ELECTROMAGNETICALLY INDUCED TRANSPARENCY IN RUBIDIUM 85

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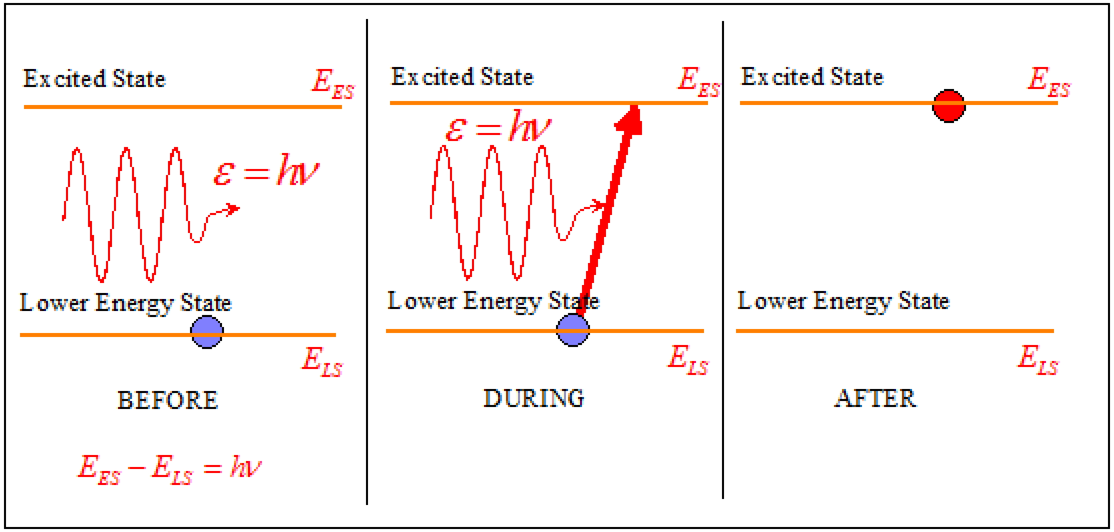
Electromagnetically induced transparency

- The concept of EIT was first given by Harris et al in 1990. When a strong coupling laser field is used to drive a resonant transition in a three-level atomic system, the absorption of a weak probe laser field can be reduced or eliminated provided the two resonant transitions are coherently coupled to a common state.
- EIT was first observed in lambda type system of strontium vapors using high pulsed laser in 1991.

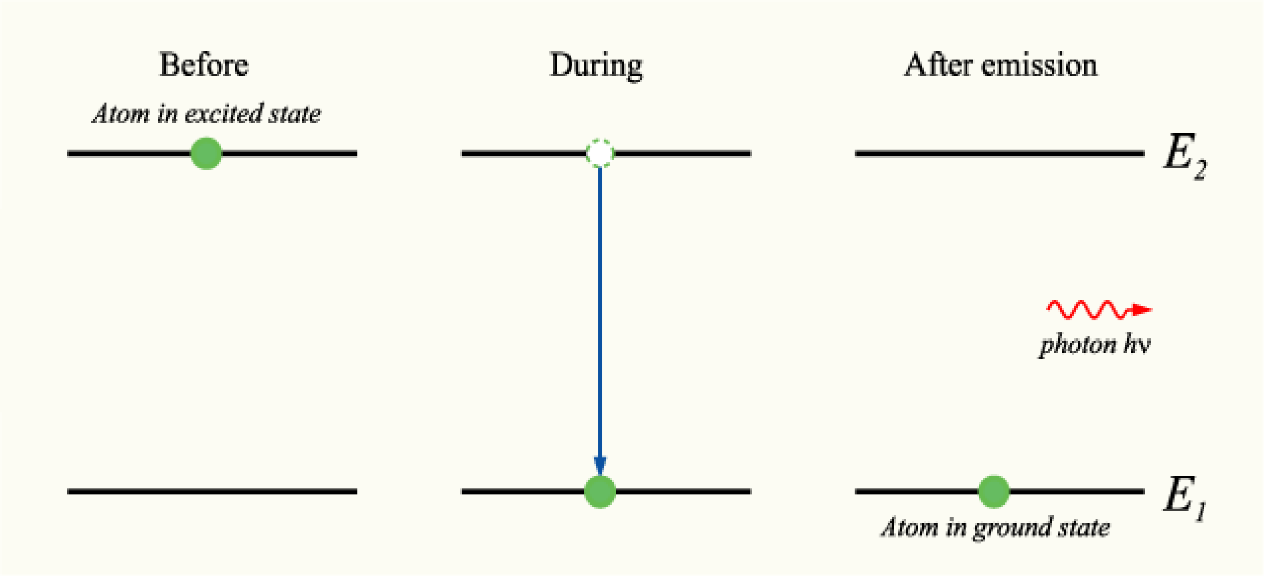
Stimulated absorption

- **Stimulated Absorption:** When a photon of energy equals to the energy difference between two atomic levels interacts with the atom. The atom in the lower energy state, absorbs the photon and jumps to the higher energy state (excited state).



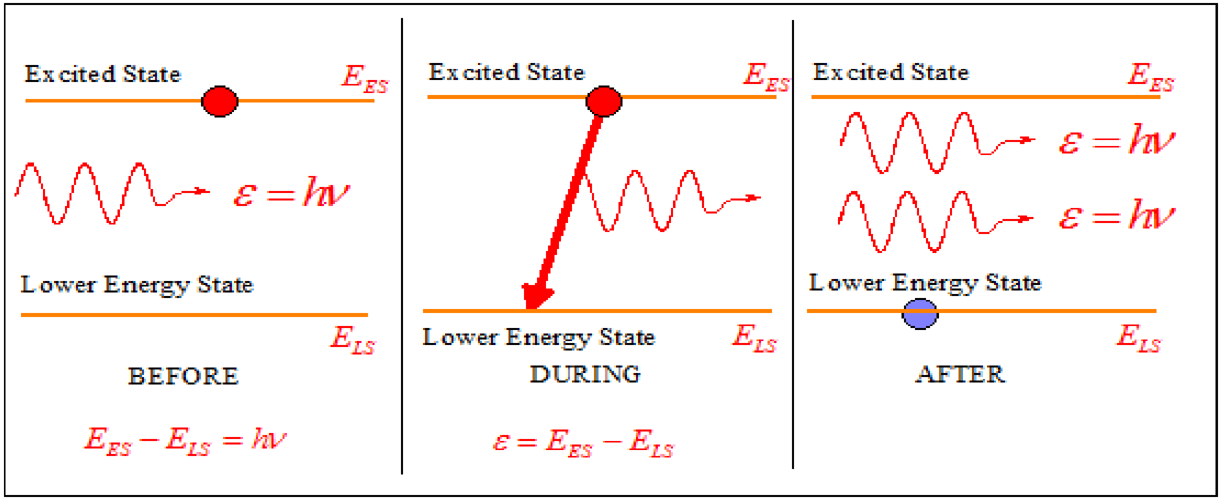
Spontaneous emission

- **Spontaneous emission:** A process by which an atom in the excited state undergoes a transition to the lower energy state (ground state) and emits a photon.



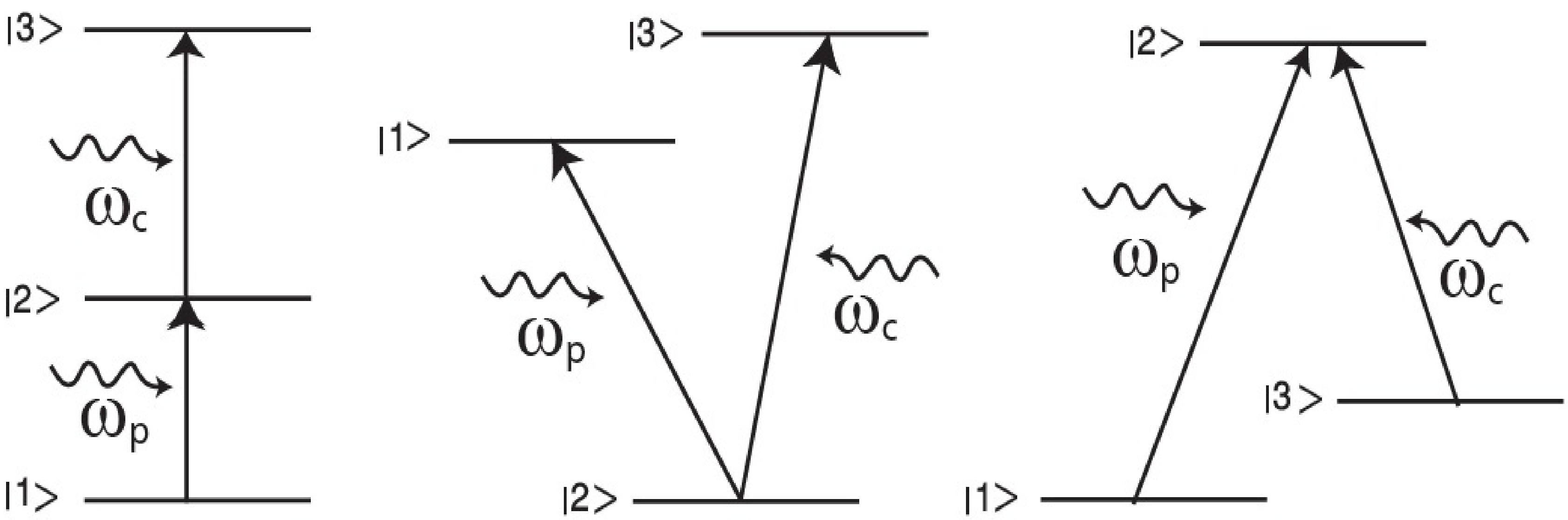
Stimulated emission

- **Stimulated emission:** A process by which an atomic electron (in excited state) interacts with an electromagnetic wave drop to a lower energy level transferring its energy to that field



Electromagnetically induced transparency

- **EIT configurations:**



Ladder V-type Lambda-type

Rubidium 85

- Rubidium is a silvery white metallic element alkali metal group.
- **Electronic configuration:**

$$1s^2, 2s^2, 2p^6, 3s^2, 3p^6, 4s^2, 3d^{10}, 4p^6, 5s^1.$$

- **Fine structure splitting:**

For $5s^1$ electron

$$\begin{aligned} s &= 1/2, \\ l &= 0, \\ J &= |L \pm S| = |0 \pm \frac{1}{2}| = \frac{1}{2}, \\ M &= 2s+1 = 2. \end{aligned}$$

Ground state

$$\boxed{5^2\mathrm{S}_{1/2}}$$

Rubidium 85

- **First excited state:** $5S \rightarrow 5P$

$$\begin{aligned} l &= 1, \\ s &= 1/2, \\ J &= |1 \pm \frac{1}{2}| = \frac{1}{2}, \frac{3}{2}, \end{aligned}$$

Possible states

$$\boxed{5^2\mathrm{P}_{1/2}, 5^2\mathrm{P}_{3/2}}$$

- **Second excited state:** $5S \rightarrow 5D$

$$\begin{aligned} l &= 2, \\ s &= 1/2, \\ J &= |2 \pm \frac{1}{2}| = \frac{3}{2}, \frac{5}{2}, \end{aligned}$$

possible states

$$\boxed{5^2\mathrm{D}_{3/2}, 5^2\mathrm{D}_{5/2}}$$

Rubidium 85

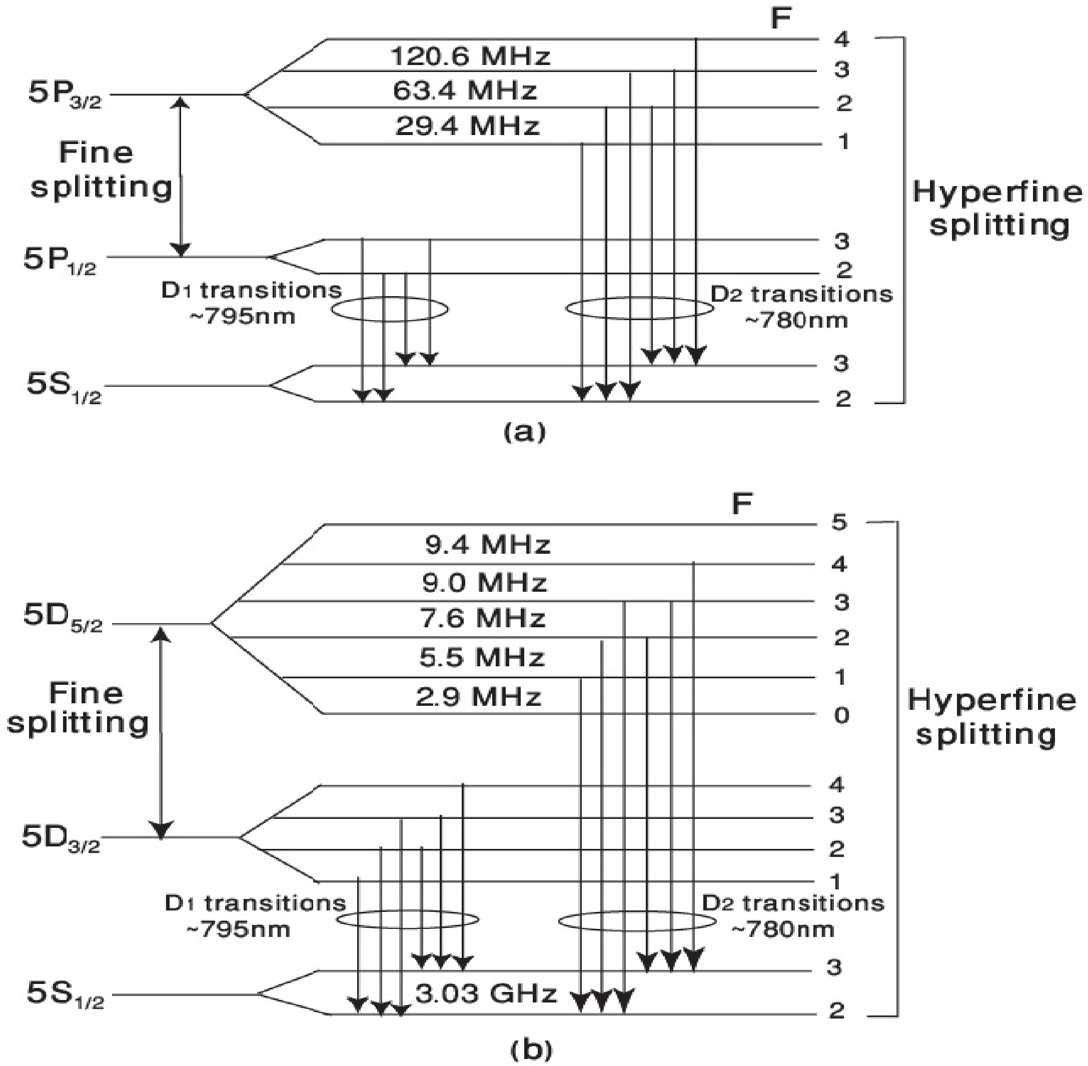
- **Hyper fine splitting:** This is interaction of nuclear spin (I) with the electron angular momentum (J).
- HF splitting of ground state is,

$$F \;=\; |J \pm I| = |\frac{1}{2} \pm \frac{5}{2}| = 2, 3.$$

- HF splitting of the 1st and 2nd excited states is,

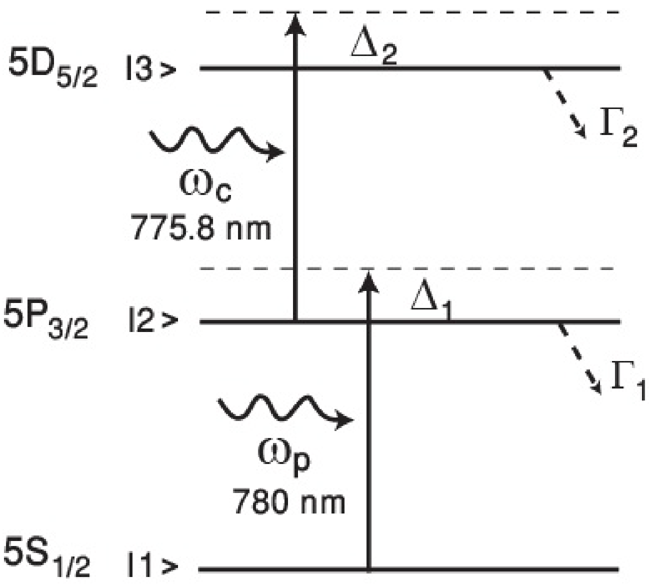
$$5^2P_{1/2}: \quad F = |\frac{1}{2} \pm \frac{5}{2}| = 2, 3,$$
$$5^2P_{3/2}: \quad F = |\frac{3}{2} \pm \frac{5}{2}| = 1, 2, 3, 4,$$
$$5^2D_{3/2}: \quad F = |\frac{3}{2} \pm \frac{5}{2}| = 1, 2, 3, 4,$$
$$5^2P_{5/2}: \quad F = |\frac{5}{2} \pm \frac{5}{2}| = 0, 1, 2, 3, 4, 5.$$

Energy level diagram of ^{85}Rb



Three level Ladder type system

- Two laser beams are used to excite the electronic transitions.
- Probe beam ω_p for the transition $|1\rangle \rightarrow |2\rangle$, coupling beam ω_c for $|2\rangle \rightarrow |3\rangle$.
- $\Delta_1 = \omega_p - \omega_{21}$ probe beam detuning while $\Delta_2 = \omega_c - \omega_{32}$ is the coupling beam detuning.



Ladder type system

- The initial state of the system is,

$$|\psi(0)\rangle \;=\; c_1(0)|1\rangle + c_2(0)|2\rangle + c_3(0)|3\rangle.$$

- The state of the system evolves after time t, so the time dependent wave function is,

$$|\psi(t)\rangle \;=\; c_1 e^{-i\omega_1 t}|1\rangle + c_2 e^{-i\omega_2 t}|2\rangle + c_3 e^{-i\omega_3 t}|3\rangle$$

Hamiltonian

- The total Hamiltonian of the system can be find out using perturbative approach,

$$\hat{H} = \hat{H}_0 + \hat{H}_I,$$

- **Unperturbed part:**

$$\hat{H}_0 = \hat{I}\hat{H}_0\hat{I}$$

$$\hat{H}_0 = (|1\rangle\langle 1| + |2\rangle\langle 2| + |3\rangle\langle 3|)\hat{H}_0(|1\rangle\langle 1| + |2\rangle\langle 2| + |3\rangle\langle 3|)$$

using orthonormality condition, i.e., $\langle n|m\rangle = \delta_{nm}$

$$\hat{H}_0 = \hbar\omega_1|1\rangle\langle 1| + \hbar\omega_2|2\rangle\langle 2| + \hbar\omega_3|3\rangle\langle 3|.$$

Hamiltonian

- **Perturbed part:** When an electromagnetic field interacts with the atom, the interaction Hamiltonian is,

$$\hat{H}_I = -eE(r,t)\hat{r}$$
$$\hat{H}_I = -eEe^{-i\omega t}\hat{r}$$

under parity conservation $\langle 1|\hat{r}|2\rangle = r_{12},\ \langle 1|\hat{r}|1\rangle = 0$

$$\begin{aligned}\hat{H}_I &= -eEe^{-i\omega t}\Big(r_{12}|1\rangle\langle 2| + r_{21}|2\rangle\langle 1| + r_{23}|2\rangle\langle 3| + r_{32}|3\rangle\langle 2|\Big)\\ &= \frac{1}{2}P_{21}E_p e^{-i\omega_p t}|2\rangle\langle 1| + \frac{1}{2}P_{32}E_c e^{-i\omega_c t}|3\rangle\langle 2| + H.c.\end{aligned}$$

$$P_{ij} = -e\langle i|\hat{r}|j\rangle$$

Hamiltonian

$$\hat{H}_I = \frac{\hbar}{2}\Omega_p e^{-i\phi_p} e^{-i\omega_p t}|2\rangle\langle 1| + \frac{\hbar}{2}\Omega_c e^{-i\phi_c} e^{-i\omega_c t}|3\rangle\langle 2| + H.c.$$

$$\Omega_p e^{-i\phi_p} = \left(\frac{P_{21}}{\hbar}\right)E_p = 2g_{21}E_p$$

$$\Omega_c e^{-i\phi_c} = \left(\frac{P_{32}}{\hbar}\right)E_c = 2g_{32}E_c$$

$$P_{ij} = 2\hbar g_{ij}$$

- **Total Hamiltonian:**

$$\begin{aligned}\hat{H} &= \hbar\omega_1|1\rangle\langle 1| + \hbar\omega_2|2\rangle\langle 2| + \hbar\omega_3|3\rangle\langle 3| \\ &+ \left(\frac{\hbar}{2}\Omega_p e^{-i\phi_p} e^{-i\omega_p t}|2\rangle\langle 1| + \frac{\hbar}{2}\Omega_c e^{-i\phi_c} e^{-i\omega_c t}|3\rangle\langle 2| + H.c\right).\end{aligned}$$

Time evolution of the density matrix

- Using density matrix has its own significance as an in physical systems the exact state of the system is hardly known but only probabilities are known.
- The density operator ρ is,

$$\rho = \Sigma_\psi P_\psi |\psi\rangle\langle\psi|$$

- The time evolution of the density matrix can be find out by Liouville (Von Neumann) equation,

$$\frac{d\rho}{dt} = -\frac{i}{\hbar}[H,\rho]$$

$$\frac{d\rho}{dt} = -\frac{i}{\hbar}[H,\rho] - \frac{1}{2}\{\Gamma,\rho\} \quad \langle n|\Gamma|m\rangle = \gamma_n\delta_{nm}$$

Time evolution of the density matrix

- **Generalized form of the density operator equation of motion:**

$$\frac{d\rho_{ij}}{dt} = \frac{1}{i\hbar}\sum_{k=1,2,3}(H_{ik}\rho_{kj} - \rho_{ik}H_{kj}) - \frac{1}{2}\sum_{k=1,2,3}(\Gamma_{ik}\rho_{kj} + \rho_{ik}\Gamma_{kj}).$$

and the density matrix elements are,

$$\begin{aligned}\frac{d\rho_{21}}{dt} = & -\frac{i}{\hbar}(H_{21}\rho_{11} - \rho_{21}H_{11}) - \frac{1}{2}(\Gamma_{21}\rho_{11} + \rho_{21}\Gamma_{11}),\\ & -\frac{i}{\hbar}(H_{22}\rho_{21} - \rho_{22}H_{21}) - \frac{1}{2}(\Gamma_{22}\rho_{21} + \rho_{22}\Gamma_{21}),\\ & -\frac{i}{\hbar}(H_{23}\rho_{31} - \rho_{23}H_{31}) - \frac{1}{2}(\Gamma_{23}\rho_{31} + \rho_{23}\Gamma_{31}).\end{aligned}$$

Time evolution of the density matrix

$$
\begin{aligned}
\frac{d\rho_{32}}{dt} &= -\frac{i}{\hbar}(H_{31}\rho_{12} - \rho_{31}H_{12}) - \frac{1}{2}(\Gamma_{31}\rho_{12} + \rho_{31}\Gamma_{12}), \\
&\quad -\frac{i}{\hbar}(H_{32}\rho_{22} - \rho_{32}H_{22}) - \frac{1}{2}(\Gamma_{32}\rho_{22} + \rho_{32}\Gamma_{22}), \\
&\quad -\frac{i}{\hbar}(H_{33}\rho_{32} - \rho_{33}H_{32}) - \frac{1}{2}(\Gamma_{33}\rho_{32} + \rho_{33}\Gamma_{32}).
\end{aligned}
$$

$$
\begin{aligned}
\frac{d\rho_{31}}{dt} &= -\frac{i}{\hbar}(H_{31}\rho_{11} - \rho_{31}H_{11}) - \frac{1}{2}(\Gamma_{31}\rho_{11} + \rho_{31}\Gamma_{11}), \\
&\quad -\frac{i}{\hbar}(H_{32}\rho_{21} - \rho_{32}H_{21}) - \frac{1}{2}(\Gamma_{32}\rho_{21} + \rho_{32}\Gamma_{21}), \\
&\quad -\frac{i}{\hbar}(H_{33}\rho_{31} - \rho_{33}H_{31}) - \frac{1}{2}(\Gamma_{33}\rho_{31} + \rho_{33}\Gamma_{31}).
\end{aligned}
$$

Time evolution of density matrix

The matrix elements are,

$$\begin{aligned}
H_{11} &= \langle 1|\hat{H}|1\rangle = \hbar\omega_1. \\
H_{22} &= \langle 2|\hat{H}|2\rangle = \hbar\omega_2. \\
H_{33} &= \langle 3|\hat{H}|3\rangle = \hbar\omega_3. \\
H_{21} &= \langle 2|\hat{H}|1\rangle = \frac{\hbar}{2}\Omega_p e^{-i\omega_p t} e^{-i\phi_p t} = H_{12}^*. \\
H_{32} &= \langle 3|\hat{H}|2\rangle = \frac{\hbar}{2}\Omega_c e^{-i\omega_c t} e^{-i\phi_c t} = H_{23}^*. \\
H_{31} &= \langle 3|\hat{H}|1\rangle = 0 = H_{13}^*.
\end{aligned}$$

$$\begin{aligned}
\gamma_{ij} &= \frac{\Gamma_i + \Gamma_j}{2}, \\
\omega_{21} &= \omega_2 - \omega_1, \\
\omega_{32} &= \omega_3 - \omega_2, \\
\omega_{31} &= \omega_3 - \omega_1,
\end{aligned}$$

Time evolution of the density matrix

$$
\begin{aligned}
\frac{d\rho_{21}}{dt} &= -(i\omega_{21}+\gamma_{21})\rho_{21} + \frac{i}{2}\Omega_p e^{-i\omega_p t} e^{-i\phi_p t}(\rho_{22}-\rho_{11}) - \frac{i}{2}\Omega_c e^{i\omega_c t} e^{i\phi_c t}\rho_{31}. \\
\frac{d\rho_{32}}{dt} &= -(i\omega_{32}+\gamma_{32})\rho_{32} + \frac{i}{2}\Omega_c e^{-i\omega_c t} e^{-i\phi_c t}(\rho_{33}-\rho_{22}) + \frac{i}{2}\Omega_p e^{i\omega_p t} e^{i\phi_p t}\rho_{31}. \\
\frac{d\rho_{31}}{dt} &= -(i\omega_{31}+\gamma_{31})\rho_{31} - \frac{i}{2}\Omega_c e^{-i\omega_c t} e^{-i\phi_c t}\rho_{21} - \frac{i}{2}\Omega_p e^{-i\omega_p t} e^{-i\phi_p t}\rho_{32}.
\end{aligned}
$$

initial conditions are,

$$
\rho_{11}^{(0)} = 1,\ \rho_{22}^{(0)} = 0,\ \rho_{33}^{(0)} = 0,\ \rho_{32}^{(0)} = 0.
$$

substituting,

$$
\begin{aligned}
\Omega_p e^{-i\phi_p t} &= 2g_{21}E_p, \\
\Omega_c e^{-i\phi_c t} &= \Omega_c e^{-i\phi_c t}.
\end{aligned}
$$

Time evolution of the density matrix

Now the equation of motion becomes,

$$\frac{d\rho_{21}}{dt} = -(i\omega_{21}+\gamma_{21})\rho_{21} - ig_{21}e^{-i\omega_p t} - \frac{i}{2}\Omega_c e^{i\omega_c t}e^{i\phi_c t}\rho_{31}.$$

$$\frac{d\rho_{32}}{dt} = -(i\omega_{32}+\gamma_{32})\rho_{32} + ig_{21}E_p e^{i\omega_p t}\rho_{31}.$$

$$\frac{d\rho_{31}}{dt} = -(i\omega_{31}+\gamma_{31})\rho_{31} - \frac{i}{2}\Omega_c e^{-i\omega_c t}e^{-i\phi_c t}\rho_{21}.$$

Introducing the transformations,

$$\rho_{21} = \tilde{\rho}_{21}e^{-i\omega_p t},$$

$$\rho_{31} = \tilde{\rho}_{31}e^{-i(\omega_p+\omega_c)t},$$

Time evolution of the density matrix

$$\frac{d\tilde{\rho}_{21}}{dt} = -(\gamma_{21} - i\Delta_1)\tilde{\rho}_{21} - ig_{21}E_p - \frac{i}{2}\Omega_c e^{i\phi_c t}\tilde{\rho}_{31}.$$
$$\frac{d\tilde{\rho}_{31}}{dt} = -(\gamma_{31} - i(\Delta_1 + \Delta_2))\tilde{\rho}_{31} - \frac{i}{2}\Omega_c e^{-i\phi_c t}\tilde{\rho}_{21}.$$

These equations are of the form,

$$\frac{dR}{dt} = MR + A,$$

$$R = \begin{bmatrix} \tilde{\rho}_{21} \\ \tilde{\rho}_{31} \end{bmatrix}, \; M = \begin{bmatrix} -(\gamma_{21} - i\Delta_1) & -\frac{i}{2}\Omega_c e^{i\phi_c t} \\ -\frac{i}{2}\Omega_c e^{-i\phi_c t} & -(\gamma_{31} - i(\Delta_1 + \Delta_2)) \end{bmatrix}, \text{and } A = \begin{bmatrix} -ig_{21}E_p \\ 0 \end{bmatrix}$$

Steady state solution can be obtained by solving,

$$R = -M^{-1}A.$$

Time evolution of the density matrix

$$
\begin{aligned}
M^{-1} &= \frac{adj[M]}{|M|} \\
adj[M] &= B^T, \\
B_{ij} &= (-1)^{i+j} det M_{ij}.
\end{aligned}
$$

Here B is a matrix of cofactors. $R = -M^{-1}A.$

$$
\begin{bmatrix} \tilde{\rho}_{21} \\ \tilde{\rho}_{31} \end{bmatrix} = \frac{\begin{bmatrix} -(\gamma_{31} - i(\Delta_1 + \Delta_2)) & \frac{i}{2}\Omega_c e^{i\phi_c t} \\ \frac{i}{2}\Omega_c e^{-i\phi_c t} & -(\gamma_{21} - i\Delta_1) \end{bmatrix}}{(\gamma_{21} - i\Delta_1)(\gamma_{31} - i(\Delta_1 + \Delta_2)) + \frac{\Omega_c^2}{4}} \begin{bmatrix} ig_{21}E_p \\ 0 \end{bmatrix}
$$

$$
\tilde{\rho}_{21} = \frac{-ig_{21}E_p(\gamma_{31} - i(\Delta_1 + \Delta_2))}{(\gamma_{21} - i\Delta_1)(\gamma_{31} - i(\Delta_1 + \Delta_2)) + \frac{\Omega_c^2}{4}},
$$

Time evolution of the density matrix

$$\rho_{21} = \frac{-ig_{21}E_p\, e^{-i\omega_p t}}{(\gamma_{21} - i\Delta_1) + \frac{\Omega_c^2/4}{(\gamma_{31} - i(\Delta_1 + \Delta_2))}}.$$

similarly,

$$\tilde{\rho}_{31} = \frac{\frac{i}{2}\Omega_c(ig_{21}E_p)e^{-i\phi_c t}}{(\gamma_{21} - i\Delta_1)(\gamma_{31} - i(\Delta_1 + \Delta_2)) + \frac{\Omega_c^2}{4}}$$

$$\rho_{31} = \frac{-ig_{32}E_c e^{-i\omega_c t} e^{-i\phi_c t}}{(\gamma_{31} - i(\Delta_1 + \Delta_2))}\, \rho_{21}$$

Complex susceptibility

- The real and imaginary parts of the complex susceptibilty are used to define the dispersion β and absorption α coefficients,

$$\alpha = \frac{\omega_p n_0 \chi''}{c}, \qquad \beta = \frac{\omega_p n_0 \chi'}{2c},$$

- The complex susceptibility and polarization is related by,

$$P = \frac{1}{2}\varepsilon_0 E_p[\chi(\omega_p)e^{-i\omega_p t} + c.c].$$

- Atomic polarization for N number of atoms per unit volume is,

$$P = -2\hbar g_{21} N \rho_{21} + c.c.$$

Complex susceptibility

$$\chi(\omega_p) = -\frac{4\hbar g_{21}\rho_{21}}{\varepsilon_0 E_p e^{-i\omega_p t}},$$

$$\chi(\omega_p) = \frac{4i\hbar g_{21}^2/\varepsilon_0}{(\gamma_{21} - i\Delta_1) + \left(\frac{\Omega_c^2/4}{\gamma_{31} - i(\Delta_1 + \Delta_2)}\right)}.$$

- **Doppler broadening effects:** The atom moving towards the probe beam will feel an up shift in its frequency by an amount $(\frac{\omega_p \nu}{c})$ and downshifted for the coupling beam by the factor $(-\frac{\omega_c \nu}{c})$.
- As number of atoms per unit volume is $N_\nu d\nu$, and $N(\nu)$

is of Maxwellian form,

Complex Susceptibility

$$N(\nu) = \frac{N_o}{u\sqrt{\pi}} e^{(-\frac{\nu^2}{u^2})} d\nu,$$

$$u = \sqrt{\frac{2k_BT}{m}}.$$

$$\chi(\nu)d\nu = \frac{4i\hbar g_{21}^2 N_o \ e^{(\frac{-\nu^2}{u^2})} d\nu}{u\varepsilon_0\sqrt{\pi}\left[(\gamma_{21} - i\Delta_1) - i\frac{\omega_p}{c}\nu + \left(\frac{\Omega_c^2/4}{\gamma_{31} - i(\Delta_1+\Delta_2) - i\frac{(\omega_p-\omega_c)\nu}{c}}\right)\right]}.$$

substituting, $\frac{\nu^2}{u^2} = x^2 \qquad dv = udx$

$$\chi(x)dx = \frac{4i\hbar g_{21}^2 N_o \ e^{-x^2} dx}{\varepsilon_0\sqrt{\pi}\left[(\gamma_{21} - i\Delta_1) - i\frac{\omega_p}{c}ux + \left(\frac{\Omega_c^2/4}{\gamma_{31} - i(\Delta_1+\Delta_2) - i\frac{(\omega_p-\omega_c)ux}{c}}\right)\right]}.$$

Complex susceptibility

- **Ignoring two photon transition:** $\frac{(\omega_p-\omega_c)ux}{c} = 0$

$$\chi(x)dx \;=\; \frac{4i\hbar g_{21}^2 N_o\, e^{-x^2}dx}{\varepsilon_0\sqrt{\pi}\left(\frac{\omega_p u}{c}\right)\left[\frac{c}{\omega_p u}\left((\gamma_{21}-i\Delta_1)+\frac{\Omega_c^2/4}{(\gamma_{31}-i(\Delta_1+\Delta_2)}\right)-ix\right]}.$$

$$\chi(x)dx \;=\; \frac{4i\hbar g_{21}^2 N_o}{\varepsilon_0\sqrt{\pi}(\frac{\omega_p u}{c})[z-ix]}\, e^{-x^2}dx. \qquad \int_{-\infty}^{\infty}\frac{C}{z-ix}dx,$$

$$\text{Integrate}\left[\frac{C\, e^{-x^2}}{z-ix}, \{x,-\infty,\infty\}, \text{Assumptions} \to z \notin \text{Reals}\right].$$

$$\chi \;=\; \frac{4i\hbar g_{21}^2 N_o}{\varepsilon_0\sqrt{\pi}(\frac{\omega_p u}{c})}\left[e^{z^2}\pi\left(-1+\sqrt{\frac{1}{z^2}}\,(z)+\text{erfc}(z)\right)\right]$$

Complex susceptibility

- The error function is the integral of normalized gaussian function,

$$\mathrm{erf}(z) = \frac{2}{\sqrt{\pi}} \int_0^z e^{-t^2} dt.$$

$$\mathrm{erfc}(z) = 1 - \mathrm{erf}(z),$$

and the susceptibility is,

$$\chi = \frac{4i\hbar g_{21}^2 N_o \sqrt{\pi}}{\varepsilon_0 (\frac{\omega_p u}{c})} e^{z^2} (1 - \mathrm{erf}(z)).$$

Complex susceptibility

- **Including two photon transition:**

$$\chi(x)dx = \frac{4i\hbar g_{21}^2 N_o \, e^{-x^2} dx}{\varepsilon_0\sqrt{\pi}(\frac{\omega_p u}{c})\left[(\frac{(\gamma_{21}-i\Delta_1)}{\omega_p u/c} - ix) + \frac{\Omega_c^2/4}{(\frac{\omega_p u}{c})((\omega_p-\omega_c)u/c)(\frac{\gamma_{31}-i(\Delta_1+\Delta_2)}{((\omega_p-\omega_c)u/c)} - ix)}\right]}.$$

$$= \frac{4i\hbar g_{21}^2 N_o(\frac{\gamma_{31}-i(\Delta_1+\Delta_2)}{((\omega_p-\omega_c)u/c)} - ix) \, e^{-x^2} dx}{\varepsilon_0\sqrt{\pi}(\frac{\omega_p u}{c})[-x^2 - i(\frac{\gamma_{21}-i\Delta_1}{\omega_p u/c} + \frac{\gamma_{31}-i(\Delta_1+\Delta_2)}{((\omega_p-\omega_c)u/c)})x + \frac{\Omega_c^2/4}{\omega_p(\omega_p-\omega_c)u^2/c^2} + (\frac{\gamma_{21}-i\Delta_1}{\omega_p u/c})(\frac{\gamma_{31}-i(\Delta_1+\Delta_2)}{(\omega_p-\omega_c)u/c})]}.$$

$$\int_{-\infty}^{\infty} \frac{A+Bx}{ax^2+bx+c} \, e^{-x^2} dx,$$

$$A = \frac{4i\hbar g_{21}^2 N_o(\frac{\gamma_{31}-i(\Delta_1+\Delta_2)}{(\omega_p-\omega_c)u/c})}{\varepsilon_0\sqrt{\pi}(\frac{\omega_p u}{c})},$$

$$B = \frac{4i\hbar g_{21}^2 N_o(-i)}{\varepsilon_0\sqrt{\pi}(\frac{\omega_p u}{c})},$$

$$a = -1,$$

$$b = -i\left(\frac{\gamma_{21} - i\Delta_1}{\omega_p u/c} + \frac{\gamma_{31} - i(\Delta_1+\Delta_2)}{((\omega_p-\omega_c)u/c)}\right),$$

$$c = \left(\frac{\gamma_{21} - i\Delta_1}{\omega_p u/c}\right)\left(\frac{\gamma_{31} - i(\Delta_1+\Delta_2)}{((\omega_p-\omega_c)u/c)}\right) + \frac{\Omega_c^2/4}{\omega_p(\omega_p-\omega_c)u^2/c^2}$$

Complex susceptibility

so two complex roots are,

$$
\begin{aligned}
z_{1,2} &= -\frac{i}{2}\left(\frac{\gamma_{21}-i\Delta_1}{\omega_p u/c}+\frac{\gamma_{31}-i(\Delta_1+\Delta_2)}{(\omega_p-\omega_c)u/c}\right) \\
&\pm \frac{i}{2}\left[\left(\frac{\gamma_{21}-i\Delta_1}{\omega_p u/c}-\frac{\gamma_{31}-i(\Delta_1+\Delta_2)}{(\omega_p-\omega_c)u/c}\right)^2-\frac{\Omega_c^2}{\omega_p(\omega_p-\omega_c)u^2/c^2}\right]^{1/2}.
\end{aligned}
$$

The integral can be written as,

$$
\int_{-\infty}^{\infty}\frac{A+Bx}{ax^2+bx+c}\,e^{-x^2}dx = \int_{-\infty}^{\infty}\frac{C_1}{x-z_1}e^{-x^2}dx-\int_{-\infty}^{\infty}\frac{C_2}{x-z_2}e^{-x^2}dx,
$$

Mathematica gives,

$$
\begin{aligned}
\chi &= C_1\,e^{-z_1^2}\left(-\pi\,(-i\,\mathrm{erf}(iz_1))+\mathrm{Log}(-1/z_1)+\mathrm{Log}(z_1)\right) \\
&\quad + C_2\,e^{-z_2^2}\left(-\pi(-i\,\mathrm{erf}(iz_2))+\mathrm{Log}(-1/z_2)+\mathrm{Log}(z_2)\right)
\end{aligned}
$$

Complex susceptibility

the constants C_1 and C_2 are,

$$C_1 = \frac{A + Bz_1}{a(z_1 - z_2)} = \frac{2i\hbar g_{21}^2 N_o}{\varepsilon_o \sqrt{\pi}(\frac{\omega_p u}{c})} \left(\frac{-(1-d)}{i} \right),$$

$$C_2 = \frac{A + Bz_2}{a(z_1 - z_2)} = \frac{2i\hbar g_{21}^2 N_o}{\varepsilon_o \sqrt{\pi}(\frac{\omega_p u}{c})} \left(\frac{1+d}{i} \right).$$

where, $$d = \frac{i}{(z_1 - z_2)} \left[\frac{\gamma_{21} - i\Delta_1}{\omega_p u/c} - \frac{\gamma_{31} - i(\Delta_1 + \Delta_2)}{(\omega_p - \omega_c)u/c} \right],$$

Logarithm of any complex number is,

$$\mathrm{Log(z)} = \log|z| + i\mathrm{Arg}(z).$$

and,

$$\mathrm{Log}\left(-\frac{1}{z_1}\right) + \mathrm{Log}(z_1) = i\pi,$$

$$\mathrm{Log}\left(-\frac{1}{z_2}\right) + \mathrm{Log}(z_2) = -i\pi,$$

Complex susceptibility

Defining the function,

$$
\begin{aligned}
s_1 &= -\text{sgn}[\text{Im}(z_1)] = -1, \qquad \text{Im}(z_1) > 0 \\
s_2 &= -\text{sgn}[\text{Im}(z_2)] = 1, \qquad \text{Im}(z_2) < 0
\end{aligned}
$$

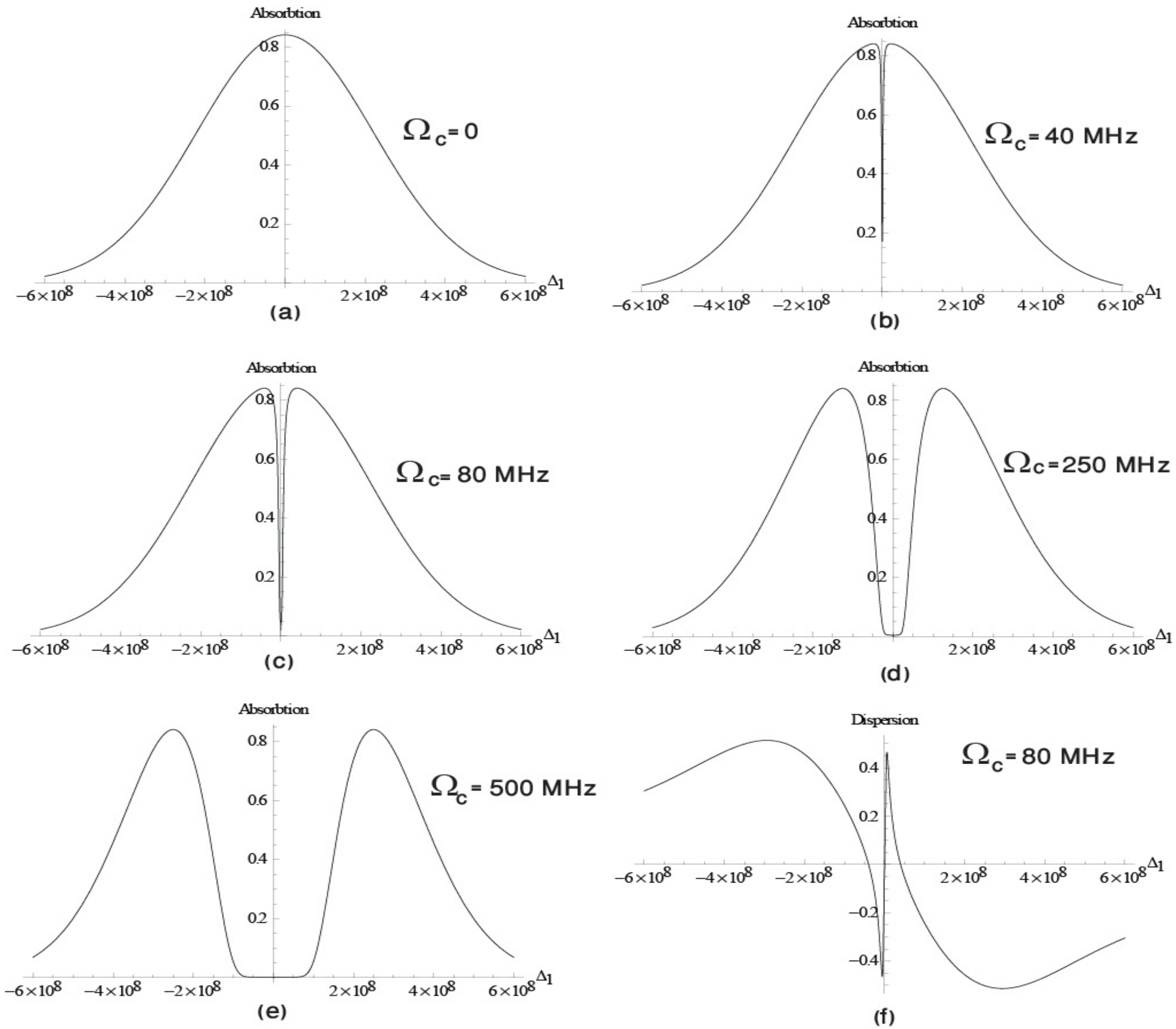
where sgn is the signum function defined as,

$$
\text{sgn}(x) = \begin{cases} -1 & \text{if } \; x < 0 \\ 0 & \text{if } \; x = 0 \\ 1 & \text{if } \; x > 0 \end{cases}
$$

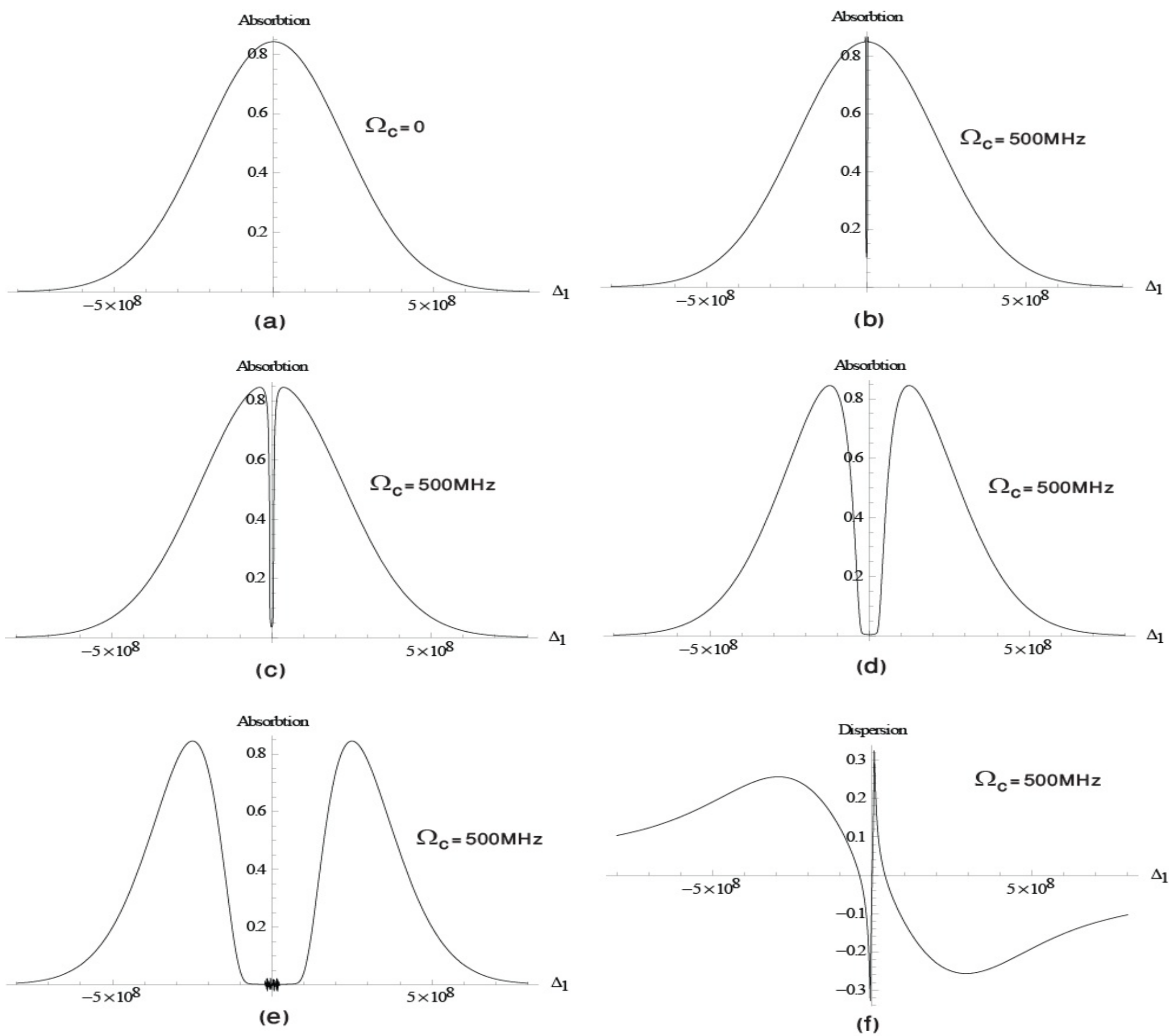
and the susceptibility expression becomes,

$$
\chi = \frac{2i\hbar c g_{21}^2 N_o \sqrt{\pi}}{\varepsilon_o \omega_p u} \left[(1-d) s_1 e^{-z_1^2} (1 - s_1 \text{erf}(iz_1)) + (1+d) s_2 e^{-z_2^2} (1 - s_2 \text{erf}(iz_2)) \right].
$$

Results


Absorbtion
$\Omega_c = 0$
(a)
Absorbtion
$\Omega_c = 40$ MHz
(b)
Absorbtion
$\Omega_c = 80$ MHz
(c)
Absorbtion
$\Omega_c = 250$ MHz
(d)
Absorbtion
$\Omega_c = 500$ MHz
(e)
Dispersion
$\Omega_c = 80$ MHz
(f)
Δ_1

Results


Absorbtion
$\Omega_c = 0$
(a)
Absorbtion
$\Omega_c = 500MHz$
(b)
Absorbtion
$\Omega_c = 500MHz$
(c)
Absorbtion
$\Omega_c = 500MHz$
(d)
Absorbtion
$\Omega_c = 500MHz$
(e)
Dispersion
$\Omega_c = 500MHz$
(f)
Δ_1
-5×10^8
5×10^8

Applications of EIT

- Fundamental and commercial applications of EIT in atomic physics and quantum optics include,
- Lasing without inversion,
- Reduction of the speed of light,
- Quantum memory,
- Optical switches,
- All optical wavelength converters for telecommunications,
- Quantum information processing.