Spring Pendulum[*]

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Oscillations are a commonly observed physical phenomenon. In this experiment, we will use our video motion analysis technique to investigate the oscillatory motion of a mass-spring system. We will calculate frequency and damping coefficient of the oscillatory motion.

Essential pre-lab reading: "*Physics for Scientists and Engineers with Modern Physics; 3rd Edition*" by Fishbane, Gasiorowicz and Thornton; (Sections 13.1-13.4 and 13.6).

1 Test your understanding

1. The displacement of an undamped simple mass spring system is given by Equation (1).

   $$x = x_o \sin\left(\omega_o t + \phi\right) \tag{1}$$

   where x_o is the initial amplitude, ω_o is the natural frequency and ϕ is the phase. Can you derive an expressions for velocity and acceleration?

2. What is the relationship between the position and the acceleration of a simple harmonic oscillator?

2 The Experiment

A body of known mass is attached and suspended from a spring which is clamped to a retort stand (as shown in Figure 1). The spring is stretched vertically downwards and then released. A high speed video of the oscillations is captured and then analyzed in Matlab using PhysLab's video tracking library "PhysTrack". Subsequent position data generated is used to further investigate different aspects of the motion.

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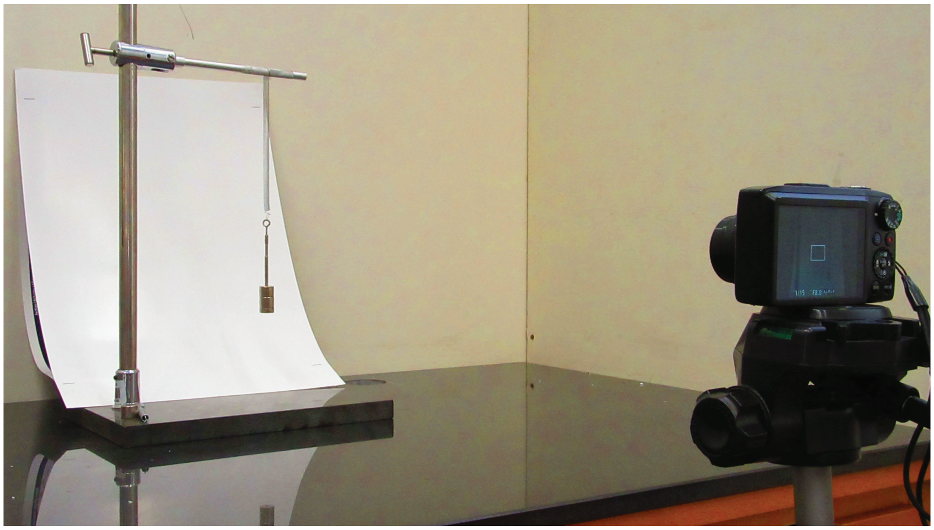

Figure 1: The setup used to investigate simple harmonic motion.

Measure the mass of the slotted weight and attach it to a retort stand using a spring. Also measure the height of the slotted weight. Place the apparatus against a seamless white background.

Set up the camera to high-speed video mode at $240fps$ and fix it on a tripod stand. Place the tripod stand 5 to 6 feet away from the apparatus. Use bull's eye level on the camera's body to confirm that the camera is placed horizontally. The image in the camera's view finder should look similar to the one shown in Figure 2. Adjust the height of the camera such that the view covers most of the portion where the body is likely to oscillate.

Take a set of readings with different masses attached to the spring each time. For a single reading, follow these steps.

1. Adjust the mass of the slotted weight and note it down in your notebook.
2. Pull the slotted weight vertically downwards and with your partner's help, start the video recording. Once the recording starts, release the weight and let the system oscillate.
3. Stop the video recording after a couple of seconds.
4. If the oscillations don't seem visually vertical, discard the video and record a new one.

3 Analysis

3.1 Preparing Data for Quantitative Analysis

I hope the reader has already studied our "Primer on video tracking" uploaded on the website.

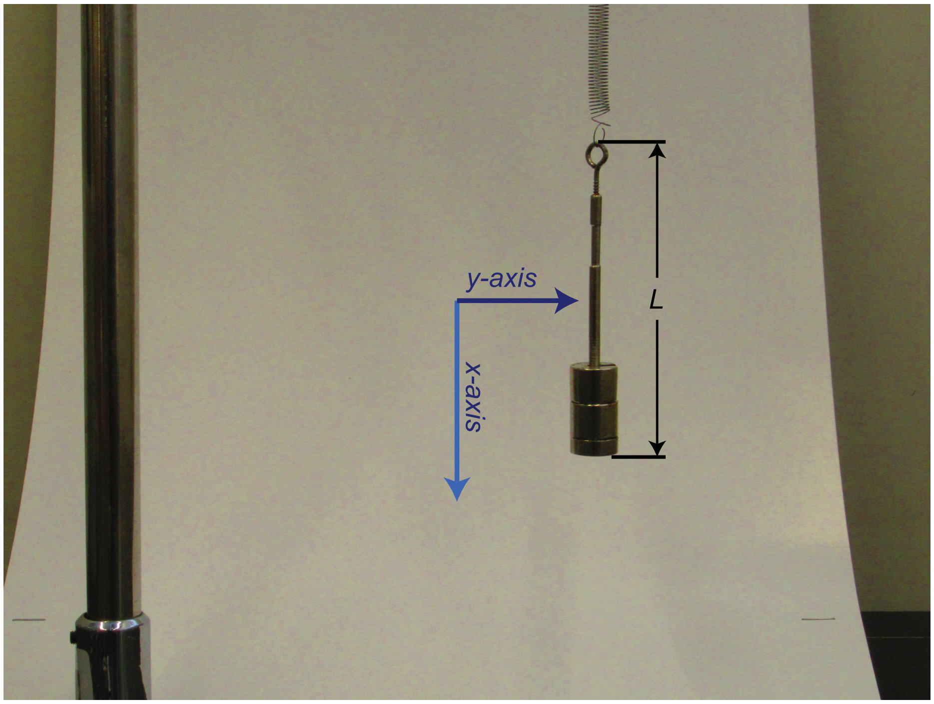

Figure 2: A sample frame showing the slotted weight. An Appropriate choice for the reference coordinate system is also shown. Note that the image has been artificially enhanced for visual clarity.

Copy the videos to some appropriate location in the hard-drive. Download and extract on hard-drive the latest release of PhysTrack from our website. In Matlab, browse to the extracted PhysTrack directory where you should see the "+PhysTrack" folder and some "analyze motion scripts" in the "Current Folder" window. Run the script `analyze1DSHM` which presents a series of GUI tools to setup and perform video tracking. When asked by the interface, browse and select your video file. The script will automatically show a tool to trim and crop the video. Use the `slider` and the `go-to buttons` to seek different frames of the video. Mark as `In-Frame` the moment when the you want to start tracking the oscillations and as `Out-Frame` the moment you want to stop the process. Preview the trimmed video if necessary and close the tool afterwards.

The script will now present a coordinate system tool. Follow the on-screen instructions to setup a coordinate system having the origin located anywhere on the line of motion of the body. The `x-axis` should be collinear with the oscillations. Clearly, the directions of both the `x-axis` and the `y-axis` are irrelevant. Use the `Reset Origin`, `Reset x-axis` and `Toggle Direction` buttons to move the origin, rotate the x-axis about the origin and toggle the y-axis direction respectively. See figure 2 which shows a preferred way of setting up the coordinate system. Enter the value of the known length of the slotted weight in text-box labelled `Marked Distance` and click on the `Reset Unit Marking` button to draw a line on the slotted weight along the length, followed by the `enter` key. Repeat any of these steps as many time as needed to achieve the desired results. Once everything is final, close the tool.

The script will now open an object selection window for marking the oscillating mass. Click

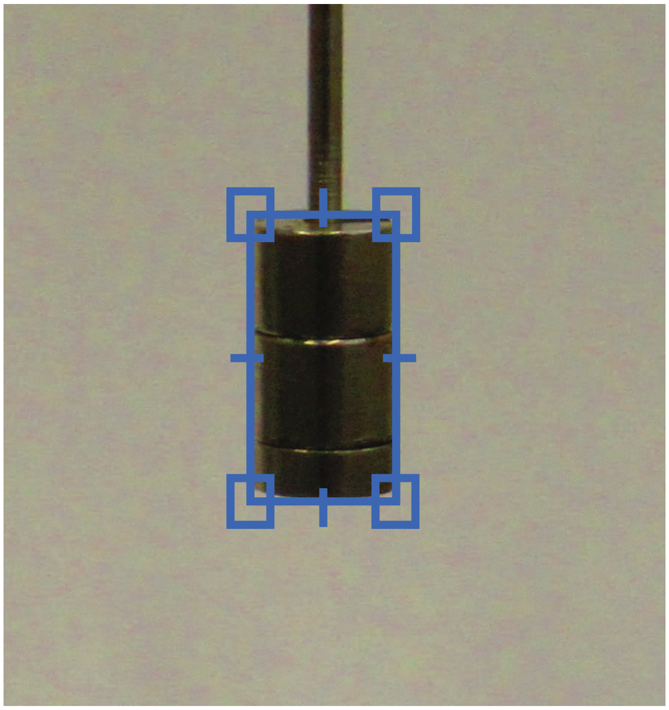

Figure 3: Identifying the body in the motion tracker software. Note that the image has been artificially enhanced for visual clarity.

`Manually Mark an Object` and drag and draw a tight square around the weighted as shown in Figure 3. Marking the rectangle again on the same place replaces the previous one automatically. When you close the tool to start the tracking process, there should only be one object in the objects listbox.

When the tracking is finished the script will open up a track point filter tool to manually remove any stray points. Use the on-screen controls to seek different instances of the video and filter out the unwanted track points by drawing rectangles around them. Each step in this tool can be undone using the `Undo` button. At the end of a successful run, the script will leave some variables in the workspace. See Table 1 for details of these variables.

Physical Quantity	Variable Names
Trajectory of the body	*trajectory*
`xdata` of the trajectory	dx
`ydata` of the trajectory	dy
Time stamps	t

Table 1: Base workspace variables generated by the script.

Now we need to use this positional data and the respective time stamps to calculate the velocity and acceleration.

3.2 Quantitative Analysis

Plot `xdata` against `t` to visually observe if the system has followed a sinusoidal motion. To confirm our observation, we need to fit the `xdata` and the `t` into a model.

Q 1. Why do you think that Equation (2) given below is a valid model for the displacement of our mass spring system? What do the coefficients a_0, a_1, a_2 and a_3 represent?

$$x = a_0 + a_1 \sin\left(a_2 t + a_3\right) \tag{2}$$

You will fit the data to this model using the least-squares fitting function `PhysTrack.lsqCFit`. The syntax for using this function is

```
% [0 0 0.5 10] are the start values for the three coefficients. Sometimes, the
% algorithm doesn't converge without an initial guess. You may choose to skip this
% parameter or to use some different start values instead.
fitResult = PhysTrack.lsqCFit(t, xdata, 'x', 'a0 + a1 sin(a2 * t + a3)', 't',...
             [0 0 0.5 10]);
```

The variable `fitResult` is a `cfit` object which contains separate variables for each unknown parameter. The syntax to evaluate the value of the model function at a specific value of x is

```
y_At_x = fitResult(x);
```

Q 2. Display the fit result along with the original data in a single plot to verify that the fit was completed successfully.

You will now differentiate the `xdata` and the `ydata` with respect to time `t` to obtain the velocity using the `PhysTrack.deriv` function. Here is an example on how to use this function.

```
[td, xd] = PhysTrack.deriv(t, xdata, order);
```

where order is 1 for computing a first-order derivative and 2 for a second-order derivative.

Q 3. Compute the first order derivatives for each dataset. Plot your results to obtain the graphs for the vertical and horizontal velocities.

Q 4. Using the `lsqfun3` function, fit the velocity data to Equation (3) and plot the results.

$$v = b_0 \sin\left(b_1 t + b_2\right) \tag{3}$$

Q 5. What do the coefficients b_0, b_1 and b_2 in Equation (3) represent?

Q 6. Using the mass of the slotted weight, calculate the spring constant for the spring used in this experiment.

Q 7. What is, theoretically, the natural frequency of the mass spring system? How does the experimentally determined frequency compare with the theoretical value?

Q 8. How can you verify the energy balance of the system? Using the data collected from the experiment, explain how the kinetic and potential energies inter-convert into each other.