Investigating Polarization of Light through Jones Calculus

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1 Abstract

In our experiment, we looked at the effects of a polarizer and a Half Wave Plate (HWP) on a laser beam by passing it through a Polarized Beam Splitter (PBS). We then measured the intensity of the transmitted and reflected beams, which represented the horizontal and vertical components of the oscillations respectively. We used Jones Calculus, which is mathematical description of the effects of optical instruments on light, to corroborate and verify the obtained readings against theoretical predictions.

2 Introduction

The Jones formalism uses the principles of Linear Algebra to calculate the effect of various optical components on light in various states of polarization. It is a convenient and simple way to predict the outcomes of optics experiments performed in the lab.

The polarization state of light is expressed as a 2×1 column vector, whereas the effect of a given optical instrument, such as a polarizer or a Half Wave Plate, is expressed as a 2×2 square matrix that represents a linear transformation, applied on the above mentioned column vector to produce a new 2×1 column vector.

The usefulness of this approach is evidenced by the fact that the effect of multiple optical instruments in a sequence can also be calculated by taking the product of their respective Jones matrices, owing to the associativity of matrix multiplication, to form a single new square matrix that represents the entire transformation.

It must be noted however, that this approach fails when dealing with light that isn't fully polarized. In such a case, we would need to employ a different method, known as the Mueller Calculus.

3 Theoretical background

We can represent linearly polarized light a using a normalized column vector as follows:

$$\vec{E} = \frac{1}{\sqrt{E_{0x}^2 + E_{0y}^2}} \begin{pmatrix} E_{0x} \\ E_{0y} \end{pmatrix},$$

where E_{0x} and E_{0y} represent the amplitudes of the horizontal component and vertical component respectively.

In terms of the angle α that the orientation of the oscillation makes with the x-axis, we can write it as[1]:

$$\vec{E} = \begin{pmatrix} \cos\alpha \\ \sin\alpha \end{pmatrix}, \tag{1}$$

which is also known as the Jones vector.

We now talk about the matrix representation of the optical instruments used in the experiment.

1. The action of a polarizer at an arbitrary angle θ between its Transmission Axis (TA) and the x-axis is given by the following Jones Matrix[1]:

$$P = \begin{pmatrix} \cos^2\theta & \sin\theta\cos\theta \\ \sin\theta\cos\theta & \sin^2\theta \end{pmatrix}. \tag{2}$$

   For the special case of a horizontal polarizer with $\theta = 0$, P reduces to $\left(\begin{smallmatrix} 1 & 0 \\ 0 & 0 \end{smallmatrix}\right)$.

2. The action of a Half Wave Plate (HWP) at an arbitrary angle θ between its Fast Axis (FA) and the x-axis is given by the following Jones Matrix[2]:

$$HWP = e^{-i\frac{\pi}{2}} \begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix}. \tag{3}$$

   For the special case where $\theta = 0$, HWP reduces to $-i\left(\begin{smallmatrix} 1 & 0 \\ 0 & -1 \end{smallmatrix}\right)$.

4 The Experiment

4.1 Experimental Set up

In the first part of the experiment, horizontally polarized light was passed through a Half Wave Plate (HWP) at varying angles between its Fast Axis (FA) and the postive x-axis. The transmitted beam was then passed through a Polarized Beam Splitter (PBS) to seperate the horizontal component of the oscillations from the vertical component, and their intensites were measured by two detectors, A and B respectively.

In the second part of the experiment, unpolarized light was passed through a polarizer, at varying angles between its Transmission Axis (TA) and the positive x-axis. The polarized transmitted beam was then passed through a HWP set at a fixed angle of $0°$. As before, this output beam was passed through a PBS and the intensities of its horizontal and vertical components measured at Detector A and B respectively.

4.2 Important optical instruments used

In order to fully understand the experiment we must first give a brief description of the Half Wave Plate which plays a central role in our experimental setup.

The HWP is made from a birefringent material, which means that the indices of refraction experienced by a beam depends on its angle of polarization[3]. Light whose polarization is aligned with what is known as the Fast Axis (FA) experiences a lower index of refraction, n_f, and travels faster, with a larger wavelength.

On the other hand, light polarized along the Slow Axis (SA) of the HWP experiences a higher index of refraction, n_s and hence travels slower, with a smaller wavelength.

These two axes form an orthonormal basis in the plane perpendicular to the direction of propagation of the wave.

The HWP is used to change the angle of polarization of light by introducing a phase difference of $180°$ between the horizontal component and the vertical component of the oscillations of linearly polarized light.

We can thus use the HWP set at an angle $\theta°$ relative to the polarization angle of incident light to produce a rotation of $2\theta°$ in the transmitted light. Hence, over a full $360°$ rotation of the HWP, light polarization is rotated over a range of $720°$, four times producing completely horizontal polarization and four times producing completely vertical polarization.

Further, when light polarization is aligned with either the fast axis or slow axis, no rotation is produced.

4.3 Theoretical predictions

For the first part of the experiment, we apply the HWP matrix from Eq.(3) to the horizontally polarized light as follows:

$$-i\begin{pmatrix}\cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta\end{pmatrix}\begin{pmatrix}1 \\ 0\end{pmatrix}$$

and we get:

$$-i\begin{pmatrix}\cos 2\theta \\ \sin 2\theta\end{pmatrix},$$

which represents the amplitudes of the x-component and y-component of the transmitted beam, which will be detected at detector A and detector B respectively. Since intensity is proportional to the square of the amplitude, the intensities recorded at the two detectors will be

$$I_A = \cos^2(2\theta), \tag{4}$$

$$I_B = \sin^2(2\theta). \tag{5}$$

In the second part of the experiment, since the HWP remains fixed at $\theta = 0°$, we can use the matrix $-i\left(\begin{smallmatrix}1 & 0 \\ 0 & -1\end{smallmatrix}\right)$ to represent its effect. Here, it is the angle of polarization of light which varies, which we represent as $\left(\begin{smallmatrix}\cos\theta \\ \sin\theta\end{smallmatrix}\right)$ from Eq.(1).

Using matrix multiplication:

$$-i\begin{pmatrix}1 & 0 \\ 0 & -1\end{pmatrix}\begin{pmatrix}\cos\theta \\ \sin\theta\end{pmatrix} = -i\begin{pmatrix}\cos\theta \\ -\sin\theta\end{pmatrix}$$

As before, the intensities recorded at the detectors will be given by the following:

$$I_A = \cos^2\theta, \tag{6}$$

$$I_B = \sin^2\theta. \tag{7}$$

5 Results and Discussion

5.1 Rotating the Half Wave Plate

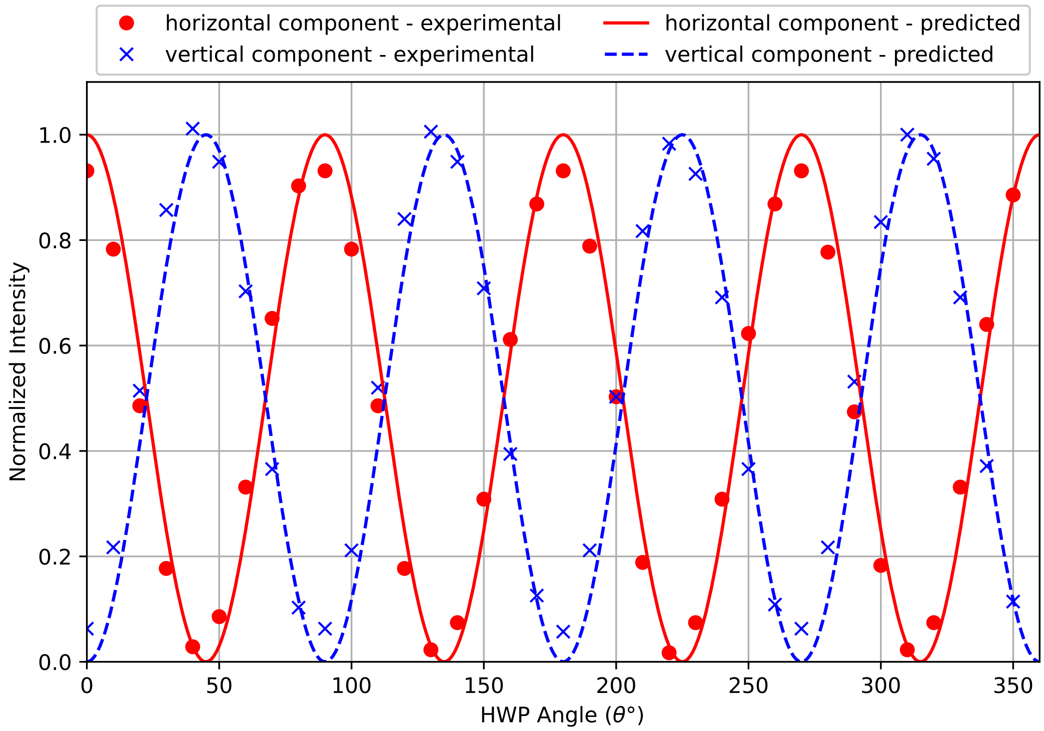

Figure 1: HWP angle against Intensity.

The results from the first experiment (reported in Table 1) have been plotted alongside the theoretically predicted values based on Eqs. (4) and (5) in Fig. (1). We observe a good match between experiment and theory, with 4 complete cycles of variation in intensity recorded at each detector over θ ranging from 0° to 360°.

This also confirms two of our qualitative predictions: firstly, that a rotation of θ of the HWP results in a rotation in the polarization angle of light equal to $\alpha = 2\theta$. Secondly, that on four positions of the HWP plate, the polarization angle of light is unaffected.

Consider the following: the incident light has a fixed polarization angle of $\alpha = 0°$. The HWP is initially set at $\theta = 0°$, so no rotation of polarization angle is produced, and the graph shows a maximum value for the horizontal component and a minimum value for the vertical one. As θ increased from 0° to 45°, the polarization angle of light was rotated from $\alpha = 0°$ to $\alpha = 90°$. Hence, the polarization vector gradually went from being completely horizontal to completely vertical, and this is reflected in the graph by a decrease in the horizontal component and an increase in the vertical component.

The reverse of this happens as the HWP angle θ goes from 45° to 90°. The polarization angle shows a corresponding two-fold increase from 90° to 180°, implying that the polarization vector gradually goes from being vertical to being horizontal. This is shown in the graph as an increase in the horizontal component from zero to its maximum value and vice versa for the vertical component.

This process occurs a total of four times over the course of a complete rotation of the HWP, illustrating our second qualitative prediction: that the polarization angle remains unchanged at 4 points, i.e., at θ values of 0°, 90°, 180°, and 270°, where the polarization vector aligns with either the FA or the SA of the HWP.

5.2 Rotating the Polarizer

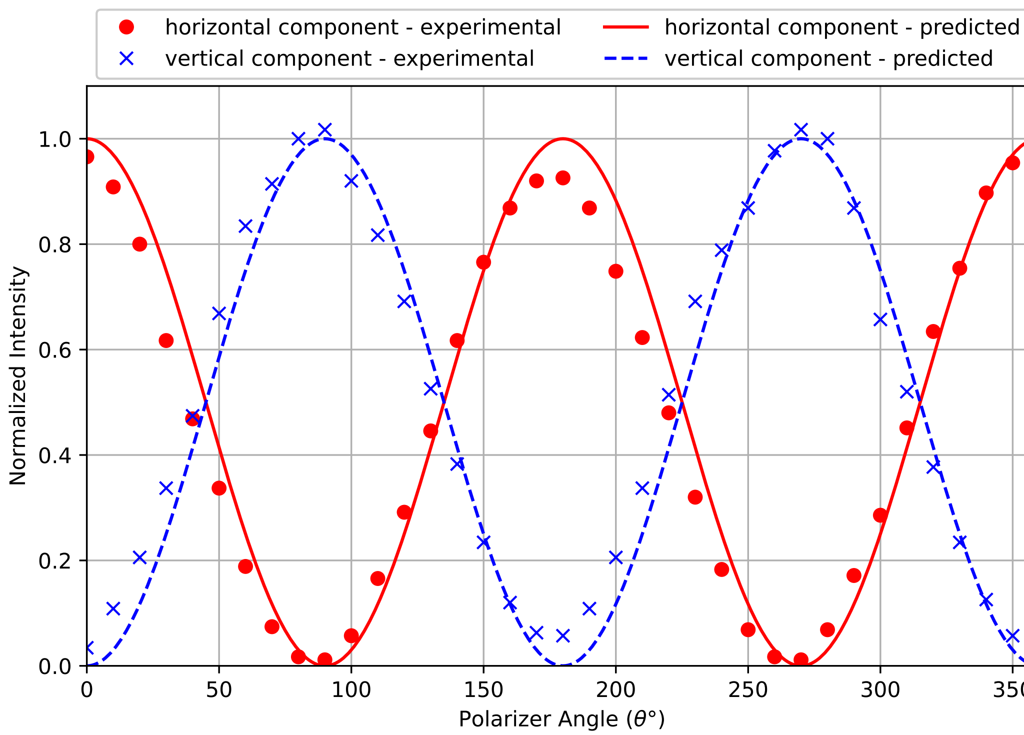

Figure 2: Polarizer angle against Intensity.

Next, the results of the second experiment (reported in Table 2) have been plotted alongside the theoretically predicted values based on Eqs. (6) and (7) in Fig. (2).

Again, we see that Jones Calculus very accurately predicts experimental results. An important difference from the previous part of the experiment is that, here, we see two full cycles on the graph rather than four. This difference may seem odd, because in both experiments, we were merely changing the relative angle between the TA of the polarizer and the FA of the HWP. The answer to this lies in the fact that we measure the polarization angle of the transmitted beam with respect to the positive x-axis rather than the polarization angle of the *incident* beam.

The total rotation of the polarization angle is the difference between the polarization angle of the transmitted beam and the incident beam, the latter parameter being varied over the course of this part of the experiment. This accounts for the factor of 2 which appeared to be missing in Fig. (2)

6 Conclusions

Based on our comparison of theoretically predicted intensity values with the experimentally obtained values, we conclude that Jones Calculus is an effective tool for predicting the action of optical instruments on polarized light. We also confirmed that rotating the HWP by an angle θ produces a rotation of polarized light by 2θ. In addition, we showed that light whose polarization is aligned with either the Fast Axis or the Slow Axis of the HWP, does not undergo a change in its state of polarization. This experiment also gives evidence that light is a transverse wave as a polarizer would not otherwise have an effect on its state of polarization.

References

[1] Frank L Pedrotti, Leno M Pedrotti, and Leno S Pedrotti, "*Introduction to Optics*", Cambridge University Press, (2017).

[2] Jordan Edmunds. (2019, August 19). *Rotated Waveplates and Jones Matrix* [Video]. YouTube. `https://www.youtube.com/watch?v=HYj3h5ncStE&list=PLQms29D1RqeIJUzgzFweiuVFWPOtSNCUP&index=7`

[3] Understanding Waveplates and Retarders. (n.d.). Retrieved November 17, 2022, from `https://www.edmundoptics.com/knowledge-center/application-notes/optics/understanding-waveplates/`

A Appendix: HWP Readings

HWP Angle[1] (θ°)	Horizontal component intensity (V) (Detector A)[2]	Vertical component intensity (V) (Detector B)[2]	Normalized horizontal component[3]	Normalized vertical component[3]
0	1.63	0.11	0.93	0.06
10	1.37	0.38	0.78	0.22
20	0.85	0.90	0.49	0.51
30	0.31	1.50	0.18	0.86
40	0.05	1.77	0.03	1.01
50	0.15	1.66	0.09	0.95
60	0.58	1.23	0.33	0.70
70	1.14	0.64	0.65	0.37
80	1.58	0.18	0.90	0.10
90	1.63	0.11	0.93	0.06
100	1.37	0.37	0.78	0.21
110	0.85	0.91	0.49	0.52
120	0.31	1.47	0.18	0.84
130	0.04	1.76	0.02	1.01
140	0.13	1.66	0.07	0.95
150	0.54	1.24	0.31	0.71
160	1.07	0.69	0.61	0.39
170	1.52	0.22	0.87	0.13
180	1.63	0.10	0.93	0.06
190	1.38	0.37	0.79	0.21
200	0.88	0.88	0.50	0.50
210	0.33	1.43	0.19	0.82
220	0.03	1.72	0.02	0.98
230	0.13	1.62	0.07	0.93
240	0.54	1.21	0.31	0.69
250	1.09	0.64	0.62	0.37
260	1.52	0.19	0.87	0.11
270	1.63	0.11	0.93	0.06
280	1.36	0.38	0.78	0.22
290	0.83	0.93	0.47	0.53
300	0.32	1.46	0.18	0.83
310	0.04	1.75	0.02	1.00
320	0.13	1.67	0.07	0.95
330	0.58	1.21	0.33	0.69
340	1.12	0.65	0.64	0.37
350	1.55	0.20	0.89	0.11

Table 1: Readings for Rotating HWP

[1] Error in angle reading of $\pm 1^{\circ}$
[2] Error in multimeter reading of ± 0.005 V
[3] Values normalized by dividing by 1.75 (obtained by taking the sum of the detector A and B readings for each angle and then calculating their average)

B Appendix: Polarizer Readings

Polarizer angle[1] ($\theta°$)	Horizontal component intensity (V) (Detector A)[2]	Vertical component intensity (V) (Detector B)[2]	Normalized horizontal component[3]	Normalized vertical component[3]
0	1.69	0.06	0.97	0.03
10	1.59	0.19	0.91	0.11
20	1.40	0.36	0.80	0.21
30	1.08	0.59	0.62	0.34
40	0.82	0.83	0.47	0.47
50	0.59	1.17	0.34	0.67
60	0.33	1.46	0.19	0.83
70	0.13	1.60	0.07	0.91
80	0.03	1.75	0.02	1.00
90	0.02	1.78	0.01	1.02
100	0.10	1.61	0.06	0.92
110	0.29	1.43	0.17	0.82
120	0.51	1.21	0.29	0.69
130	0.78	0.92	0.45	0.53
140	1.08	0.67	0.62	0.38
150	1.34	0.41	0.77	0.23
160	1.52	0.21	0.87	0.12
170	1.61	0.11	0.92	0.06
180	1.62	0.10	0.93	0.06
190	1.52	0.19	0.87	0.11
200	1.31	0.36	0.75	0.21
210	1.09	0.59	0.62	0.34
220	0.84	0.90	0.48	0.51
230	0.56	1.21	0.32	0.69
240	0.32	1.38	0.18	0.79
250	0.12	1.52	0.07	0.87
260	0.03	1.71	0.02	0.98
270	0.02	1.78	0.01	1.02
280	0.12	1.75	0.07	1.00
290	0.30	1.52	0.17	0.87
300	0.50	1.15	0.29	0.66
310	0.79	0.91	0.45	0.52
320	1.11	0.66	0.63	0.38
330	1.32	0.41	0.75	0.23
340	1.57	0.22	0.90	0.13
350	1.67	0.10	0.95	0.06

Table 2: Readings for Rotating Polarizer

[1] Error in angle reading of $\pm 1°$

[2] Error in multimeter reading of ± 0.005 V

[3] Values normalized by dividing by 1.75 (obtained by taking the sum of the detector A and B readings for each angle and then calculating their average)