unknown line tension's magnitude T, we can decide whether the line will break. We draw a sketch of the situation (Fig. 5–4a).

Strategy An important first step is to adapt the sketch and make it into a free-body diagram for the box (Fig. 5–4b). With the aid of this diagram we can write Newton's law for the box and use the condition that the mass is stationary—that is, that there is no acceleration, so that the net force is zero—to find T.

Working It Out The free-body diagram shows us that the tension and the force of gravity are the only forces acting on the box. Thus the net force on it is zero:

$$\vec{F}_{\text{net}} = \vec{T} - M\vec{g} = 0.$$

We solve this for T:

$$T = Mg.$$

The sign is positive because we want only the magnitude of the tension force. With $M = 2.0$ kg,

$$T = Mg = (2.0\,\text{kg})(9.8\,\text{m/s}^2) = 19.6\,\text{N}$$
$$= (19.6\,\cancel{\text{N}})\left(\frac{1\,\text{lb}}{4.45\,\cancel{\text{N}}}\right) = 4.4\,\text{lb}.$$

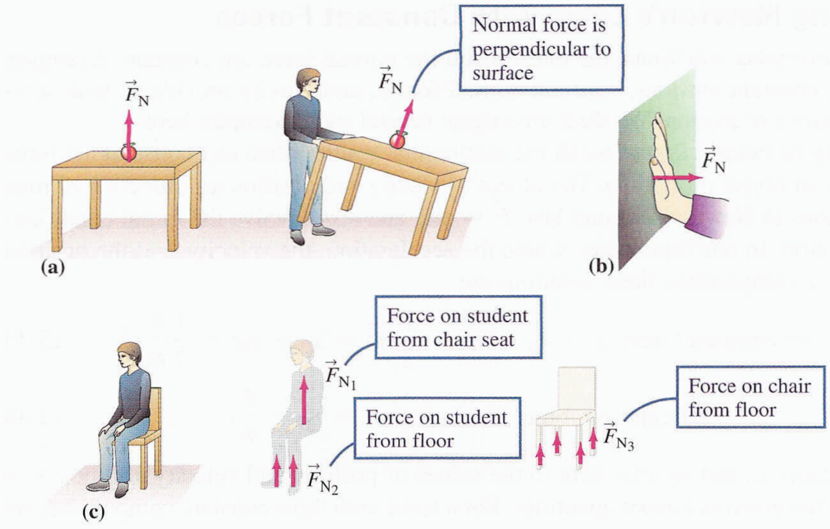
The line will hold. But with $M = 5.0$ kg,

$$T = (5.0\,\text{kg})(9.8\,\text{m/s}^2) = 49\,\text{N} = (49\,\cancel{\text{N}})\left(\frac{1\,\text{lb}}{4.45\,\cancel{\text{N}}}\right) = 11\,\text{lb}.$$

The line may not hold with the heavier mass suspended from it as the tension within it exceeds its breaking tension.

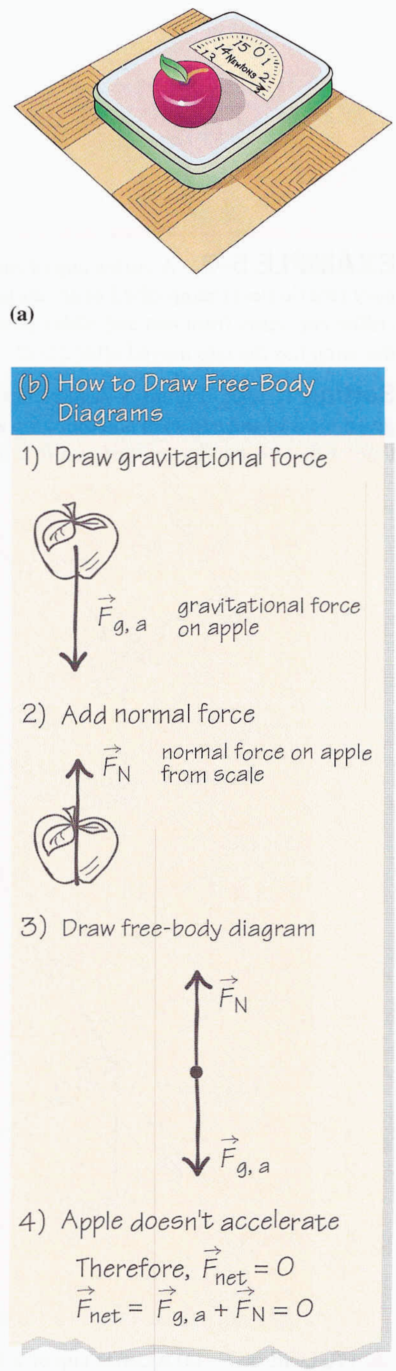
What Do You Think? Why do fishermen use fishing line that is rated much higher than the weight of any fish they are likely to catch? *Answers to **What Do You Think?** questions are given in the back of the book.*

How can we measure the tension within a rope at a given point? One way would be to cut the rope at the given point and insert a calibrated spring scale. The scale will stretch by an amount corresponding to the tension in the rope. This experiment could be performed at different points along the rope to show that the tension is the same throughout.



◀ **FIGURE 5–5** Some normal forces. These forces are always perpendicular to the contact surface, keeping the object on which the force acts from entering the surface.

▼ **FIGURE 5–6** (a) A scale can be used to measure the normal force on an apple. (b) We can draw the forces acting on the apple.



(b)

Normal Force

Let's look at the forces acting on an apple placed on a table (Fig. 5–5). In addition to the force of gravity, a *normal force* $\vec{F}_N$ acts on the apple. In Chapter 4, we were introduced to this force and acknowledged that such a normal force must exist—without it, gravity would cause the apple to accelerate into the surface of the table. The normal force is the result of the complex interaction between the molecules of the table. We call a material that can give rise to a normal force a *solid*. The normal force on an object acts only when the object is in contact with the table. The normal forces on the objects shown in Fig. 5–5 adjust themselves to cancel the components of forces perpendicular to the surface. If the weight is too large, however, the supporting surface will collapse. The normal force pushes but never pulls. It acts perpendicular to and away from the surface at the object's point of contact. If this were not true, an apple on a table would accelerate to one side of the table or the other. In fact, whatever the tilt of the table, the apple does not move into its surface because *the normal force always acts perpendicular to and away from the surface*. Because of this property, the normal force cannot counteract any forces parallel to the surface. For example, the normal force cannot oppose the frictional force parallel to the table's surface in the case of the tilted table in Fig. 5–5b and cannot keep the apple from rolling down the table. All the normal force can do is keep the apple from penetrating the table.

We can see how the magnitude of $\vec{F}_N$ is determined in a situation by the use of Newton's first law. An apple of 0.25 kg sitting on a table must experience an upwardly directed normal force that is equal and opposite to the downward force of gravity; the magnitude of this force is $mg = (0.25\ \text{kg})(9.8\ \text{m/s}^2) = 2.5\ \text{N}$. If a pumpkin of mass 2.5 kg sits on the table, the normal force is 25 N. As we'll see in the examples below, the normal force on the apple will differ in magnitude from the apple's weight if we tilt the table.

An ordinary bathroom scale gives us a way to read directly the magnitude of the normal force. The spring in the scale opposes the force of gravity in exactly the same way as does the normal force and therefore gives the magnitude of the normal force via a calibrated scale. In Fig. 5–6, a scale measures the normal force of 2.5 N that acts on the apple (although the force is often expressed in units other than newtons). The scale could equally well be placed on other surfaces that exert normal forces. For example, the normal force exerted on a hand by a wall can be measured directly if a scale is inserted between the wall and the hand.

There are other forces similar to the tension and the normal force. Among these we could include the support forces supplied by hooks, connection points, bearings, and so forth. The direction and magnitude of such forces are determined by the requirement that the attachment point in question does not accelerate. For example, a hook from which a 20-lb weight is suspended must supply an upward force of 20 lb if the mass is not to fall.

Applying Newton's Laws with Constant Forces

In many everyday situations, the tension and the normal force are constant. Examples involving constant tensions, constant normal forces, and gravity provide us with solvable equations of motion. We shall investigate several such examples here.

It may be helpful first to recall the motion that results when *any* constant net force $\vec{F}$ acts on an object of mass m. The object's velocity and position as a function of time are solutions to Newton's second law, $\vec{F} = m\vec{a}$, and also involve the initial conditions of the motion. In one dimension, where the acceleration, the velocity, and the position have only x-components, these solutions are

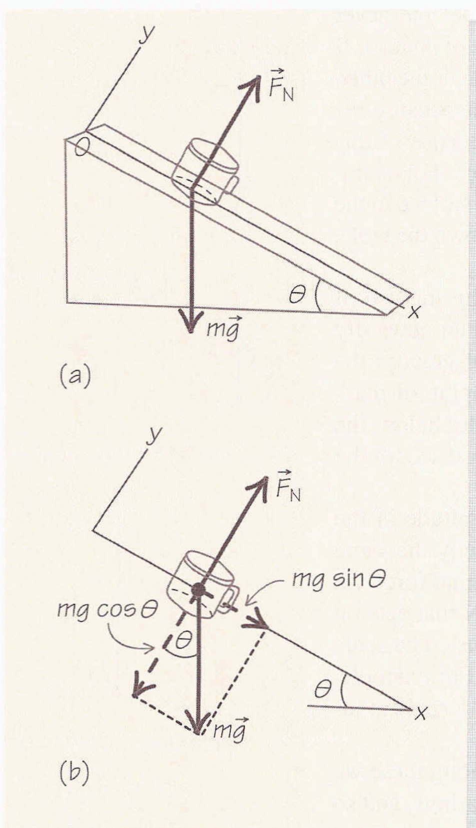
$$\text{for constant force: } x = x_0 + v_0 t + \frac{1}{2}at^2 = x_0 + v_0 t + \frac{1}{2}\frac{F}{m}t^2; \qquad (5\text{–}8)$$

$$\text{for constant force: } v = v_0 + at = v_0 + \frac{F}{m}t. \qquad (5\text{–}9)$$

The constants x_0 and v_0 refer here to the values of position and velocity at time $t = 0$ and are often given or known quantities. For a force with three constant components, we must apply the solution to each component separately. Remember that these formulas apply only when $\vec{F}$ is constant.

EXAMPLE 5–3 A coffee cup of mass 75 g is placed on a slippery (frictionless) ramp tilted at an angle 20° to the horizontal. The coffee cup starts from rest and slides down the ramp. How far down the ramp has the cup moved after 2.0 s?

Setting It Up Figure 5–7a sketches the situation. We label the given mass of the cup as m and the ramp angle as θ. We want the distance x that the cup has moved as a function of time t.



▲ **FIGURE 5–7** (a) A coffee cup on a frictionless, tilted ramp. (b) Free-body diagram for the coffee cup, including the vector decomposition of the forces.

Strategy We first prepare a free-body diagram, choose a convenient set of axes, and decompose the forces into their components along these axes (Fig. 5–7b). We then write Newton's second law, which will determine acceleration. At that point kinematic equations can be used to find the displacement. Because there is no friction, the only forces acting on the cup are gravity $\vec{F}_g$ and the normal force $\vec{F}_N$. Note that $\vec{F}_N$ is perpendicular to the surface of the inclined plane and is *not* oriented in the vertical direction.

The origin is placed at the starting point of the coffee cup. We choose the axes shown in Fig. 5–7b because we know from experience the acceleration $\vec{a}$ of the cup will be along the ramp, with no component perpendicular to the ramp. Accordingly, the x-direction points down along the ramp and the acceleration will have only an x-component a_x, $\vec{a} = a_x\hat{i}$. We must then decompose the forces acting along this set of axes and express Newton's second law. While only the component of forces along x will act to accelerate the cup, the constraint that there is *no* acceleration perpendicular to the surface may still be helpful, and so we will write Newton's law for that direction as well. (Although we may not use this in this example, writing equations for all vector components is a good habit to get into and will be necessary when friction becomes an issue.)

Finally, it is always useful to have a way to check our result. Here we can use the fact that we expect no movement if the plane is horizontal.

Working It Out With our choice of axes, the normal force has only a y-component:

$$\vec{F}_N = F_N\hat{j}.$$

The magnitude F_N is as yet unknown, but we will find it below from the requirement that the acceleration has no y-component. The force of gravity, however, has both x- and y-components (Fig. 5–7b):

$$\vec{F}_g = (mg \sin\theta)\hat{i} - (mg \cos\theta)\hat{j}.$$

Newton's second law has the vector form $\vec{F}_g + \vec{F}_N = m\vec{a}$, or

$$(mg \sin\theta)\hat{i} - (mg \cos\theta)\hat{j} + F_N\hat{j} = ma_x\hat{i}.$$

Each component of this equation is a separate equation of motion:

$$\text{for the } x\text{-component: } mg \sin\theta = ma_x; \qquad (5\text{–}10)$$

$$\text{for the } y\text{-component: } -mg \cos\theta + F_N = 0. \qquad (5\text{–}11)$$

Now we solve the equations of motion. Equation (5–10) gives

$$a_x = g \sin\theta. \tag{5–12}$$

Equation (5–11) shows that $\vec{F}_N$ has magnitude $mg \cos\theta$.

To find how the position of the coffee cup changes with time, we use the kinematic equation for constant acceleration in one dimension: Eq. (5–8). In this equation, the cup starts at the origin, so $x_0 = 0$; moreover, $v_0 = 0$ because the cup is initially at rest. We then insert Eq. (5–12) into Eq. (5–8):

$$x = \tfrac{1}{2}a_x t^2 = \tfrac{1}{2}(g \sin\theta)t^2.$$

At $t = 2.0$ s,

$$x = (0.5)(9.8\ \mathrm{m/s^2})(\sin 20°)(2.0\ \mathrm{s})^2 = 6.7\ \mathrm{m}.$$

We find the cup has moved 6.7 m after 2.0 s. Does this seem far to you? The lack of friction is an important effect! As is typical for problems involving gravity, the answer is independent of the mass of the cup. The angle $\theta = 0°$ presents a special limit; in this case, the plane is horizontal, and there should be no acceleration whatsoever. This is ensured by the $\sin\theta$ factor in the acceleration, which is zero when $\theta = 0°$. In this limit the normal force has magnitude mg, as expected. A second limit is the case $\theta = 90°$, when $\sin\theta = 1$. Here, $a_x = g$ and $F_N = 0$, also as expected.

What Do You Think? It seems unwise to put a coffee cup on an inclined frictionless ramp. What would change in this example if we put other objects—a golf ball or a car—on the ramp?

EXAMPLE 5–4 Because of a wager, a woman wishes to lift a professional football player off his feet. The player is a large interior lineman (a tackle) with a mass of 149 kg. (He weighs 328 lb.) The woman has devised a system for the task, which is shown in Fig. 5–8a. We will assume that all pulleys, ropes, and miscellaneous gear in the apparatus have negligible mass and are frictionless. What is the magnitude of the downward force the woman must exert on the end of the rope in order to lift the lineman?

Setting It Up The given mass of the lineman is M; the mass of the woman is denoted by m. The downward force the woman exerts at the point that the lineman is lifted is just the tension T in the rope, and this is the quantity we want to find.

Strategy Even a smooth lift at constant velocity is sufficient—the lineman does not need to accelerate continuously to be lifted. This is a condition that the net force of the ropes on the lineman balances his weight. The tension in the rope, which is the force that must be supplied by the woman pulling on the rope, is the same throughout the rope.

If we examine Fig. 5–8a, we see that all four rope segments, 1 through 4, pull upward on the lineman. Figure 5–8b illustrates the external forces on the isolated system more clearly. Since it is a single massless rope passing around all the pulleys, the tension in each one is the same, namely T. The free-body diagram for the system is Fig. 5–8c. When the woman begins to pull on the rope, it acquires a tension of magnitude T, and the sum of the tensions $T_1 + T_2 + T_3 + T_4 = 4T$ increases from zero. As long as this sum is less than the weight of the lineman, Mg, he will remain on the ground. But when this sum becomes equal to Mg, there is no net force on the lineman, and any additional tension—no matter how small—will start him accelerating upward. Thus the condition to lift the lineman is

$$4T = Mg.$$

We need only plug in the numbers.

Working It Out We have

$$T = \tfrac{1}{4}Mg = \tfrac{1}{4}(149\ \mathrm{kg})(9.80\ \mathrm{m/s^2}) = 365\ \mathrm{N}.$$

This is the magnitude of the smallest downward force that the woman must apply to her end of the rope. Provided that she weighs more than 365 N (82.1 lb), which corresponds to a mass $m = W/g = 37.3$ kg, she can apply this force just by hanging on the rope. (If she weighs more than 82.1 lb, then she can "partially" hang from the rope, meaning that her feet don't completely lose contact with the ground.)

The arrangement described here is called a *block and tackle* and is used, for example, to enable a single person to lift an engine from a car.

What Do You Think? Why did we make the assumption that all pulleys, ropes, and miscellaneous gear in the apparatus have negligible mass and are frictionless?

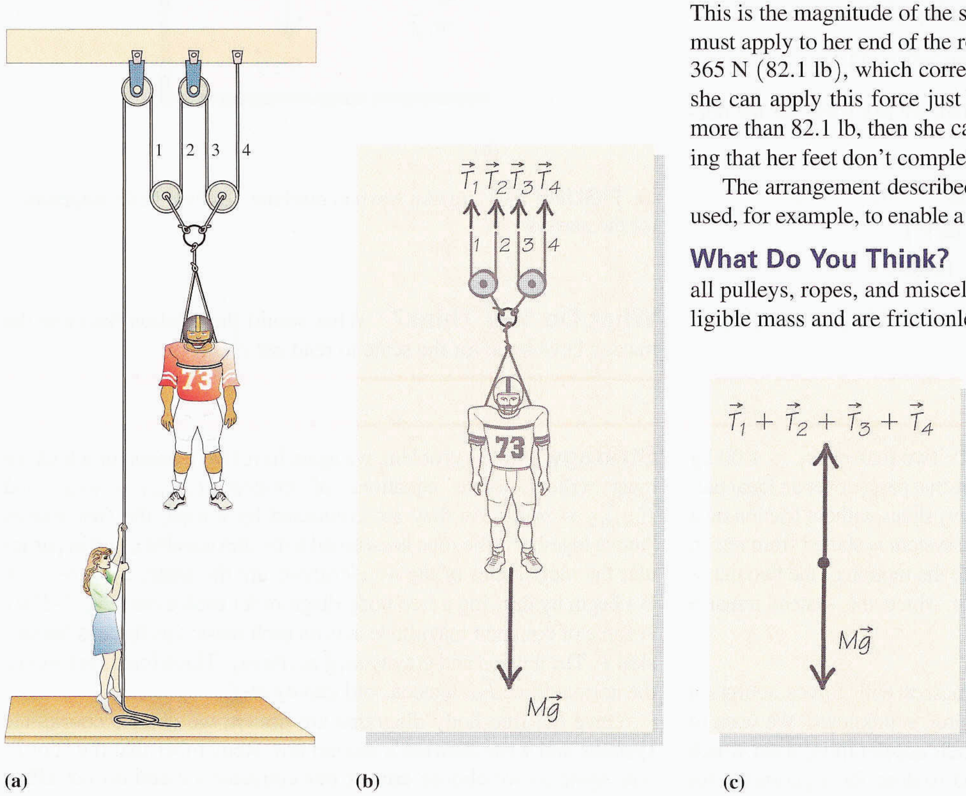


FIGURE 5–8 (a) A lineman lifted by a rope via a system of pulleys. (b) The lineman is clearly isolated, with the external forces shown. (c) Free-body diagram for the lineman.

In Examples 5–5 and 5–6 we investigate further all three forces discussed so far in this section—tension, the normal force, and gravity. These examples also introduce something new, in that two masses are involved. Each mass requires an identification of the forces acting on it, a free-body diagram, and an equation of motion.

EXAMPLE 5–5 Masses $m_1 = 1.1$ kg and $m_2 = 2.3$ kg are attached to opposite ends of a massless rope draped over a pulley (Fig. 5–9a). (This device is called an *Atwood machine.*) Mass m_2 rests on a scale that measures the normal force exerted on m_2. There is no motion. What is the reading on the scale, and what is the tension in the rope?

Setting It Up We want the rope tension, magnitude T, and the normal force, magnitude F_N, that the scale exerts on m_2.

Strategy We start with a free-body diagram for each mass (Fig. 5–9b), including a coordinate, here y, and find the equations of motion for each mass. Because there is no acceleration, these equations express the fact that net force on each mass is zero. All three forces—gravity, tension, and normal force—are vertical, so we have only one component (the y-component) to consider and we can drop the vector notation. There are two masses, hence two equations, and these should be enough to determine the two unknown forces.

Working It Out Take the upward direction as positive. The forces acting on m_1 are gravity $(-m_1 g)$ and the rope tension (T), and the net force on m_1 is

$$F_{\text{net},1} = -m_1 g + T = 0. \tag{5–13}$$

Three forces act on mass m_2: $-m_2 g, T$, and F_N. Notice that the same value of T acts on each mass, pulling (acting upward) in each case. The net force on m_2 is

$$F_{\text{net},2} = -m_2 g + T + F_N = 0. \tag{5–14}$$

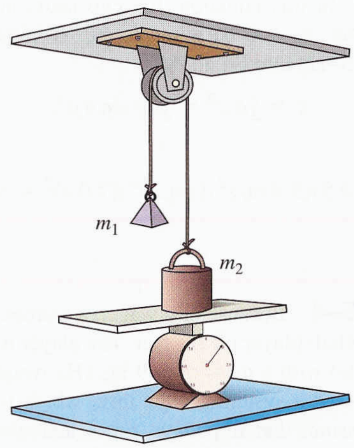
We then can solve Eqs. (5–13) and (5–14) for the tension and the normal force (the scale reading). Solving Eq. (5–13),

$$T = m_1 g = (1.1\text{ kg})(9.8\text{ m/s}^2) = 11\text{ N}.$$

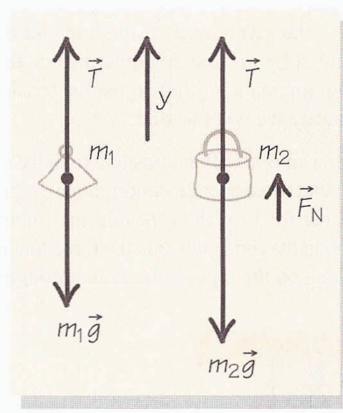
If we insert this value of T into Eq. (5–14), we can solve for the normal force and hence the scale reading:

$$\begin{aligned} F_N &= m_2 g - T = m_2 g - m_1 g = (m_2 - m_1)g \\ &= (2.3\text{ kg} - 1.1\text{ kg})(9.8\text{ m/s}^2) \\ &= (1.2\text{ kg})(9.8\text{ m/s}^2) \cong 12\text{ N}. \end{aligned}$$

For comparison, the weight of m_2 alone is $(2.3\text{ kg})(9.8\text{ m/s}^2) = 23$ N.



(a)



(b)

▲ **FIGURE 5–9** (a) An Atwood machine. (b) Free-body diagrams for the masses.

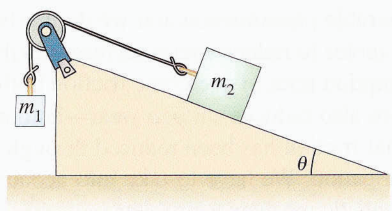
What Do You Think? What would the relation between the masses have to be for the scale to read zero?

EXAMPLE 5–6 Consider Fig. 5–10: Two masses $m_1 = 1.00$ kg and $m_2 = 2.00$ kg are connected by a rope that passes over an ideal pulley. Mass m_1 hangs straight down, while m_2 slides without friction on a ramp inclined at an angle θ. At $t = 0$, the system is started from rest in the position shown in Fig. 5–10a. Describe the motion of the two masses for $\theta = 25.0°$. Find the angle θ' for which the system remains motionless.

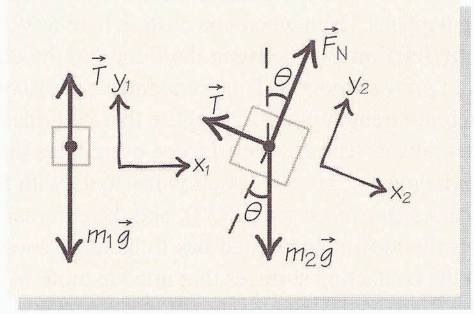
Setting It Up We are given two masses with forces acting on them. One of those forces, the rope tension, is unknown. We want to find the motion of the masses, that is, their acceleration, from which we can work out position. We also want to describe a geometry for which there is no acceleration.

Strategy In this problem, we again have two masses for which we must write separate equations of motion: $\vec{F}_{\text{net on }1} = m_1\vec{a}_1$ and $\vec{F}_{\text{net on }2} = m_2\vec{a}_2$. As they are connected by a rope, the two masses "move together" (the rope is assumed to be inextensible), and in particular the *magnitudes of the accelerations* are the same, $a_1 = a_2 = a$. We begin by drawing a free-body diagram for each mass (Fig. 5–10b). A force of common magnitude acts on each mass; this force is the tension $\vec{T}$. The tension and gravity $m_1\vec{g}$ act on m_1. Three forces act on m_2: the normal force $\vec{F}_N$, tension, and gravity $m_2\vec{g}$.

Once the free-body diagrams are drawn, we choose coordinate systems and write Newton's second law. Remember that the coordinate systems we choose are for our convenience and do not affect the result, and we *do not have to choose the same coordinate system*



(a)



(b)

▲ **FIGURE 5–10** (a) Two masses connected by a rope via a pulley. (b) Free-body diagrams for masses m_1 and m_2.

for the two masses. In fact, choosing different systems makes this problem easier to solve (Fig. 5–10b). We will have to count equations and determine if there are enough to solve for the unknowns, in particular a.

As for finding the angle for which $a = 0$, if we had found enough equations to allow us to solve for a in the first part, we could see whether there is an angle for which $a = 0$. This is much simpler if we leave our expressions in algebraic form, a very good general rule.

Working It Out We choose the origin of each coordinate system to be at the location of the respective mass at $t = 0$. Because m_1 moves only in the vertical direction, labeled y_1, its acceleration $\vec{a}_1$ is aligned with y_1. For m_1, then, we need look only at the component of Newton's second law along y_1,

$$T - m_1 g = m_1 a_1. \qquad (5\text{–}15)$$

Although the mass m_2 has forces acting in two directions, it moves only in the direction labeled x_2 in Fig. 5–10b. Thus $\vec{a}_2$ is aligned with x_2. We must now decompose the forces acting on m_2 into components in this coordinate system. The normal force $\vec{F}_N$ is in the $+y_2$-direction and $\vec{T}$ is in the $-x_2$-direction. The third force, gravity, has two components; we can determine these components by recalling from geometry that the angle θ indicated in Fig. 5–10b is the same as the ramp angle θ in Fig. 5–10a. Then the force of gravity $m_2\vec{g}$ has x_2-component $m_2 g \sin\theta$ and y_2-component $-m_2 g\cos\theta$. Thus Newton's second law for mass m_2, $\vec{F}_{\text{net on 2}} = \vec{F}_N + \vec{T} + m_2\vec{g} = m_2\vec{a}_2$, breaks down into two component equations:

$$\text{for the } x_2\text{-component: } -T + m_2 g\sin\theta = m_2 a_2; \qquad (5\text{–}16)$$

$$\text{for the } y_2\text{-component: } F_N - m_2 g\cos\theta = 0. \qquad (5\text{–}17)$$

The three equations (5–15), (5–16), and (5–17) are not enough to solve for the four unknowns T, F_N, a_1, and a_2. The needed fourth equation is the expression that states that the the two masses have accelerations of the same magnitudes,

$$a_1 = a_2 \equiv a. \qquad (5\text{–}18)$$

The single acceleration magnitude a is then substituted in Eqs. (5–15), (5–16), and (5–17), which become three equations for the three unknowns a, T, and F_N. Equation (5–17) gives the normal force,

$$F_N = m_2 g\cos\theta.$$

The tension cancels in the sum of Eqs. (5–15) and (5–16):

$$T - m_1 g - T + m_2 g\sin\theta = m_1 a + m_2 a.$$

We are left with an equation for a with solution

$$a = \frac{m_2\sin\theta - m_1}{m_1 + m_2}g. \qquad (5\text{–}19)$$

Now that we have found the accelerations we can find positions as a function of time by using the constant-acceleration equations (5–8). Each mass starts from rest at the origin of its respective coordinate system, so $v_0 = 0$ for both masses, and both y_{10} ($\equiv y_1$ at $t = 0$) and x_{20} ($\equiv x_2$ at $t = 0$) are zero. Thus Eq. (5–8) gives

$$\text{for } m_1\text{: } y_1 = \tfrac{1}{2}at^2;$$

$$\text{for } m_2\text{: } x_2 = \tfrac{1}{2}at^2.$$

The masses move only in these directions. Together with Eq. (5–19), these equations describe the motion fully.

For the case $\theta = 25.0°$, we use $\sin 25.0° = 0.423$, so

$$a = \frac{(2.00\ \cancel{\text{kg}})(0.423) - 1.00\ \cancel{\text{kg}}}{1.00\ \cancel{\text{kg}} + 2.00\ \cancel{\text{kg}}}(9.80\ \text{m/s}^2) = -0.503\ \text{m/s}^2.$$

Thus $y_1 = (-0.503\ \text{m/s}^2)t^2/2 = x_2$. Note the minus sign in the acceleration: The sign indicates that m_1 drops and m_2 moves up the ramp. Equation (5–19) shows that the acceleration—including its sign—depends on the masses and the ramp angle.

Finally, to find the angle for which the acceleration is zero, we note that, according to Eq. (5–19), the acceleration is zero at an angle θ' for which

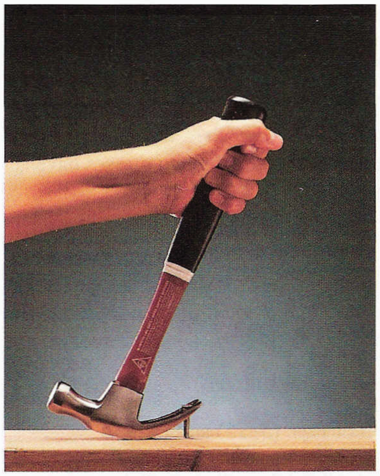
$$m_2\sin\theta' - m_1 = 0, \qquad (5\text{–}20)$$

or $\sin\theta' = m_1/m_2$. For this particular problem the forces will balance and acceleration will be zero for $\sin\theta' = (1.00\ \text{kg})/(2.00\ \text{kg})$, or $\theta' = 30.0°$.

What Do You Think? Consider the system at the angle $\theta' = 30.0°$. (a) What happens for masses $m_1 = 3.0$ kg and $m_2 = 6.0$ kg? (b) What happens for masses $m_1 = 4.0$ kg and $m_2 = 6.0$ kg? (c) What happens for masses $m_1 = 3.0$ kg and $m_2 = 5.0$ kg?

5–2 Friction

Friction is a familiar concept. It is a contact force that impedes sliding, and we experience it in all aspects of our lives. Sometimes friction is useful to us: It is friction that holds nails and screws in place (Fig. 5–11); if there were no friction between our feet and the ground, we could not walk, and if there were no friction between the wheels of a car and the road, the engine would cause the wheels to spin but there would be no forward or backward

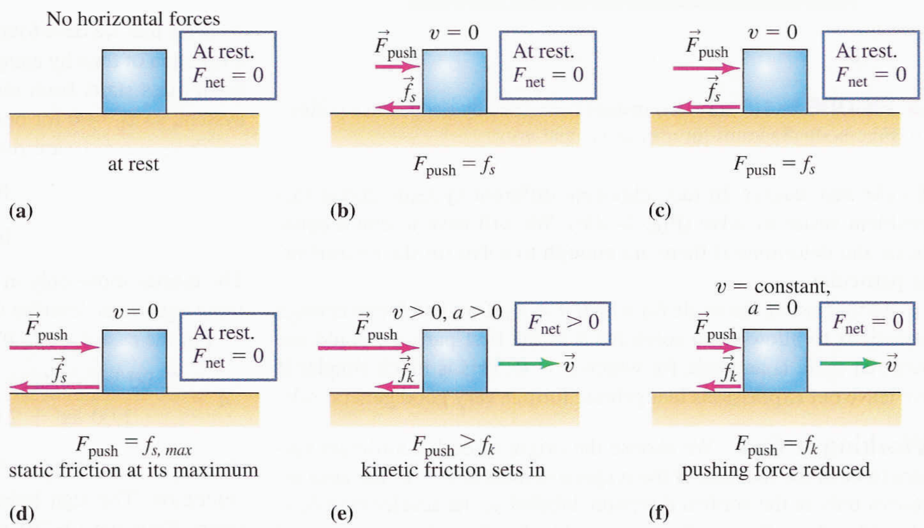


▲ **FIGURE 5–11** Nails are held in place by the force of friction, which can be quite substantial.

motion. In other situations friction is not a desirable phenomenon and we do our best to minimize it. Even with the oil added to a car's motor to reduce frictional forces in the engine, as much as 20 percent of gasoline consumption goes to overcome friction in the engine. Lubrication reduces friction and therefore also reduces surface wear—automobile engines now tend to last longer because internal friction has been reduced through more precise manufacturing and more effective lubrication. We have to take into account the forces of friction if we are to understand any realistic mechanics problem.

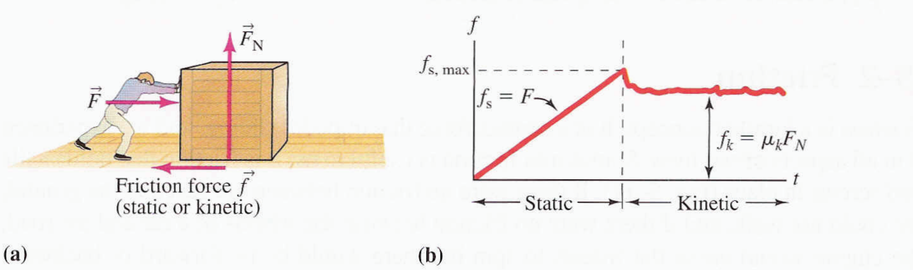
Static and Kinetic Friction

Suppose you want to slide a crate full of books from one place to another. You push on it with a small horizontal force but nothing happens. Even when you push as hard as you can, the crate does not move. Why not? **Static friction** acts between the floor and the crate in the absence of motion in such a way as to *prevent* motion. This force must be variable because it balances each of your own different-strength pushes. Suppose that you finally get the crate moving with the help of another person. The combined force overcomes the static friction because *static friction has a maximum magnitude*. Friction has to do with the interlocking of microscopically rough surfaces (for more see p. 133), and the external force that overcomes the interlocking and gets the movement started has to be large enough to bend or break the tiny protuberances on the contacting surfaces that impede motion.



▶ **FIGURE 5–12** Sequence in which an increasing pushing force is opposed by an equal and opposite static friction force. Static friction can increase only to a certain point, after which the crate accelerates. Kinetic friction, which applies when the crate actually moves, is smaller than the maximum size of static friction, so it takes less pushing force to move the crate once its motion has begun.

Once the crate is moving, it is easier to keep it moving at a constant speed. There is still friction opposing your push, but it is now **kinetic** (or **sliding**) **friction**, that is, friction associated with motion. Experiment shows that the magnitude of kinetic friction is smaller than the maximum value of static friction. The entire sequence of getting the crate started and keeping it moving is illustrated in Fig. 5–12. Static friction acts in a direction *opposite* to the component of an applied force *along* the surface; sliding friction acts *opposite* to the direction of the velocity of a sliding object at its point or points of contact. In each case the friction force is parallel to the surface (Fig. 5–13a). Figure 5–13b illustrates the magnitude of friction when an object such as the crate starts from rest and is pushed with a steadily increasing external force. The magnitude of static friction increases as the external force increases until the "break point" arrives, at which point kinetic friction enters. This force does not vary with the external pushing force. When the applied force decreases to zero, kinetic friction continues to act until the body stops. At that point we revert to the static situation.



▶ **FIGURE 5–13** (a) Friction opposes the motion of the crate. (b) With a steadily increasing push, static friction will increase in magnitude until the crate starts to move, at which point kinetic friction, whose magnitude does not vary with the pushing force, takes over.

CONCEPTUAL EXAMPLE 5–7 In the discussion above we stated that a large enough force has to be applied to overcome static friction, and the applied force must be large enough to bend or break the small protuberances that keep a surface from being perfectly flat. Given the fact that measurements of the coefficient of static friction between a block of wood and a table give the same answer after many experiments, would you argue in favor of bending or breaking the surface imperfections?

Answer If static friction were due primarily to obstacles that have to be broken in order to facilitate motion, then after a few experiments all the obstacles would have been broken off, and static friction would decrease significantly. Because this does not happen, the bending of deformations must be the primary source of friction. We should add that if you actively smooth a surface by, for example, sanding it, then you are presumably doing some breaking of the protuberances on it.

Quantitative Properties of Friction

Quantitative tests on friction were made by Leonardo da Vinci some 200 years before Newton's work on dynamics. Leonardo experimented with a set of blocks of varying sizes sliding on table tops and discovered some surprising facts. He found that both static and kinetic friction are independent of the surface area of the blocks in contact with the table top. Moreover, both *static and kinetic friction are proportional to the magnitude F_N of the normal force exerted by the table top on the blocks*. The experiments that led Leonardo to his conclusions are quite simple: Take a given block and turn it so that faces of different areas are in contact with the table top. The friction force on the block is the same no matter what face is down.

The proportionality constant that relates the friction force and the normal force is the **coefficient of friction** μ. This (positive dimensionless) constant is determined experimentally. As the maximum value of static friction is generally not equal to the force of kinetic friction, we distinguish two coefficients: μ_s for static friction and μ_k for kinetic friction.

If we write the force of static friction as $\vec{f}_s$ and that of kinetic friction as $\vec{f}_k$, their magnitudes are given by

$$0 \leq f_s \leq \mu_s F_N; \tag{5–21}$$

STATIC FRICTION

$$f_k = \mu_k F_N. \tag{5–22}$$

KINETIC FRICTION

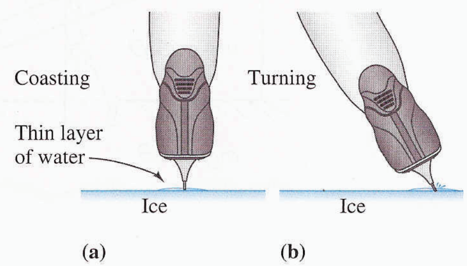
Equation (5–21) expresses a range because, as we have described above, static friction takes a value that depends on the external conditions. The experimental fact that the maximum value of static friction exceeds kinetic friction implies the inequality

$$\mu_s > \mu_k. \tag{5–23}$$

You experience this when you have to exert a greater force to get a crate of books moving than you have to exert to keep it moving. We also make the assumption, reasonably well satisfied by experiment, that *μ_k is independent of the relative speed of the two surfaces.*

The coefficients of friction depend on the two surfaces involved. We know from everyday experience that a basketball shoe on a basketball court involves a larger coefficient of friction than does the blade of an ice skate on a frozen lake. A lubricating material—such as sweat—between the basketball shoe and the court will drastically reduce the coefficient of friction. In ice skating, the lubricating material is a layer of liquid water between blade and ice (Fig. 5–14a); an ice skater can make static friction take over by adroit use of the blade—in Fig. 5–14b the skater is pushing off or making a turn by digging into the ice rather than gliding over it. Table 5–1 shows some typical values of coefficients of static friction. The materials listed are generally unlubricated ("dry"). Coefficients of kinetic friction can be anywhere from roughly 25 percent to 100 percent of the corresponding coefficients of static friction.

The values in Table 5–1 are meant only to be indicative—the coefficients of friction are sharply dependent on such things as the cleanliness of the surfaces, their roughness, and so forth. Two very rough objects may have a large coefficient of friction that can be reduced once the objects are smoothed. But if two objects of the same material



▲ **FIGURE 5–14** (a) A layer of water lubricates and decreases the action of kinetic friction for an ice skater. (b) By manipulating the blade of the skate, a skater makes static friction act in the sideways direction and can make a turn.

TABLE 5–1 • Some Coefficients of Static Friction

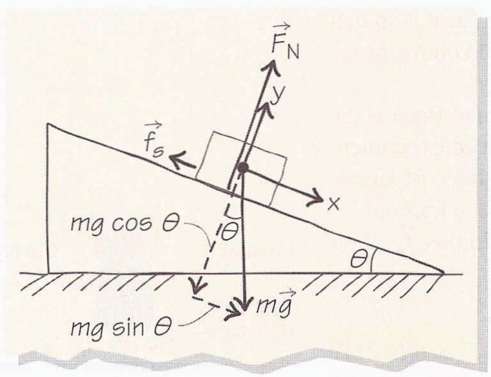
Materials	μ_s
Automobile brake shoes on a brake drum	1.2
Dry tire on dry asphalt	1.0
Hard steel on hard steel	0.8
Oak on oak, parallel to the grain	0.6
Book on a table	0.3
Wet tire on wet asphalt	0.2
Ice on wood	0.05
Teflon on steel	0.04

are smoothed too much and are free of dirt and oxidation as well, the coefficient of friction may rise virtually to infinity because the surfaces weld together! In effect, the molecules at the surface between the two objects can interact just as they do at the interior, and the two objects become one.

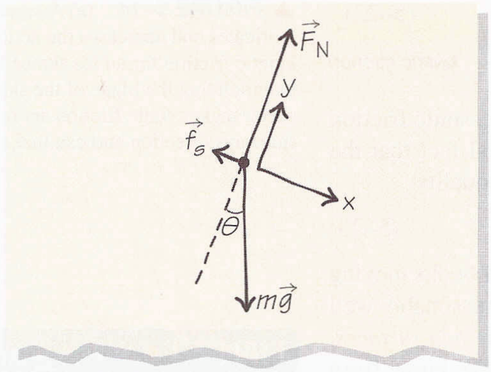
Example 5–8 illustrates one method of measuring the coefficient of static friction.

EXAMPLE 5–8 A box of mass m is set at rest on a horizontal surface; there is friction between the box and the surface. The surface is slowly raised at one side so that it becomes a ramp, making an angle θ with the horizontal. By analyzing the forces on the box, find the critical ramp angle θ_c at which the box will start to slide.

Setting It Up Figure 5–15a illustrates the situation. We include in the figure a coordinate system that simplifies the solution: The $+x$-direction is oriented down along the ramp and the y-direction is perpendicular to the ramp. The origin is placed at the initial position of the mass.



(a)



(b)

▲ **FIGURE 5–15** (a) Box of mass m experiencing a friction force on a ramp. The ramp angle θ at which the mass starts to slide yields the coefficient of static friction. (b) Free-body diagram for the box.

Strategy The free-body diagram is Fig. 5–15b. The three forces acting on the box before it starts to move are the force of gravity (vertically down), the normal force $\vec{F}_N$ (perpendicular to the surface), and the force of static friction, $\vec{f}_s$ (up the ramp). The friction force opposes the motion that would take place if there were no friction. The magnitude of the force of static friction is determined by the fact that the mass remains motionless for a sufficiently shallow ramp angle. The maximum value of static friction depends on F_N [see Eq. (5–21)], so we must also find its value. We can use the fact that there is no acceleration perpendicular to the ramp to find F_N.

As the ramp angle increases, the components of the normal and gravity forces along and perpendicular to the ramp change. As long as the box is not slipping (there is no net force—we refer to the box as being in *static equilibrium*), the friction force (along the ramp) is static friction. Once the value of f_s reaches its maximum value $\mu_s F_N$, however, the box will slip, and this determines the critical value of the ramp angle θ_c.

Working It Out We start with the box stationary, so that Newton's second law with zero acceleration reads $\vec{F}_g + \vec{F}_N + \vec{F}_s = 0$, or

$$(mg\sin\theta)\hat{i} - (mg\cos\theta)\hat{j} + F_N\hat{j} - f_s\hat{i} = 0.$$

In component form, the equations of motion are

$$\text{for the } x\text{-component: } mg\sin\theta - f_s = 0; \qquad (5\text{–}24a)$$

$$\text{for the } y\text{-component: } F_N - mg\cos\theta = 0. \qquad (5\text{–}24b)$$

The x-component equation relates the force of static friction to the force of gravity,

$$f_s = mg\sin\theta. \qquad (5\text{–}25)$$

The y-component equation determines F_N as a function of θ:

$$F_N = mg\cos\theta. \qquad (5\text{–}26)$$

As θ (and $\sin\theta$) increases, the force of friction from Eq. (5–25) that is needed to hold the box in static equilibrium also increases. Eventually, static friction reaches its maximum value, $\mu_s F_N$. Beyond that point, the box will start to slide. Setting static friction to its maximum value, $f_s = \mu_s F_N$, Eq. (5–25) determines the critical angle θ_c at which the box starts to slide. Equation (5–25) then becomes

$$\mu_s F_N = mg\sin\theta_c,$$

or, from Eq. (5–26),

$$\mu_s mg\cos\theta_c = mg\sin\theta_c.$$

Cancel the factor mg from this equation to find θ_c:

$$\frac{\sin\theta_c}{\cos\theta_c} = \tan\theta_c = \mu_s. \qquad (5\text{–}27)$$

This equation tells us that if we measure the angle at which the box begins to slip, we measure μ_s, and this is in fact a useful way to determine coefficients of static friction between surfaces.

What Do You Think? Imagine that a person pushes down on the box in a direction opposite to the normal force. How does this change the frictional force? What happens to the critical angle?

Example 5–8 suggests a related experiment that will measure the coefficient of kinetic friction. Once the box begins to slip, kinetic friction acts, given by $f_k = \mu_k F_N = \mu_k mg\cos\theta$. If we use Newton's second law and apply it along the x-direction, then

instead of Eq. (5–24a), we find

$$mg \sin\theta - \mu_k mg \cos\theta = ma_x. \qquad (5\text{–}28)$$

Because μ_k is smaller than μ_s, the ramp can be lowered back down, decreasing θ while the mass is still sliding, and the box will continue to slide. There is a second critical value of θ—call it θ_c'—for which the forces in Eq. (5–28) cancel and the object no longer accelerates. Instead, the object slides at constant velocity. This critical angle is given by

$$\frac{\sin\theta_c'}{\cos\theta_c'} = \tan\theta_c' = \mu_k. \qquad (5\text{–}29)$$

Thus θ_c' measures the coefficient of kinetic friction.

EXAMPLE 5–9 A professor with a light eraser (assume massless) in her hand leans against a blackboard. Her straight arm makes an angle of 60° with the horizontal, and the force $\vec{F}_{\text{prof}}$ exerted by her arm on the eraser has magnitude $F_{\text{prof}} = 50$ N. The coefficient of static friction between the eraser and the blackboard is $\mu_s = 0.15$. Does the eraser slip?

Setting It Up We sketch the situation in Fig. 5–16b, and we can use this figure as a free-body diagram for the eraser. We choose a coordinate system with x into the board and y vertically upward. We are interested in finding the force of friction between the board and the eraser. If that value exceeds the maximum value of static friction, the eraser will slip.

Strategy After drawing the free-body diagram to allow us to pick out the forces acting, we apply Newton's law of motion to the eraser under the assumption that it is not slipping. The components of that equation should allow us to solve for the magnitude of the friction force to find out if it exceeds its maximum value.

Working It Out If there were no friction (a perfectly slippery board), the eraser would slide up. Therefore, the static friction force must be down, in the $-y$-direction. As long as there's no acceleration, Newton's first law applies to the eraser, $\vec{F}_N + \vec{f}_s + \vec{F}_{\text{prof}} = 0$, and we have

$$-F_N\hat{i} - f_s\hat{j} + (F_{\text{prof}}\cos\theta)\hat{i} + (F_{\text{prof}}\sin\theta)\hat{j} = 0.$$

The two component equations are (θ is the angle with respect to x, here 60°)

$$\text{for the } x\text{-component:} \quad -F_N + F_{\text{prof}}\cos\theta = 0;$$

$$\text{for the } y\text{-component:} \quad -f_s + F_{\text{prof}}\sin\theta = 0.$$

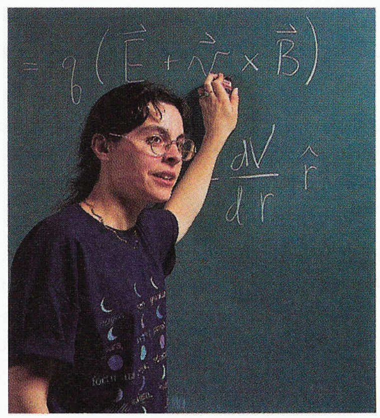
The x-component equation determines F_N from the requirement that it balances the horizontal component of the professor's force $\vec{F}_{\text{prof}}$. Once we have found that $F_N = F_{\text{prof}}\cos\theta$, we can determine the *maximum* value of static friction, $f_{s,\max} = \mu_s F_N = \mu_s F_{\text{prof}}\cos\theta$. When this maximum value is exceeded, the eraser begins to slip. Thus, when we substitute the maximum value of static friction into the y-component equation, we find a condition for the critical angle θ_c for which the eraser begins to slip:

$$-\mu_s F_{\text{prof}}\cos\theta_c + F_{\text{prof}}\sin\theta_c = 0;$$

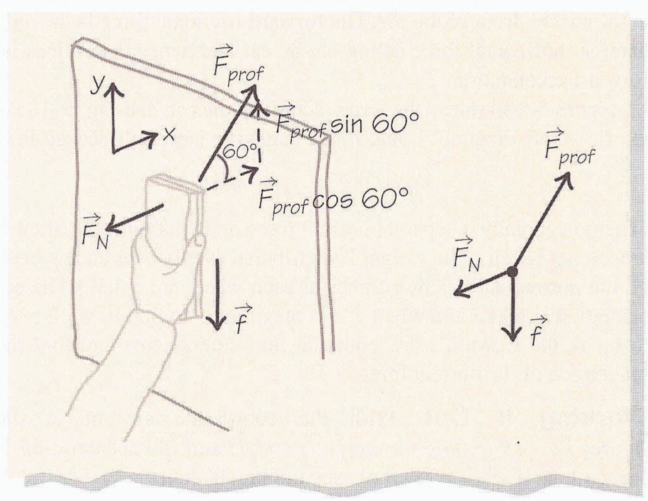
$$\frac{\sin\theta_c}{\cos\theta_c} = \tan\theta_c = \mu_s.$$

Note the striking feature that the critical angle at which the eraser starts to slip is independent of the force the professor exerts! Numerical substitution yields $\tan\theta_c = 0.15$, or $\theta_c = 8.5°$. This angle is less than the 60° angle made by the arm, so the eraser indeed slips.

What Do You Think? Suppose that the eraser were not massless. Is the critical angle θ_c still independent of the professor's force?



(a)

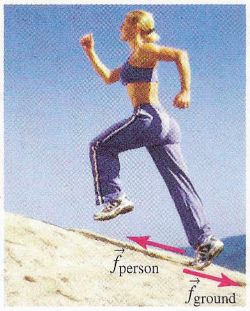


(b)

▲ **FIGURE 5–16** (a) Professor erases the blackboard. (b) Sketch of forces and free-body diagram for the eraser.

THINK ABOUT THIS . . .

FRICTION ACTS BACKWARD—HOW CAN IT ALLOW US TO WALK FORWARD?



▲ **FIGURE 5–17** A runner uses the force of static friction between shoe and ground to accelerate forward.

We have stated that friction acts against motion, but at the start of the chapter we also stated that the presence of friction allows us to walk and cars to accelerate. Friction can act to produce a positive acceleration on us. How is this possible? The answer lies in the fact that people are extended and flexible systems and for such systems static friction can act *forward on the system as a whole*, even though it will always act to *impede motion at the point of contact*. Walking or running is perhaps the most familiar example of this phenomenon (Fig. 5–17). The runner is exercising a muscle that pushes the contact foot backward, and without friction the motion of the foot at the point of contact would therefore be backward. (You can easily test this by trying to walk on a frozen puddle.) But friction opposes the backward motion at the point of contact and therefore is a force that acts in the *forward* direction on the runner's foot. With sufficient stiffness in the runner, this forward friction force acts on the entire system to move her forward. By Newton's third law, there is a corresponding backward-directed force on Earth. An automobile similarly moves forward because of static friction, as we describe more quantitatively in Example 5–10.

EXAMPLE 5–10 An automobile with four-wheel drive and a powerful engine has a mass of 1000 kg. Its weight is evenly distributed on its four wheels, whose coefficient of static friction with the dry road is 0.8. If the car starts from rest on a horizontal surface, what is the greatest forward acceleration that it can attain without spinning its wheels?

Setting It Up The situation is illustrated in Fig. 5–18a. The car mass is m and the coefficient of static friction between the wheels and the road is μ_s. The unknowns are the car's acceleration, magnitude a, and in particular maximum magnitude a_{max}.

Strategy It is static friction, magnitude f_s, that accelerates the car forward, and the automobile has its greatest forward acceleration when f_s is at its maximum. The engine of the car creates the rotational motion shown in Figure 5–18a in the wheels. If there were no friction between the wheels and the road, the wheels would simply spin. When there is no slipping, it is *static* friction that acts, and μ_s, not μ_k, enters the problem. The motion of the tires at the point of contact with the road is to the rear in the absence of friction. The force of friction opposes this rearward motion, so the direction of $\vec{F}_s$ is toward the front of the car. The forward frictional force is the *only* external horizontal force acting on the car and hence determines its forward acceleration.

Figure 5–18b shows the forces acting on the car, and Fig. 5–18c is the free-body diagram. Using it, we can write Newton's second law:

$$\vec{F}_N + m\vec{g} + \vec{f}_s = m\vec{a}.$$

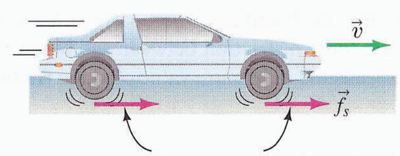
(There is actually a separate normal force and friction force at each wheel, but because the weight is distributed evenly over each wheel, all the normal and friction forces at each wheel are equal.) The acceleration is maximum when f_s is a maximum. To find it, we'll need to break the Newton's law equation into components and find the magnitude of the normal force.

Working It Out With the coordinate system in the figure, $\vec{F}_N = F_N\hat{j}$, $m\vec{g} = -mg\hat{j}$, $\vec{f}_s = f_s\hat{i}$, and the acceleration is forward, so $\vec{a} = a\hat{i}$. Newton's second law is then

$$F_N\hat{j} - mg\hat{j} + f_s\hat{i} = ma\hat{i},$$

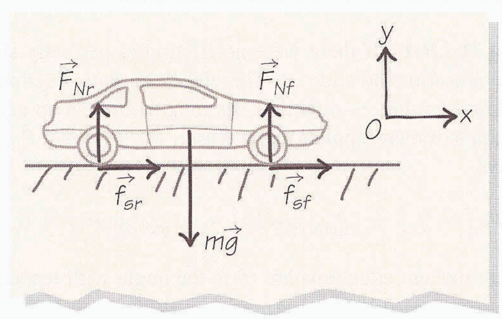
and the component equations are

$$\text{for the } x\text{-component: } f_s = ma;$$
$$\text{for the } y\text{-component: } N - mg = 0.$$

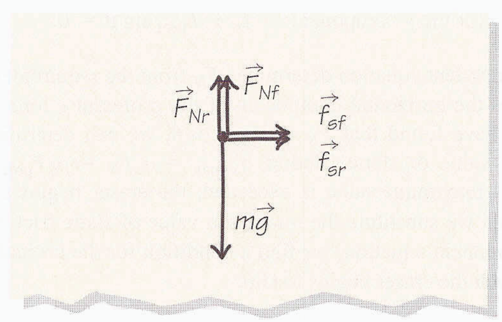


Force of static friction on rear and front tires.
The same force acts on the left side tires.

(a)



(b)



(c)

▲ **FIGURE 5–18** (a) A car accelerates forward under the influence of static friction. (b) Forces acting. (c) Free-body diagram for the car.

We find a_{max} when static friction is at its maximum value $\mu_s F_N = \mu_s mg$, where we have used $F_N = mg$ from the y-component equation. Thus

$$\mu_s mg = ma_{max}.$$

The mass of the car cancels out of this expression, leaving

$$a_{max} = \mu_s g = (0.8)(9.8\ \mathrm{m/s^2}) \cong 8\ \mathrm{m/s^2}.$$

This is quite a significant forward acceleration. Note that as μ_s decreases, the maximum acceleration decreases; in other words, when μ_s is zero, the automobile can only spin its wheels.

What Do You Think? Cars with front-wheel drive perform better in snow than those with rear-wheel drive. Why do you suppose this is?

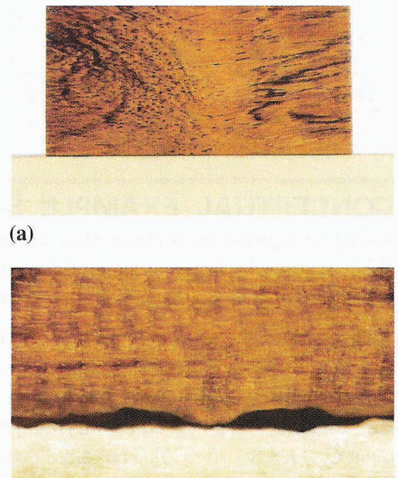
THINK ABOUT THIS . . .

WHERE DOES FRICTION COME FROM?

To understand the origins of friction, let's look closely at two surfaces that rub against one another. Figure 5–19 shows a microscopic view of two such surfaces. Because of the hills and valleys present on any rough surface—and all surfaces are rough when viewed closely enough—the area on two surfaces that actually touch together is a small fraction of the area that appears to be in contact. Friction forces are due to three major effects: the interlocking of surface irregularities, the attraction between the contact points due to forces between the molecules of the two objects (the objects "adhere"), and the "plowing out" of softer materials by harder ones. The coefficient of static friction can be greater than that of kinetic friction because the materials have a longer time to "settle in" together.

This description helps us understand why the friction force is independent of the apparent surface area that is in contact while it is dependent on the normal force. The normal force is a measure of how strongly the two surfaces are pressed together; when the normal force is large, the two surfaces are pressed strongly together. The rough surfaces shown in Fig. 5–19 mesh more closely when pressed strongly together, and the *actual* surface area in contact increases. In fact, the normal force is a good measure of the actual surface contact area. Whether we place the broad side or the narrow end of a brick on a table, approximately the same surface areas are in actual contact, even if the apparent contact areas are vastly different. So the friction force comes from the interaction of the two surfaces at the atomic level and is proportional to the real area that is in contact at this microscopic level.

The study of friction, wear, and lubrication is called *tribology*, a subject of obvious importance. Despite much effort, a truly fundamental understanding of friction remains elusive. The discovery of Teflon™—a very slippery coating material that you have likely seen in frying pans—was a happy accident, not the result of a planned development program. The discoverer has stated that he was lucky he was not blown up in the process.



(a)

(b)

▲ **FIGURE 5–19** If we use both of the figures chosen: (a) The two surfaces appear smooth, (b) but a microscopic view of the contact region reveals rough surfaces.

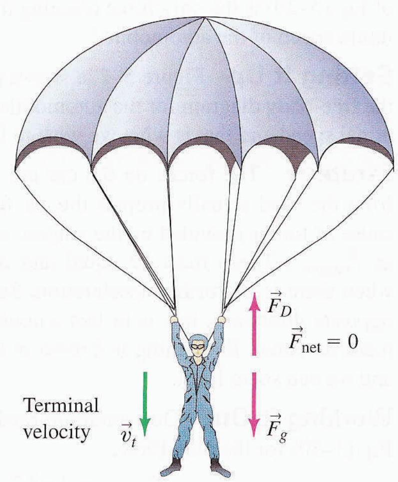
5–3 Drag Forces

A spoon dropping through molasses, an automobile moving at highway speeds, and the space shuttle using a parachute to slow down during landing are all subject to a substantial *drag force*, which is a resistive force somewhat like friction. Drag forces act like sliding friction in that *they act in a direction opposite to that of the motion*, but they differ from sliding friction in two ways: They depend on the *speed* v of the object that is moving through the medium and there is no equivalent to a normal force to set their magnitude.

In many everyday situations, an automobile moving on a highway, for example, the drag force $\vec{F}_D$ is found by experiment to have magnitude

$$F_D = \tfrac{1}{2}\rho A C_D v^2, \qquad (5\text{–}30)$$

where ρ is the mass density (the mass per unit volume) of the medium through which the object moves, A is the maximum cross-sectional area presented by the moving object, and C_D is the *drag coefficient*. The drag coefficient is dimensionless and depends on the object's shape. A highly streamlined object might have a drag coefficient as small as 0.1, whereas a particularly awkward shape will have a drag coefficient greater than 1. The most streamlined automobiles have drag coefficients around 0.25.



▲ **FIGURE 5–20** When the drag force and the force of gravity acting on a parachutist are equal and opposite, the parachutist has reached his terminal speed.

Terminal Speed

The fact that the drag force on an object increases with the speed has an important consequence. Consider a parachutist falling through the air (Fig. 5–20) and acted upon by both gravity and a drag force, as described by Eq. (5–30). When the parachutist first starts to fall,