A Better 2-D Mechanical Energy Conservation Experiment

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A variety of simple classical mechanics energy conservation experiments are used in teaching laboratories. Typical one-dimensional (1-D) setups may involve falling balls or oscillating springs. Many of these can be quite satisfying in that students can confirm—within a few percent—that mechanical energy is conserved. Students generally have little trouble identifying discrepancies such as the loss of a few percent of the gravitational potential energy due to air friction encountered by a falling ball. Two-dimensional (2-D) systems can require more sophisticated analysis for higher level laboratories, but such systems often incorporate complicating components that can make the exercise academically incomplete and experimentally less accurate. The following describes a simple 2-D energy conservation experiment based on the popular "Newton's Cradle" toy that allows students to account for nearly all of the mechanical energy in the system in an academically complete analysis.

The problem

The elementary physics curriculum often has treated one- and two-dimensional dynamics before presenting energy conservation. Rotational motion usually follows these topics in the curricular sequence. When energy conservation is considered in two-dimensional arenas, student laboratories quite commonly resemble either commercially available "Roller Coaster" experiments[1] or simple "Ball in Track" experiments.[2] Experiences in our teaching laboratories find that students must deal with ad hoc curricular considerations as well as appreciable energy losses when performing quantitative studies using such equipment.

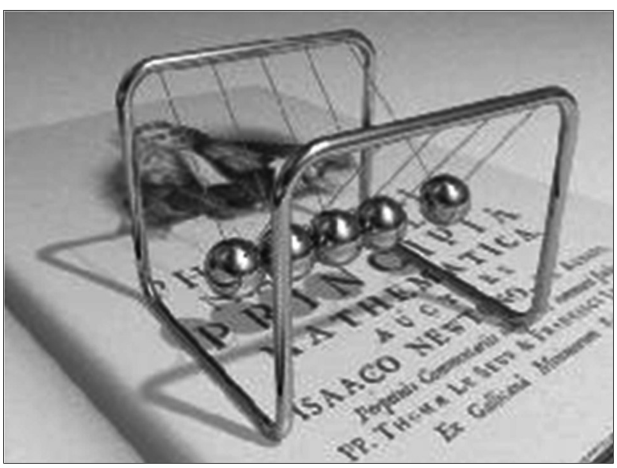

Fig. 1. Newton's Cradle.

The curricular problem with many mechanical energy conservation laboratories that involve rolling components lies in the fact that, in most cases, rotational motion has not been introduced into the curriculum when these laboratories are performed. The fraction of kinetic energy in a rolling ball, for example, must then be brought into student analysis in an ad hoc fashion. This is both quantitatively unsatisfying to the student and pedagogically unsound.

Furthermore loss mechanisms in many 2-D systems are manifest in the fact that most such experiments are not quiet, thus indicating at least one dissipative sink: sound. Air resistance and even heating of tracks and carts can likewise result in nonconservative inclusions. Indeed, when allowed to oscillate between final and initial positions, most such laboratory experiments lose as much as half of their amplitude in three or four cycles.

A "Newton's Cradle" solution

We have developed a 2-D student laboratory that introduces translational mechanical energy conservation in a manner that substantively avoids both of the pitfalls described above. General physics students in a sophomore-level university course regularly using this equipment often account for energy losses on the order of 2%—far less than previous setups used in our laboratory. In addition they need not consider topics beyond what they have already covered in class.

Newton's Cradle (Fig. 1) is a popular desktop toy often used to demonstrate energy and momentum conservation. In the ideal cradle, collisions involving balls of equal mass on separate pendulums are quite elastic, and intriguing motion

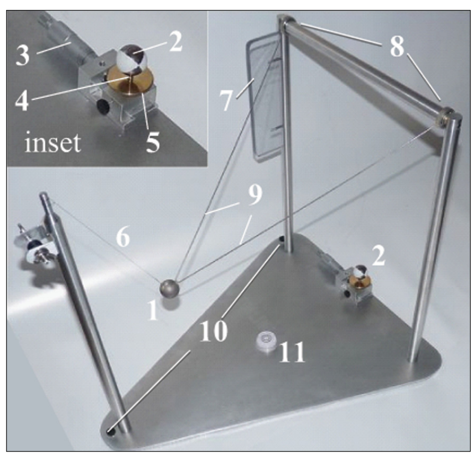

Fig. 2. Experimental setup. Components include: pendulum ball (1); struck ball (2); lateral micrometer adjustment (3); "golf tee" (4); height adjustment dial (5); thread (6); protractor (7); bearings (8); stainless rods (9); leveling screws (10); and level (11).

can result when balls are set in motion. In our laboratory version, only two balls of nearly equal mass are involved. A ball is welded to two thin stainless rods that are attached to bearings so as to form a relatively low-loss pendulum (Fig. 2), wherein this pendulum ball follows a circular arc and is not allowed to twist as it swings through the arc.[3] The pendulum ball is held at its highest position by a thread strung through a tiny eyelet on the ball. To start the ball in motion, a match is used to sever the thread.[4]

An untethered struck ball is placed on a tiny machined "golf tee" such that a collision occurs when the pendulum ball is exactly at the bottom of its swing. The collision that results when the pendulum ball arrives at the bottom of its arc causes the struck ball to fly off of its perch in a "knuckle ball" that involves no spinning. The "golf tee" is machined to be as small as possible to hold the struck ball. Because it is small, any torque delivered to the struck ball as it leaves the tee is minimized. With these precautions, rotational energy can be essentially eliminated from consideration in the subsequent analysis.

In order to achieve pure translational motion of the projectile ball, the balls must touch at the exact bottom of the swinging ball's arc and in such a way that the line connecting the center of the two balls at collision is in both the plane of the arc and in a horizontal plane. To achieve this precision: i) leveling screws and a small circular level are used to assure that the collision point is at the bottom of the swing; ii) a micrometer adjusts the position of the "golf tee" in a direction perpendicular to the plane of the pendulum's arc to ensure that the struck ball travels in a path coplanar with that arc; and iii) a screw adjusts the height of the "golf tee" to insure that the struck ball leaves the tee along an initially horizontal path.

The struck ball flies off of the lab bench and arrives at the floor, where its position is recorded when it strikes a piece of carbon paper, leaving a small dot on a piece of paper lying on the floor.

Parameters measured before the experiment is performed are L, the length of the pendulum, and h, the height of the collision point above the floor level. Variables measured when the experiment is performed include θ, the angle from the vertical above which the swinging ball's pendulum is positioned, and d, the horizontal distance the struck ball travels before hitting the floor. The linear distances are measured with a meterstick and the angle is determined from lines scribed on a protractor-like attachment at 15°, 30°, 45°, and 60°. Components of the setup are identified in Fig 2.

If the zero of mechanical energy is taken as the situation when both balls are motionless and in contact at the bottom of the pendulum's arc, the initial energy of the system, E_0, is just the potential energy of the motionless pendulum ball when the pendulum is pulled an angle θ above its lowest position. Thus,

$$E_0 = mgL(1 - \cos\theta). \tag{1}$$

When the balls collide, if the collision is elastic, this energy is fully transferred as kinetic energy to the struck ball as E_1, where

$$E_1 = \frac{1}{2}mv^2, \tag{2}$$

and v is the horizontal velocity of the struck ball. It is presumed here that m is the identical mass of both of the balls. The struck ball will leave the apparatus horizontally at a height h above the floor and will arrive at the floor at a time t, where simple kinematics allows one to write

$$h = \frac{1}{2}gt^2. \tag{3}$$

Exploiting energy conservation as expressed in Eq. (1) and Eq. (2) and using the time determined from Eq. (3), one can predict d, the horizontal distance the struck ball will travel before arriving at the floor, as

$$d = 2\sqrt{hL(1-\cos\theta)}. \tag{4}$$

The accuracy to which mechanical energy is conserved in this experiment can be ascertained by comparing the predicted distance d_p, as determined using the initial condition (the angle θ), to the measured distance d_m, directly determined upon performing the experiment.

Results

In student laboratories, the four separate angles above are used in four test runs. The predicted and measured values of d can then be compared. A typical data set is reproduced as Table I.

Table I. Typical student result table.

θ	measured d_m	predicted d_p	% error
15°	1.03 m	1.06 m	-2.9%
30°	1.41 m	1.43 m	-1.4%
45°	1.62 m	1.65 m	-1.9%
60°	1.89 m	1.95 m	-3.2%

Note that while there is some variance in the values of d as predicted and measured, all measured values are slightly (typically 2%) less than the predicted values. Students are asked to speculate as to the sinks of energy that result in these slightly smaller measured values. The sound of the "click" at the collision, slight movement of the swinging ball after the collision, and air resistance are frequently mentioned. Before performing the experiment, we distinctly decorate the struck ball with white correctional fluid (i.e., Wite-Out™) markings to make any rotational motion obvious. By eliminating rotation through adjustment of struck ball initial positioning, it then is straightforward to ensure that the collision involves only translational motion.

Upon completing the laboratory exercise, students are satisfied that they have created and observed a fully translational collision and have quantitatively demonstrated the near-elasticity of that collision. The setup was constructed in the department instrument shop at minimal cost, with an inexpensive micrometer, two bearings, and a small circular level being the only separately purchased items.

Acknowledgments

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References

1. See, for example, "Roller Coaster" demos and student experiments available at www.pasco.com.
2. See, for example, "Ball in Track" demo #1M40.30 at demoroom.physics.ncsu.edu/.
3. Thin rods were used because attempts to construct the setup using two threads to create the pendulum were not successful. With threads, after the collision the pendulum ball almost always twists back and forth as the two threads on either side of the ball oscillate out-of-plane. This twisting added an intolerable energy loss.
4. The thread and match are used in starting the pendulum to ensure that no additional momentum is imparted to the ball as it begins its swing. This technique was pioneered by Foucault as described in Amir D. Acsel's biography *Pendulum: Léon Foucault and the Triumph of Science* (Atria Books, New York, 2003).

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