Chaos and Non-Linear Physics

Hudaiba Soomro
Roll no: 2021-10-0290
LUMS School of Science and Engineering

Tuesday, November, 19, 2019

1 Abstract

The non-linear dynamics of an RLD circuit were explored using various data-acquisition methods in order to distinguish between higher order period doubling and chaos as well as to make precise the instance of bifurcation. These techniques ranged from obtaining time-series plots and phase portraits using an oscilloscope, peak detection via a spectrum analyzer, and the use of modulation and software techniques to obtain bifurcation diagram.

2 Introduction

Chaotic systems are exemplary across wide range phenomena from fluid flow to the spread of diseases. A chaotic system while may follow deterministic equations of motion, its detailed future behavior is highly unpredictable and non-linear [1]. A simple means to study a dissipative chaotic system is the RLD circuit which was made use of in this experiment.

The RLD circuit, in particular goes through a sequential cycle of period doubling and quadrupling before reaching chaos which is identified as a stage of absolute non-deterministic aperiodic behavior. Here, the period being the period of the voltage drop across the resistor once a varying AC voltage is fed into it. These cycles of bifurcations while follow this general pattern, allow for several other bifurcations along the way and this is substantiated in our experiment.

To obtain the periodicity of voltage drop across the resistor, we used various data acquisition methods that involved the following: time series plots, phase portraits, spectrum density plots, Poincare maps, and a bifurcation diagram. These were achieved by use of various instruments and methods that will be detailed.

3 Theoretical background

Chaotic systems disobey linearity and the principle of superposition. In an RLD circuit, this chaotic characteristic is introduced by the recovery time of the diode. The RLD Circuit goes through conducting and non-conducting cycles. In the former, the forward biased diode behaves as battery source while in the latter, it behaves as a capacitor. In the transition from a conducting to a non-conducting cycle, the diode's recovery time - which is the time taken for the current in the conducting cycle to completely stop flowing in the forward direction - contributes to the period doubling that eventually leads to chaos [2].

In our experiment, we made use of the Feigenbaum constant which is the ratio of differences of parameter values at which successive bifurcations occur[cite]. This approaches a constant value for all bifurcations and is given by the following:

$$\delta_n = \frac{\lambda_n - \lambda_{n-1}}{\lambda_{n+1} - \lambda_n} \tag{1}$$

As, $\lim_{n\to\infty} \delta_n, \delta = 4.669201...$

We calculated the Feigenbaum constant in Equation (1) when noting bifurcations from different instruments and this helped in determining which instrument is more precise in determining bifurcations.

4 Experimental procedure

We carried out the experiment in a manner that was non-sequential as opposed to the one provided by the manual. This was due to constraints that came up in the performance of the experiment. Yet, a rough sequence of the experimental procedure is as follows:

1. **Times Series Plots:**

   Obtained time series plots of Voltage output across the resistor and the diode by manually incrementing the input AC voltage sent via the function generator. The set up is as shown in (Figure 1)

   At first, we mistook certain trigger issues as instances of period doubling. This seemed to be the case to us since the wave seemed to be splitting in a period manner. However, after adjusting trigger, we were able to obtain smooth wave forms with more distinct period patterns.

   For the diode, we also introduced two DC offsets amounting to ± 1V at the instance where we first observed period doubling.

   At this step, the oscilloscope tended to pause the signal. At times, we were able to resolve by adjusting trigger. We could not locate precisely why this was occurring although we observed this was happening at times when DC

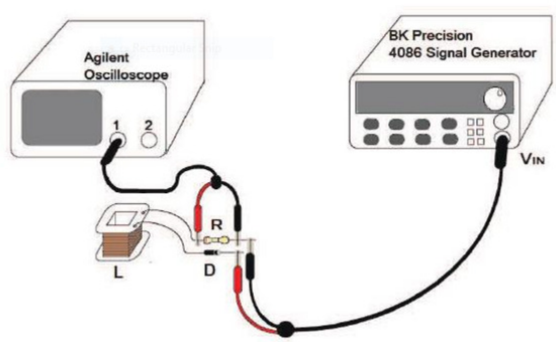

Figure 1: Experimental Setup for Generating Time Series

offset was being fed. We could not obtain the waveform for the negative offset because of this constraint.

2. **Phase Portraits:**

   Used a BNC-to-crocodile cables connection to feed data into Channel 2. As such, one of the crocodile cables were attached after the inductor and the diode. We used the toggle labelled horizontal on the oscilloscope to change to XY mode in order to obtain phase portraits.

3. **Spectrum Density Plots:** While aligning the Spectrum analyzer, we faced an error that the input voltage was too low. Not being able to locate this error in the instrument's user manual either, we attempted to resolve by replacing the wire with two BNC to crocodile cables. This did not help either but later we realized we only needed to tighten the connection.

   After calibrating, we proceeded with keeping the Bandwidth resolution at 300 Hz. We could not locate the source of one peak at 6kHz as it remained constant while the input parameter was varied. We adjusted the window of the display such that the 50kHz peak was made visible at the far right of the screen.

4. **Amplitude Modulation and obtaining the Poincare Maps**

   So far, we had been incrementing the input parameter manually. This, evidently, does not lead to precise results as the precise instance of bifurcation cannot be mapped. As such, we proceeded to use a combination of amplitude modulation and software techniques to obtain a time varying voltage across the resistor while a time varying input is given by using an amplitude modulation ramp.

   After setting the function generator at AM mode, we used it as a source for the carrier signal which was fed into the RLD circuit. The modulating signal was fed into the function generator via LabVIEW which modulated the signal that was retrieved across the resistor back to oscilloscope and stored as a .txt file by LabVIEW.

5. **Using LabVIEW for Phase Portraits, Poincare Maps, and The Bifurcation Diagram**

   Using existing LabVIEW files, we were able to obtain Poincare maps that stroboscopically sample their corresponding Phase portraits. We then made use of existing files that enabled data acquisition and peak detection across the voltage across the resistor. Peak detection uses the Savitsky Golar filter to identify turning points and in doing so, locates instances of period doubling. Obtaining two .txt files as resulted, we plotted the data sets of Voltage across resistor against the voltage fed into the circuit. This gave us the Bifurcation Diagram.

5 Results and Discussions

1. **Time Series Plots across Diode and Resistor**

   (Figure 3)on page 5 shows a comparison of selected bifurcations noted by taking voltages across resistor and diode. The general sequence of bifurcations for the resistor is 1T to 2T to 4T to 8T to 4T to 2T to 3T. For the diode, a similar pattern was observed till 8T after which multiple cycles of 2T and 4T were observed which ended with chaos while the resistor settles at 3T.

   We had difficulty identifying 8T via time-series plots clearly and had to take readings starting from 0V again due to hysteresis.

   For the diode, the waveform is marked by repetitive plateaus and sharp dips. For the resistor, it emerges as superposition of various sinusoidal waves with different frequencies. This becomes more evident when we we use the spectrum density analyzer that carries out a Fourier transform to make evident the various peaks that are present.

   After introducing a DC offset at +2V, we were able to obtain a sinusoidal waveform (Figure 1). Introducing a DC offset causes the mean about which the waveform oscillates to move up. As such, the threshold voltage of the diode is not reached and it allows all the current to pass through which is why we observe a sinusoidal waveform.

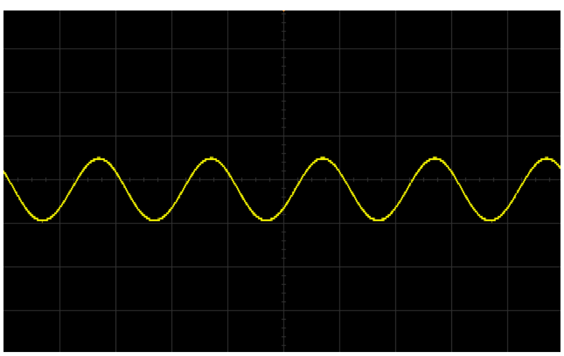

Figure 2: Waveform Obtained as a Result of DC offset

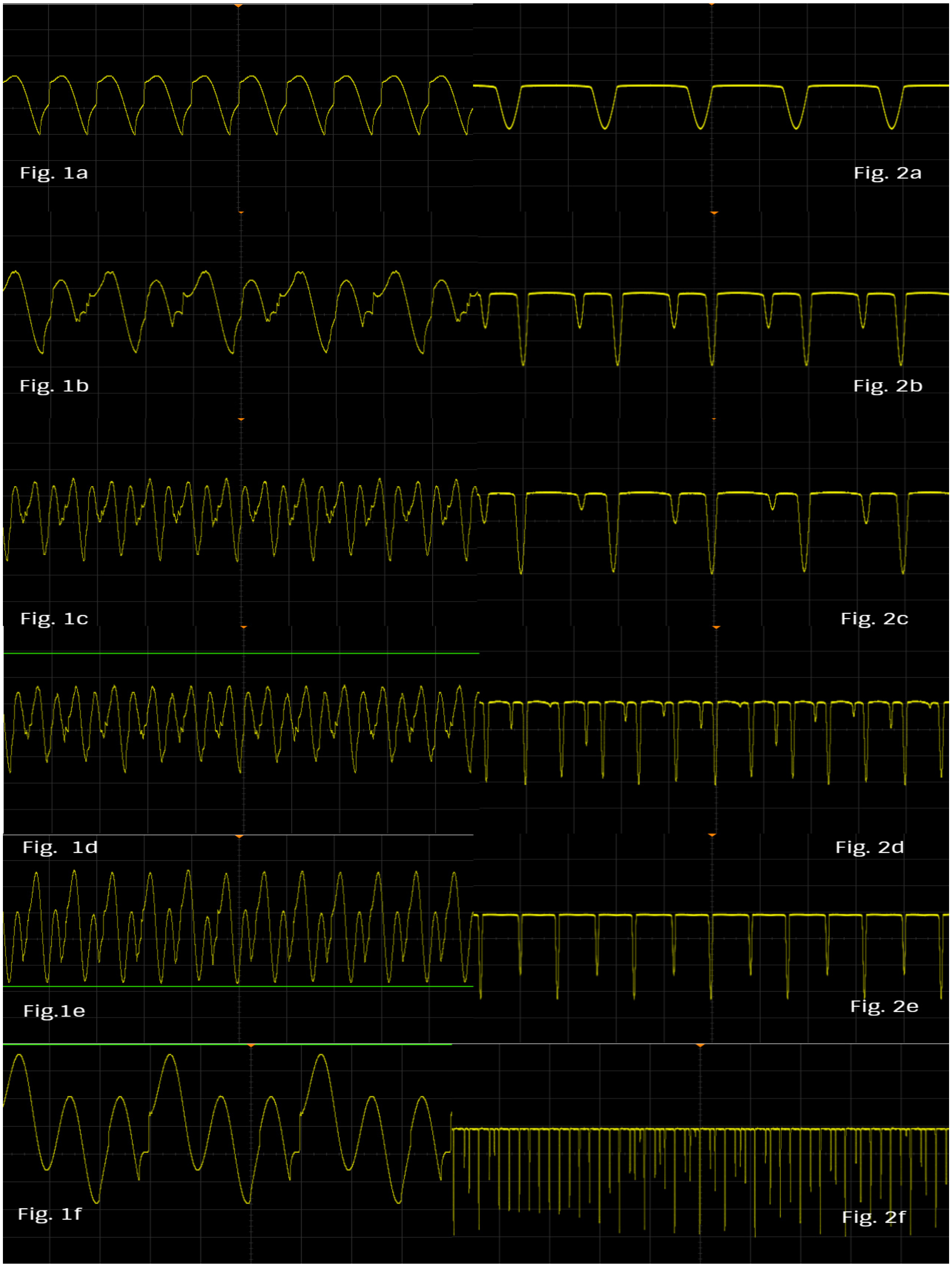

Figure 3: Sequence of Bifurcations: Resistor v/s Diode

2. **Calculating the Feigenbaum Constant**

We calculate the Feigenbaum constant from Equation (1) from the parameter values noted for both spectrum density plots as well as the time series. This will help determine which is more precise.

From 1, we find the value of δ_n to be 1.6246. From 2, we find the value of δ_n to be 2.0104. However, if we consider the more precise values of bifurcations where we have clearly identified 4T and 2T, we obtain a precise value of 4.6216677 for the Spectrum Analyzer. In either case, we are convinced of the latter's precision.

λ_n	Vpp (mV)	T	δ_n
λ_0	1.70	$1T$	0
λ_1	3.65	$2T$	1.72
λ_2	4.7	$4T$	3.05
λ_3	5.15	$8T$	1.85
λ_4	5.34	$4T$	0.633
λ_5	5.64	$2T$	0.0383
λ_6	13.47	$4T$	2.231
λ_7	16.98	$3T$	1.85

Table 1: Data from Time Series Plots.

λ_n	Vpp (mV)	Peaks Visible	δ_n
λ_0	1.50	$1T$	0
λ_1	3.22	$2T$	1.835
λ_2	4.157	$4T$	3.987
λ_3	4.392	$2T$	0.979
λ_4	4.632	$0T$	0.667
λ_5	4.993	$2T$	1.44
λ_6	5.242	$4T$	1.0504
λ_7	5.48	$3T$	5.667
λ_8	5.522	$2T$	0.009
λ_9	9.883	$4T$	2.25
λ_{10}	11.82	$2T$	4.21
λ_{11}	12.28	$4T$	1.822
λ_{12}	14.42	$3T$	0.214

Table 2: Data from Spectrum Density Plots.

3. **Period Doubling Observation across various Interfaces**

Figure (4) shows how period doubling at various orders was observed by different means. Via time series plots, we can see how the waveform evolves in a manner so that it period doubles, quadruples, and octuples. Across the phase portraits, we can observe how then number of loops increase from one to two to four to eight corresponding to the period. Across the spectrum density plots, the changing period is represented by the number of peaks. These are for different voltage values as shown in Tables (1) and (2)a.

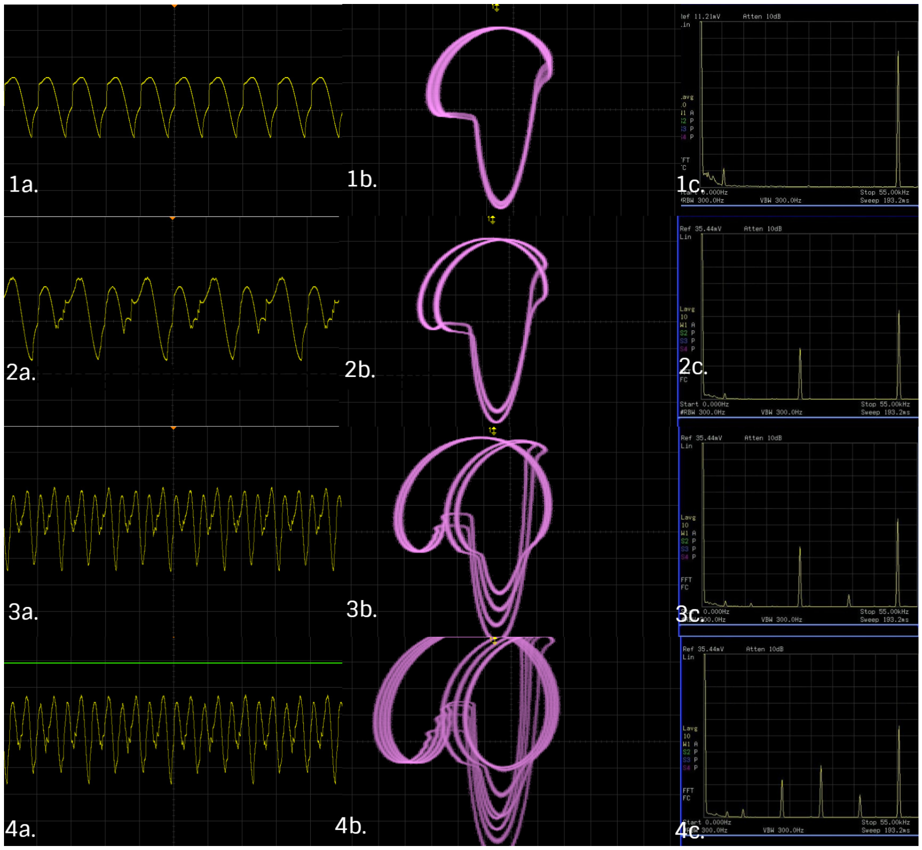

Figure 4: Bifurcations across different instruments

4. **Phase Portraits and Poincare Maps**

   Figure (5) shows Phase Portraits and Poincare Maps for 1T, 2T, 4T, and 8T respectively.

   The Poincare Map stroboscopically samples through the phase portrait and as such, maps an n-space to $(n-1)$ dimensional space. This results in clusters of data points for one period. As such, for 1T, we have one distinct cluster of data points in (Figure 5), for 2T, 2 distinct clusters in (Figure 5), for 4T, four distinct. For chaos, because there are no distinct loops on the phase portrait, a random sequence of data points emerges on the Poincare map.

5. **Modulation**

   Increasing the AM level on the oscillator causes the amplitude of the modulated waveform to decrease and the waveform becomes more spread out. This is expected as the AM level is a measure of the modulation depth and determines the extent to which the modulating signal is forged onto the carrier signal. As such, we maintain the AM level at 100 for the remainder of the experiment to obtain optimal results.

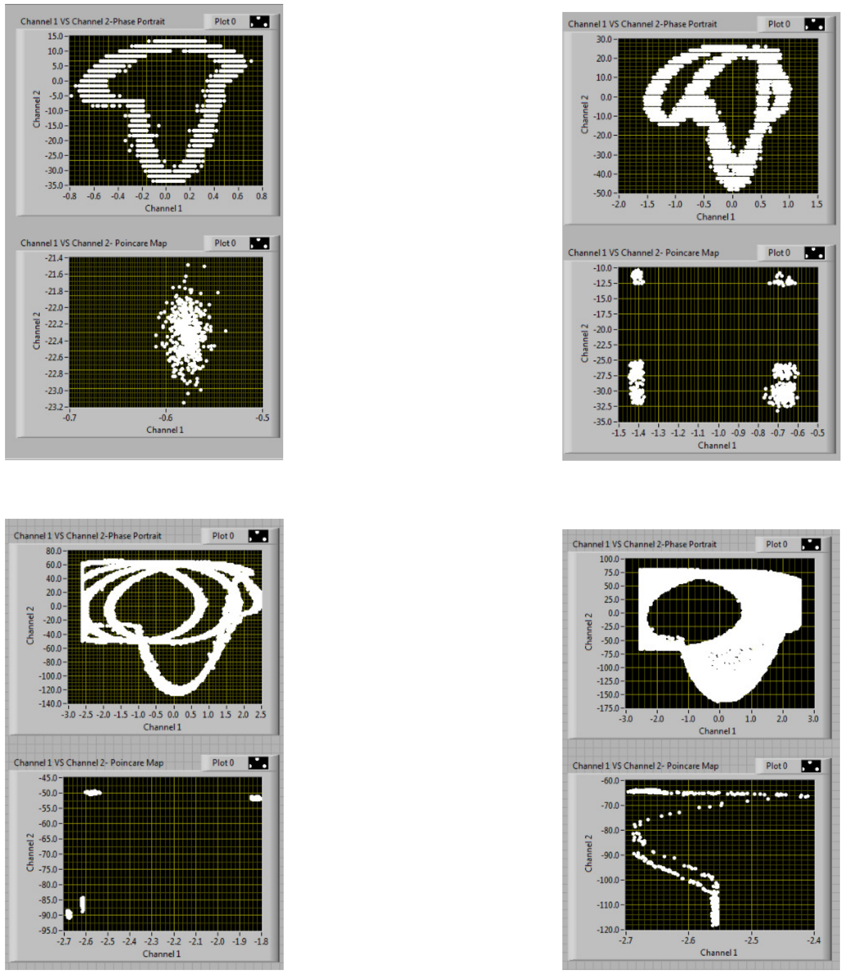

Figure 5: Phase Portraits and Poincare Maps

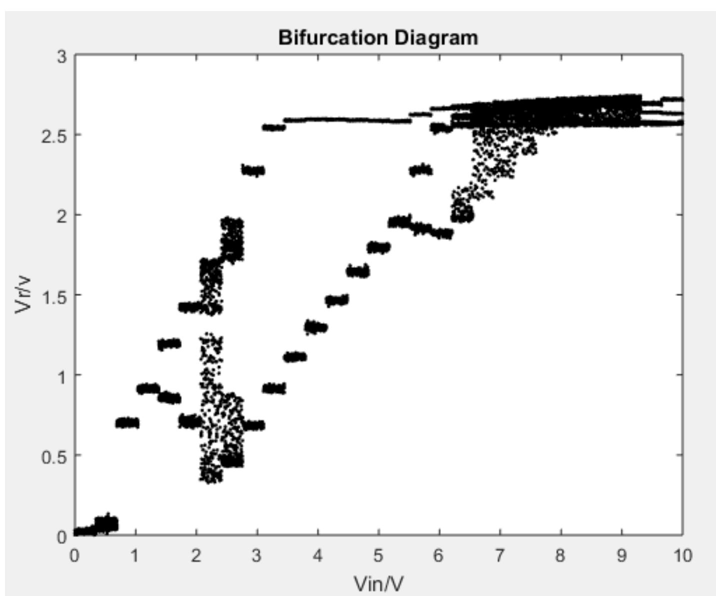

Figure 6: Bifurcation Diagram (with filter)

6. **The Bifurcation Diagram**

   Figure (6) shows that the period stays at 1T that is 20 μs till around 1.3V. This then splits into two clear peak clusters and stays at 2T till 5.5V. This, then enters the chaotic region around 6.5V after which goes to 3T around 9.2V where it remains till the maximum voltage is achieved. This also validates earlier observations made via phase portraits and time series plots.

6 Conclusion

The precision with which bifurcations are determined increased as we proceeded from time series plots to phase portraits to spectrum density plots, and finally, the bifurcation diagram.

References

[1] Taylor, John R. "*Classical Mechanics.*Sausalito, Calif: University Science Books, 2005. Print.

[2] M. P. Hanias, Z. Avgerinos, G.S. Tombras. "*Chaos, Solitons and Fractals*, 40 1050 , (2009).