Phase Sensitive Faraday Rotation in TERBIUM GALLIUM GARNET crystal and various Diamagnetic liquid Samples

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Roll. No. 0702

M.Phil (2007-2009)

Outlines

- Magneto optics
- Polarization of light
- Jones Calculus
- Faraday Rotation
- Lock-in Amplifier ; PSD
- Why PSD in Faraday rotation?
- Schematic of the experiment.
- Results
- Determination of V using higher harmonics.
- References

Magneto Optics

Historical background

- 1813 Morrichini
- 1814 Faraday
- 1826 S. H. Christie
- 1834 Faraday
- 1844 " Magnetic force and light were proved to have relation to each other".

Magneto Optics (cont…)

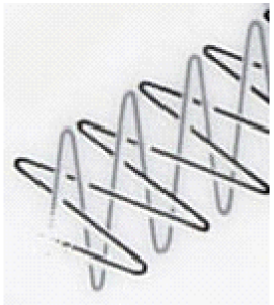
Optical radiation ⟷ Interaction ⟷ Magnetic medium or Optically inactive medium placed in Magnetic field

- Faraday rotation
- Kerr effect

Polarization of light

1.Linear polarization

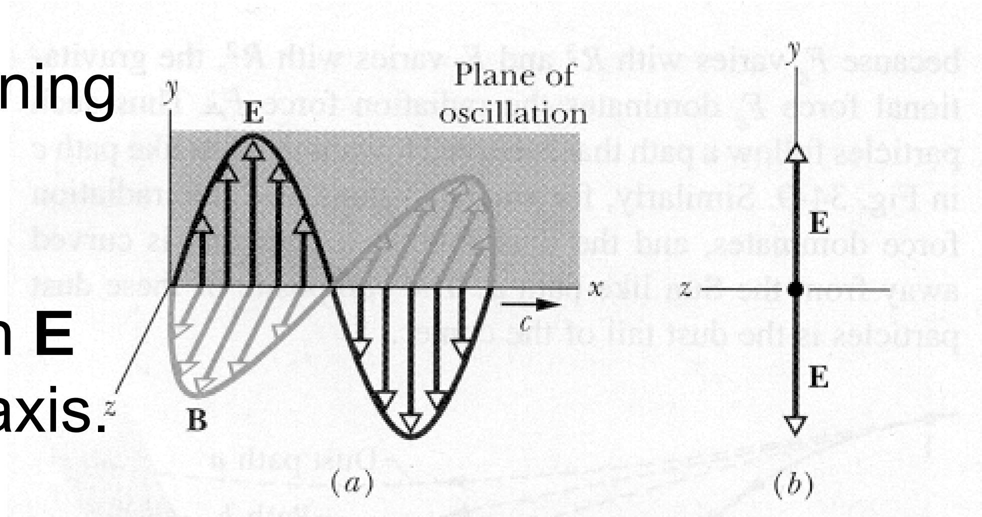
- Orientation of **E** field remains constant, magnitude and sign varies.



Unpolarized light

- Plane of oscillation, containing **E** and **K**

- Electromagnetic wave with **E** field oscillating parallel to y axis.

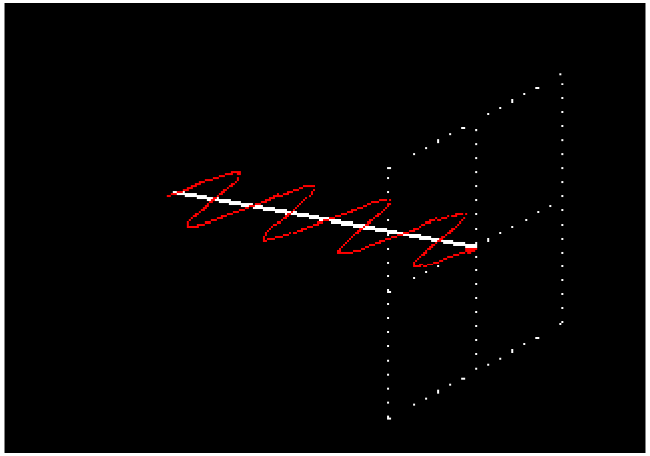


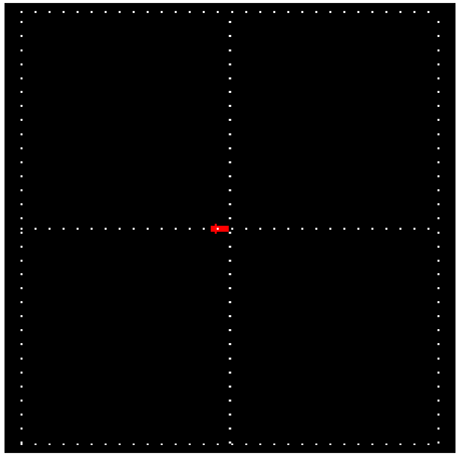
Linearly polarized

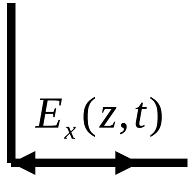
Linear polarization (cont…)

Horizontally polarized light propagating in z direction

$$E_x(z,t) = \hat{i}\, E_{ox} \cos(kz - \omega t)$$



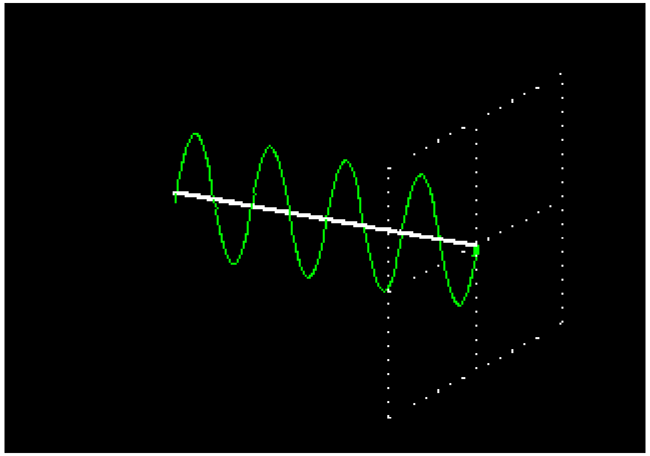


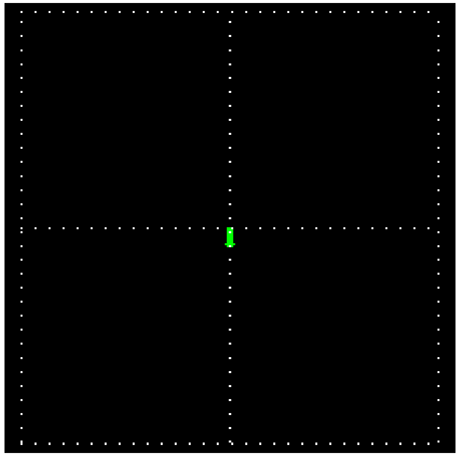


Vertically polarized at phase difference ε,

$$E_y(z,t) = \hat{j}\, E_{oy} \cos(kz - \omega t + \varepsilon)$$

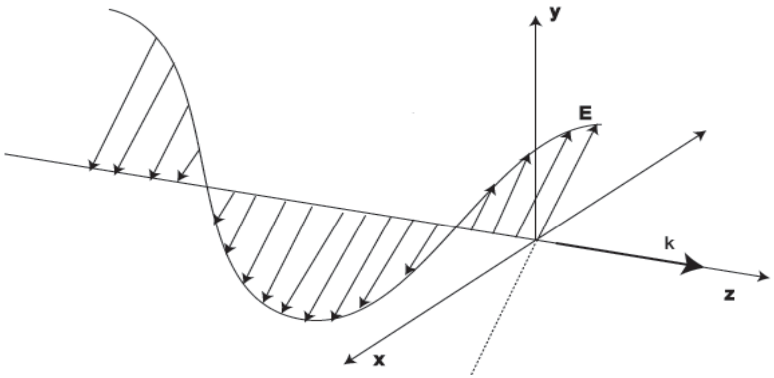
Linear polarization (cont…)

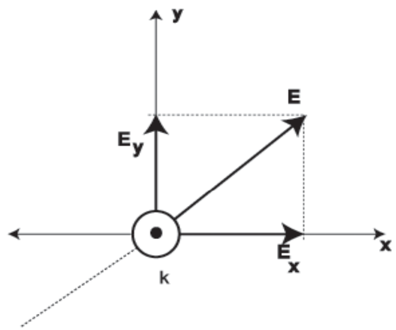




Superposition of the two

$$E(z,t) = \hat{i}\, E_{ox} \cos(kz - \omega t) + \hat{j}\, E_{oy} \cos(kz - \omega t + \varepsilon)$$





2. Circular polarization

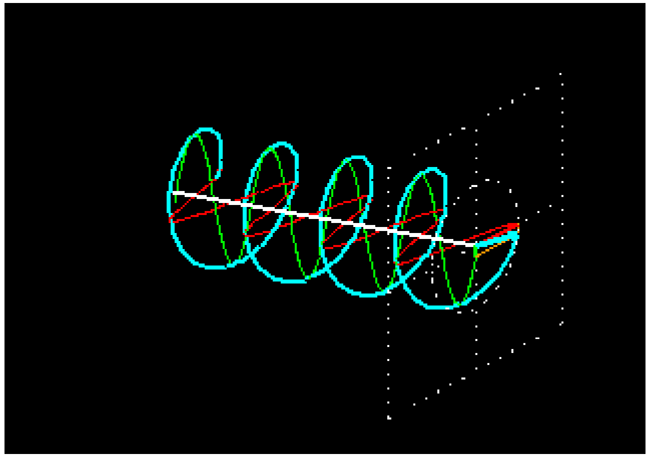
- Two orthogonal waves have equal amplitudes.
- Relative phase shift of 90^{o}
- Direction of **E** is time varying, magnitude remains constant.

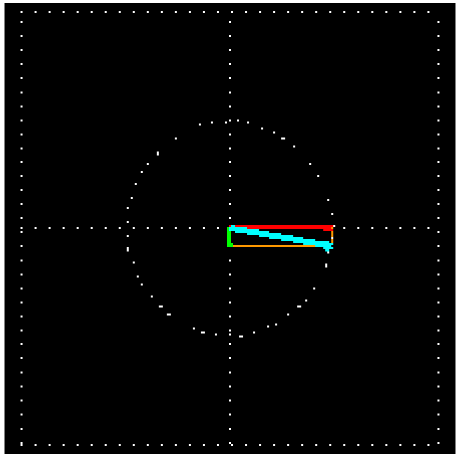
a) Right Circularly polarized light

- relative phase difference of $-90^{o} + 2m\pi$
- rotating clockwise

$$E = E_{o}[\hat{i}\cos(kz - \omega t) + \hat{j}\sin(kz - \omega t)]$$

2. Circular polarization (cont..)



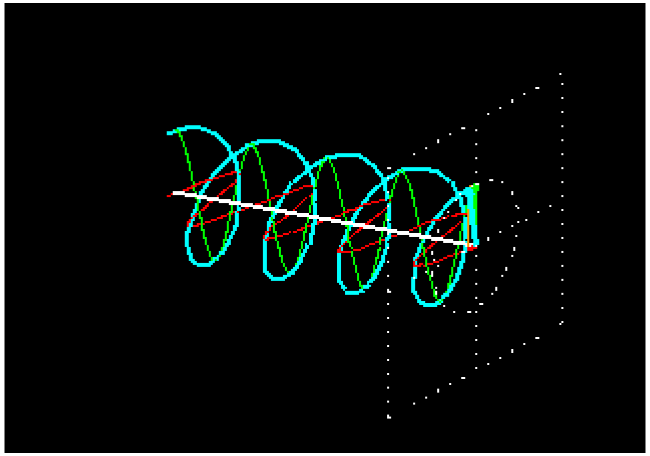


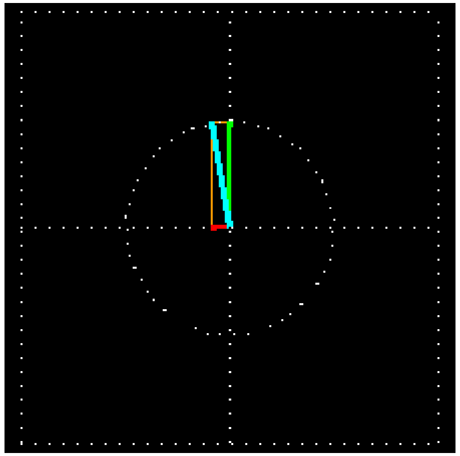
Right circular

- Left circularly polarized, phase shift of $90^{\text{o}} + 2m\pi$
- rotating anti clockwise

$$E = E_o[\hat{i}\cos(kz - \omega t) - \hat{j}\sin(kz - \omega t)]$$

2. Circular polarization (cont..)

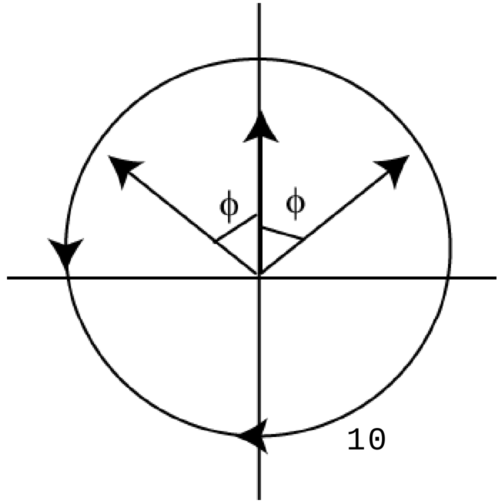




Left circular

- Linearly polarized is sum of R.C.P and L.C.P

$$E(z,t) = 2E_o \hat{i} \cos(kz - \omega t)$$



Jones calculus

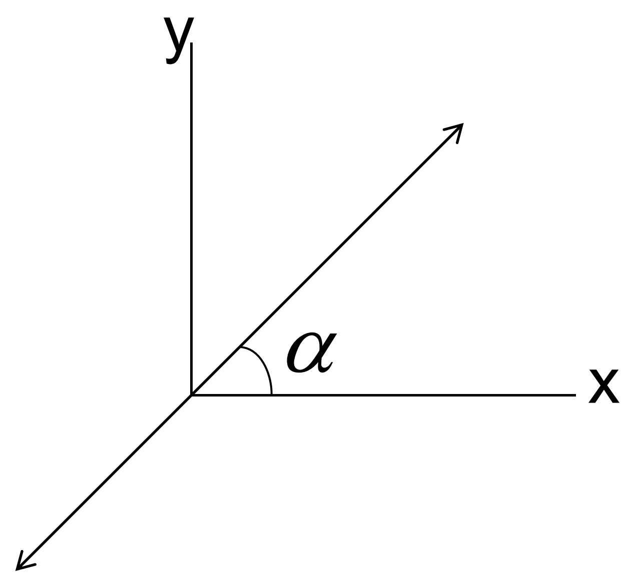
- R. Clark Jones in 1941
- For perfectly polarized light

$$E(z,t) = \hat{i}\, E_{ox} \cos(kz - \omega t) + \hat{j}\, E_{oy} \cos(kz - \omega t + \varepsilon)$$

$$E(z,t) = \begin{pmatrix} E_{ox} \\ E_{oy} e^{i\varepsilon} \end{pmatrix}$$

- For linearly polarized light

$$E(z,t) = \begin{pmatrix} \cos\alpha \\ \sin\alpha \end{pmatrix}$$



Jones calculus (cont…)

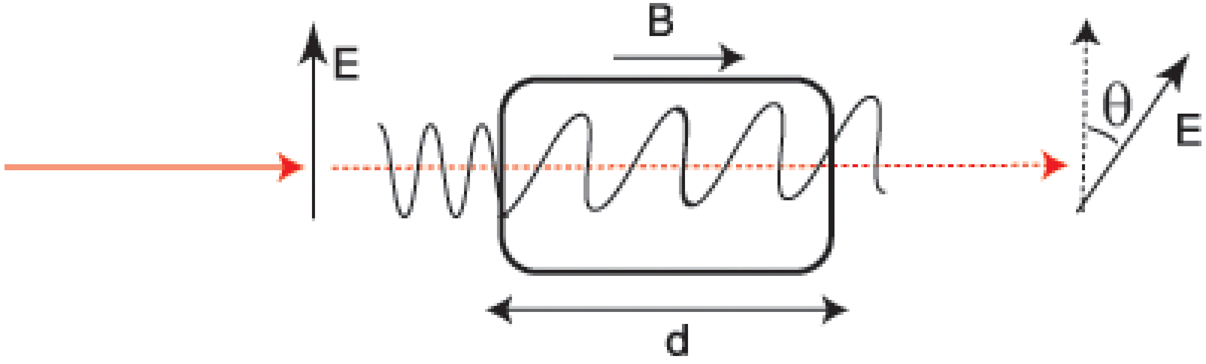
- Incident Jones vector E_i is related to transmitted jones vector E_t through matrix, J

$$E_t = JE_i$$

- For beam passing through a series of optical elements

$$E_t = J_n \dots J_3 J_2 J_1 E_i$$

Faraday rotation



- Linearly polarized monochromatic light while transmitting through an optically inactive material, under the influence of an axial magnetic field, is rotated by an angle θ.
- Is non-reciprocal.

Faraday rotation (cont…)

- For uniform B field

$$\theta = VBd$$

- For non uniform B field

$$\theta = V\int_{0}^{d} B(z).dz$$

Where,

d= length of the sample

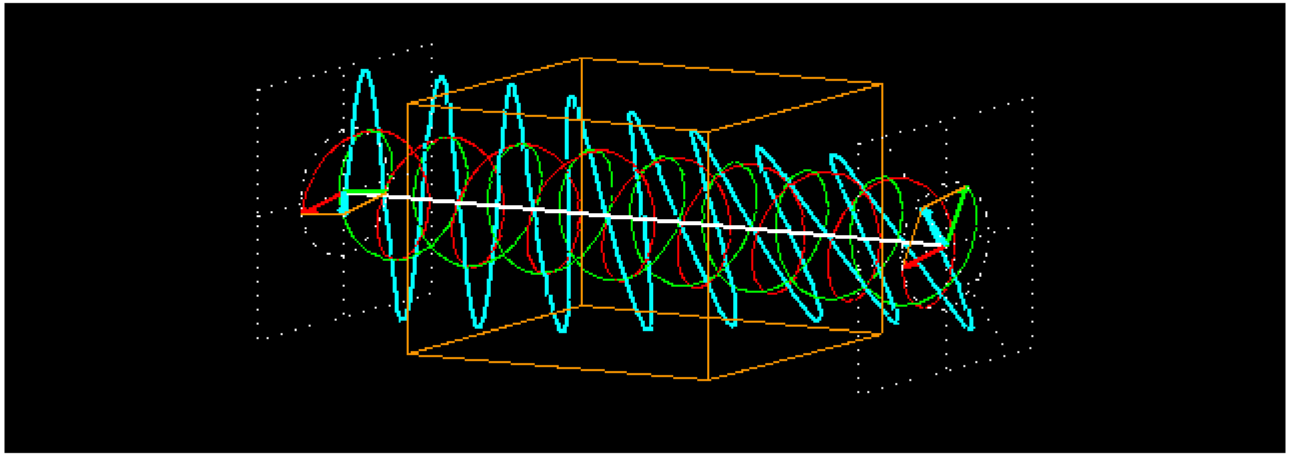
V= material parameter called Verdet constant,

- is a function of wave length of light.
- of the order of micro rad /G cm.

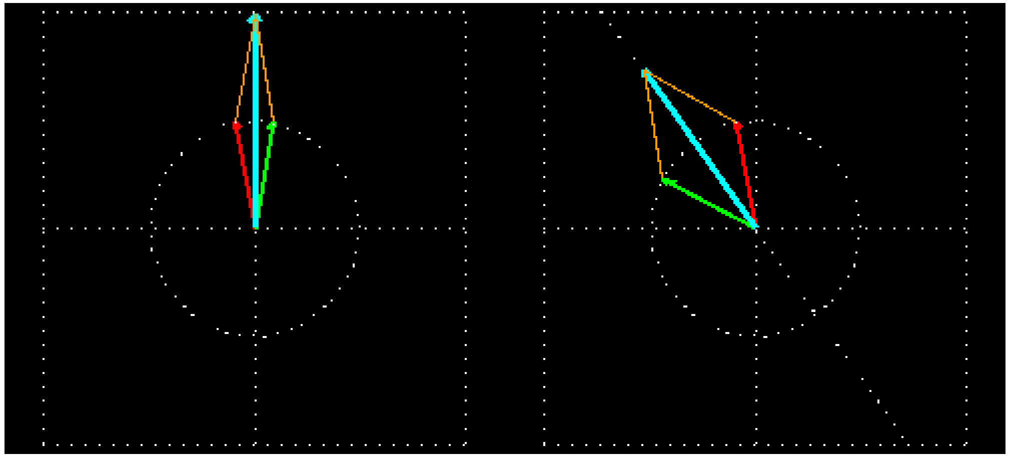
Faraday rotation (cont…)

Induced circular birefringence

$$n_r \neq n_l$$



The plane of polarization of plane-polarized light gets rotated



Phase Sensitive Detection (PSD)

$$V = 10\text{nV sin wave at 20 kHz}$$

- Amplifier

$$\text{input noise} = 4\ \text{nV}/\sqrt{\text{Hz}}$$

$$\text{Q} = 1000 \qquad \text{Bandwidth} = 1\text{MHz}$$

$$V_o = 1000 \times 10\text{nV} = 10\mu\text{V}$$

$$V_{noise} = 4\text{mV}$$

Phase Sensitive Detection (cont…)

- Band pass filter

$$\text{at } 50\text{ kHz} \qquad Q = 1000$$

$$V_{noise} = \sqrt{500}(4\text{nV})(1000)$$
$$= 89\mu\text{V}$$

- PSD

$$\text{Band width} = 0.125\text{ Hz}$$

$$V_{noise} = 1.4\mu\text{V}$$

Lock in Amplifier

- A lock-in amplifier amplifies a small frequency band around a certain reference frequency.
- A lock-in can be used to

Measure sinusoidal voltage amplitudes and phase

Measure noise around a certain frequency.

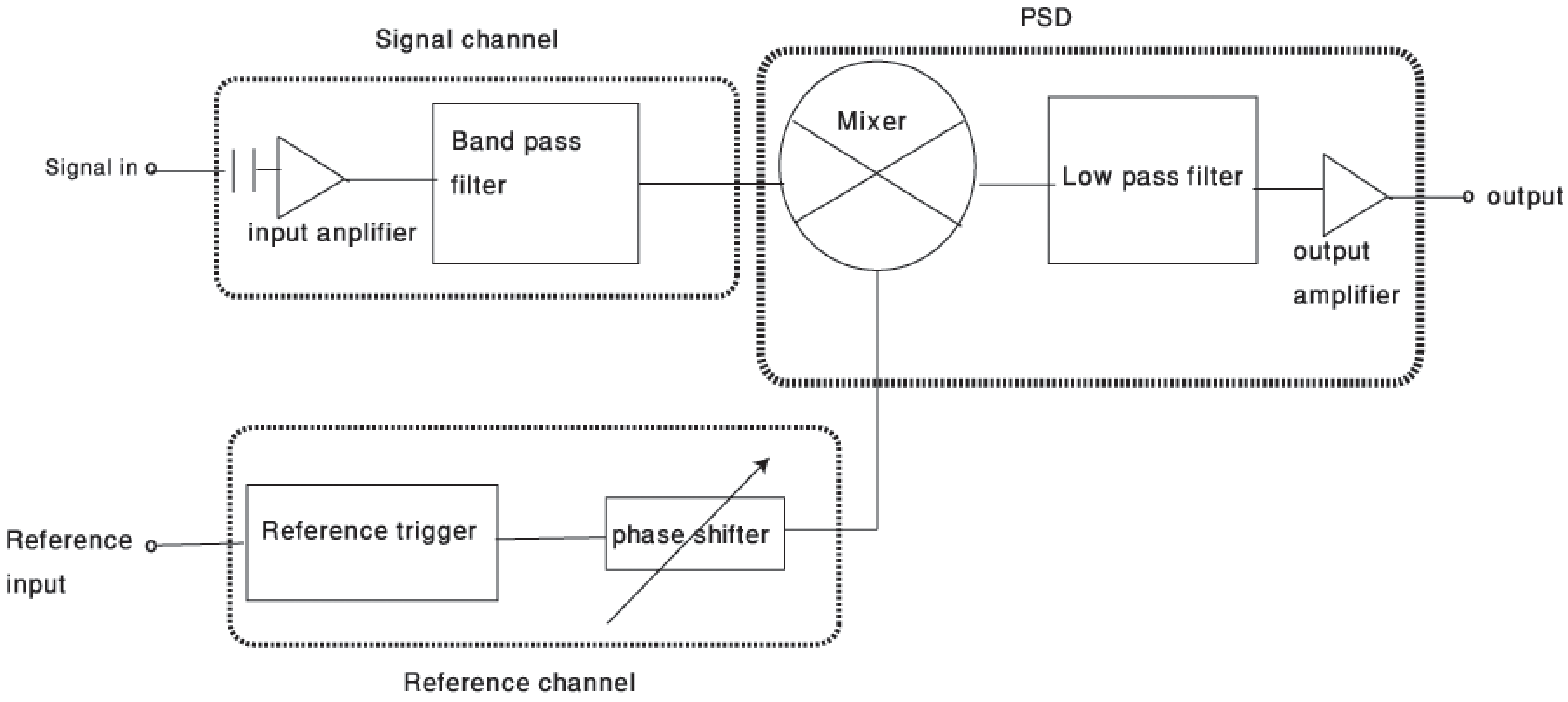
- Consists of

Signal channel

Reference channel

PSD : Heart of lock-in amplifier

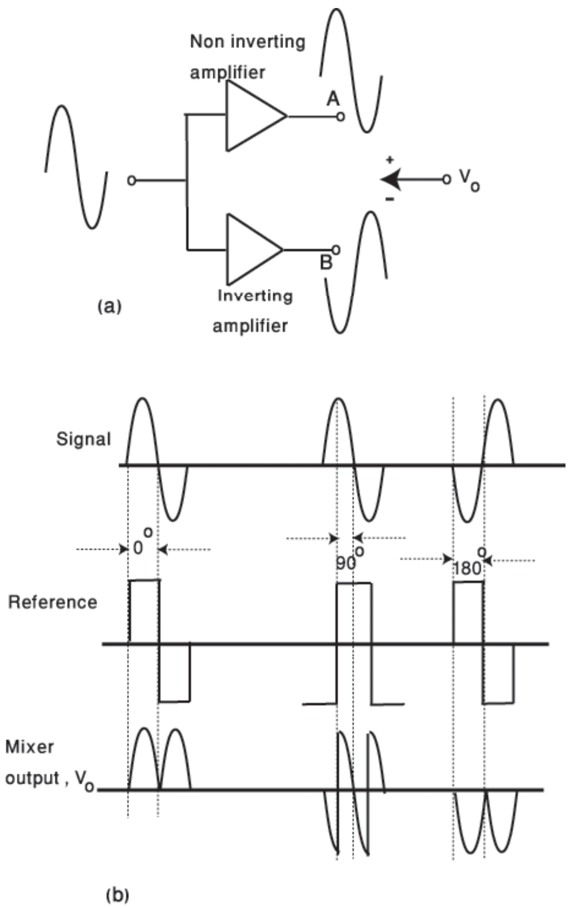
Lock in Amplifier (cont…)



Lock in amplifier, block diagram

Lock in Amplifier (cont…)

Mixer



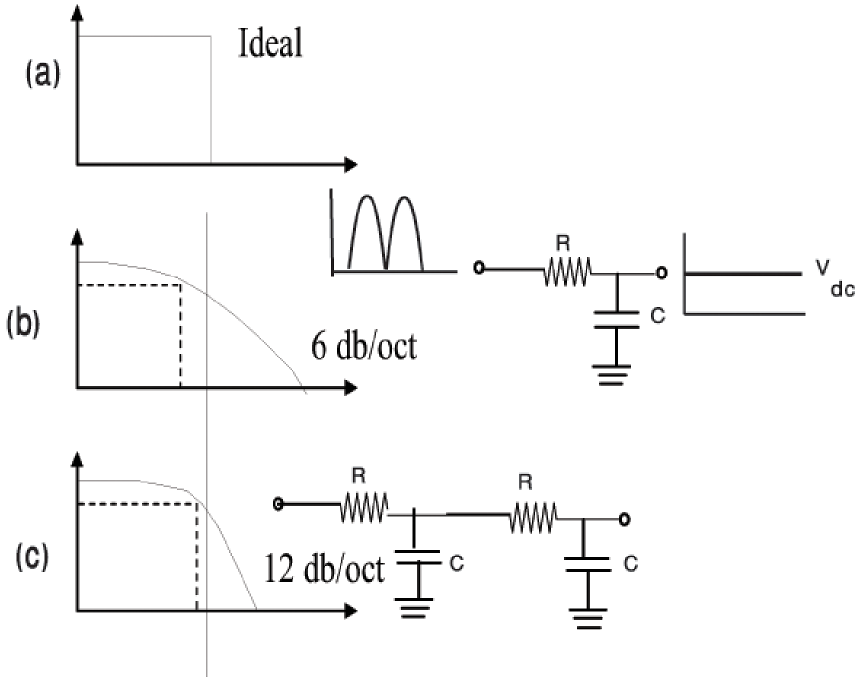
$$V_{in} = A\sin(\omega t)$$

$$V_{ref} = B\sin(\Omega t + \varphi)$$

$$V_o = \frac{AB}{2}[\cos(\varphi) - \cos(2\omega t + \varphi)]$$

Lock in Amplifier (cont…)

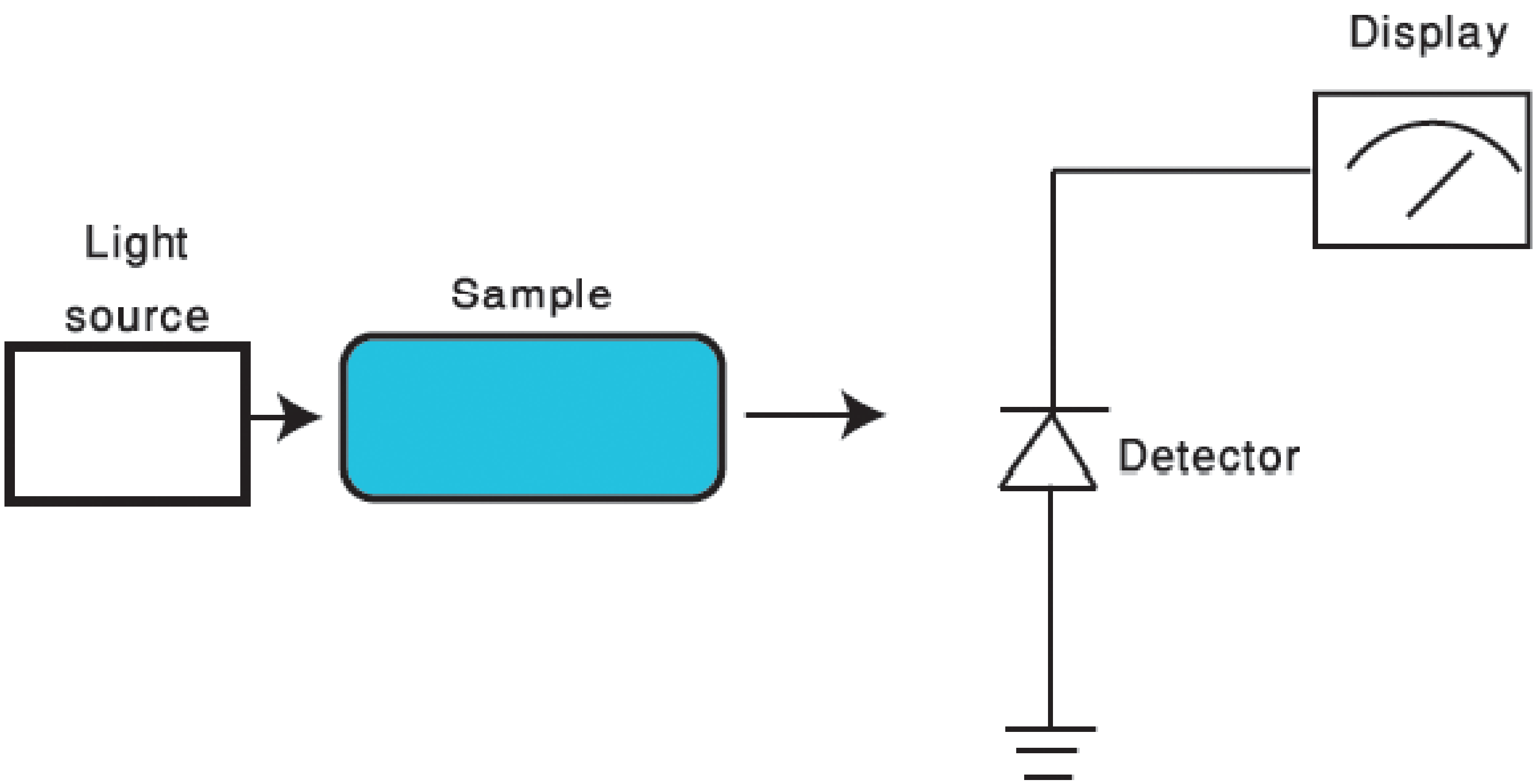
Low pass filter



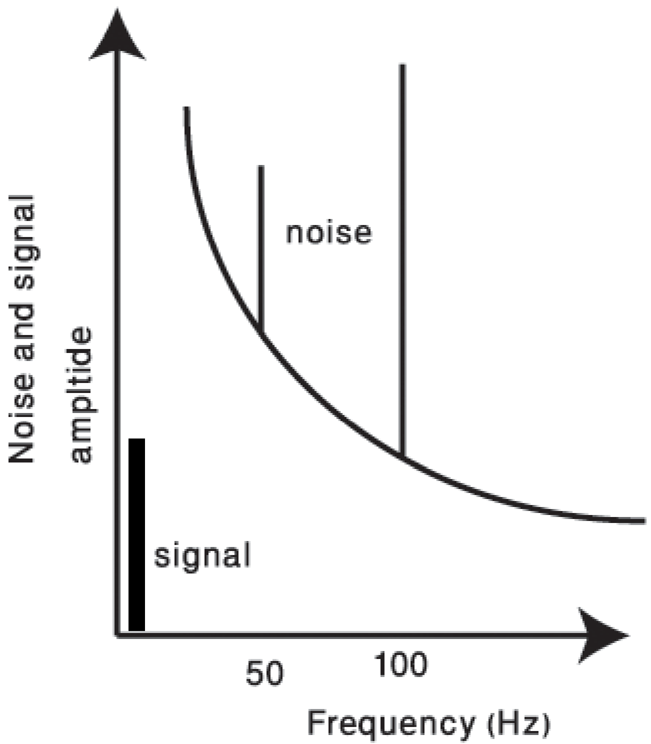
$$V_o = \frac{AB}{2}\cos(\varphi)$$

Noise close and closer to the reference frequency is removed.

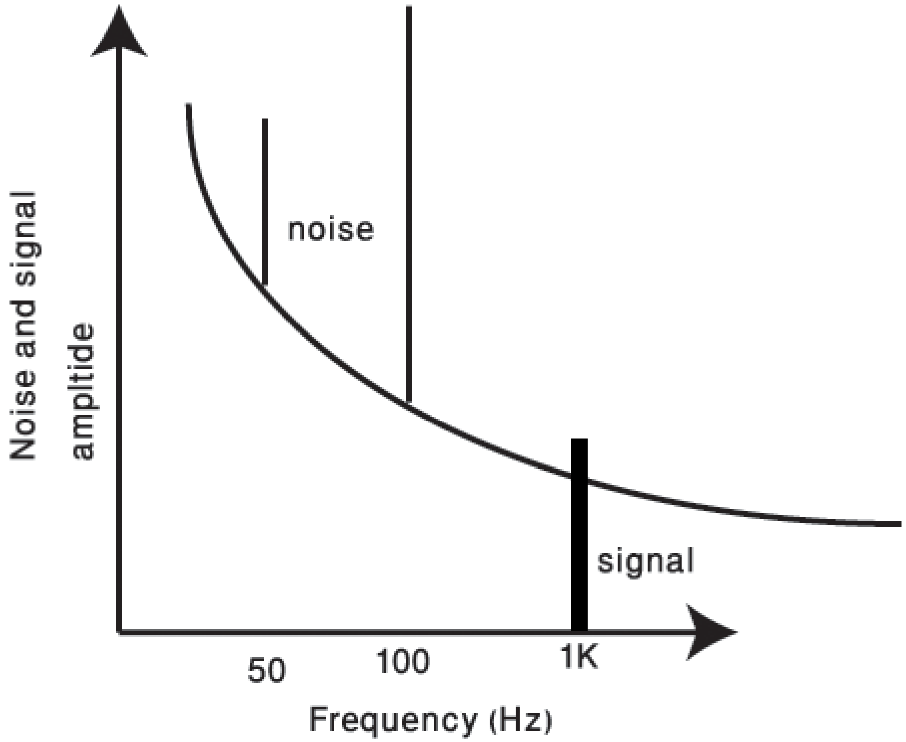
Why PSD in Faraday rotation?



Why PSD in Faraday rotation? (cont…)

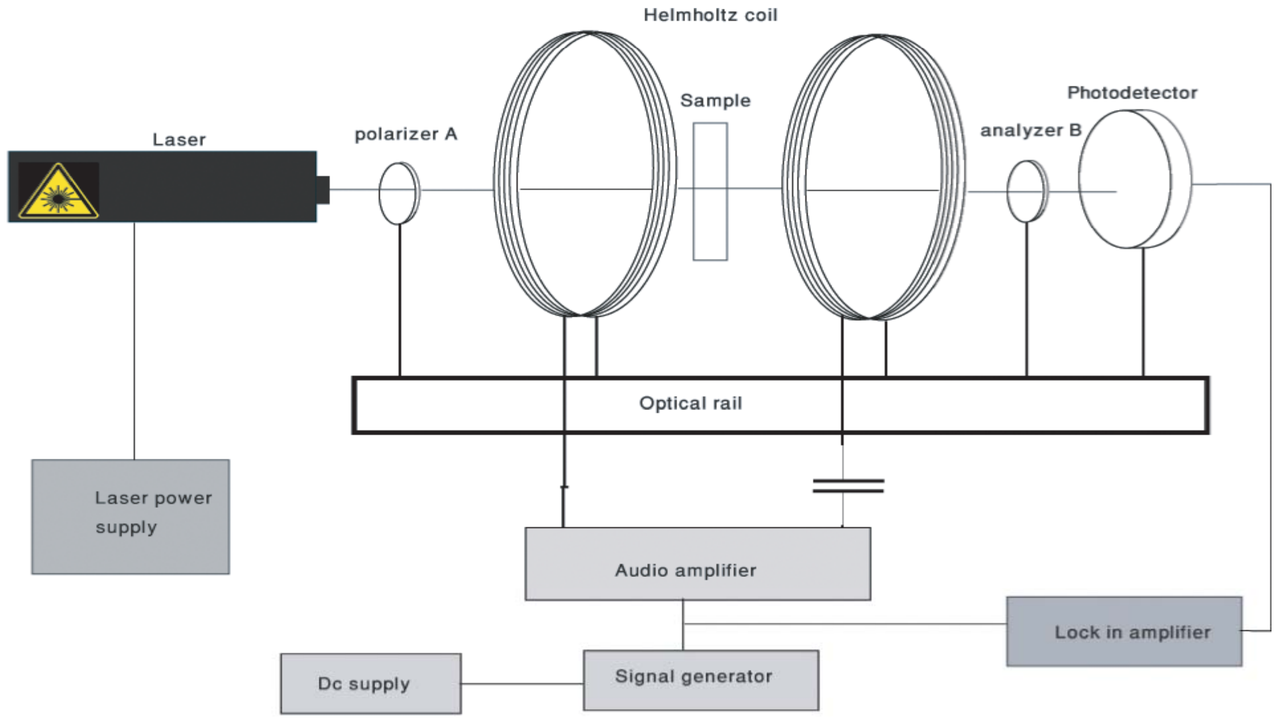


Noise and signal amplitude as a function of frequency.



Modulating the signal to a region of low noise.

Schematic of the experiment

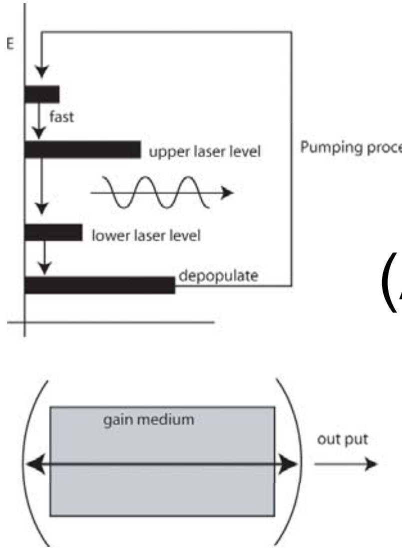


- Light source
- Source of magnetic field
- Detecting devices

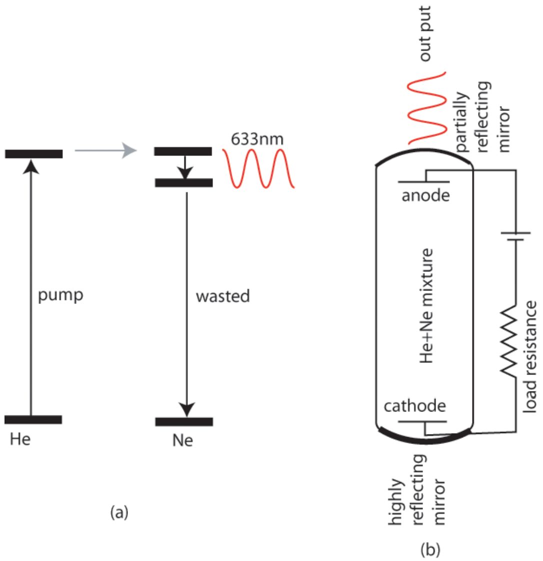
Schematic of the experiment (cont…)

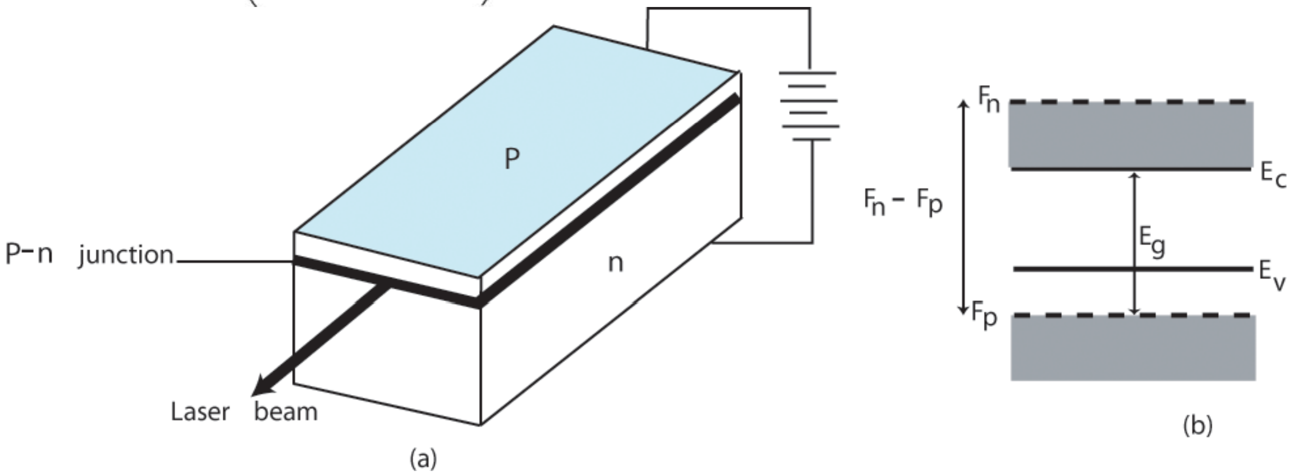
Light source

- Laser



(A)

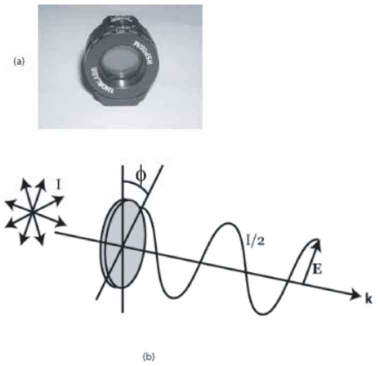




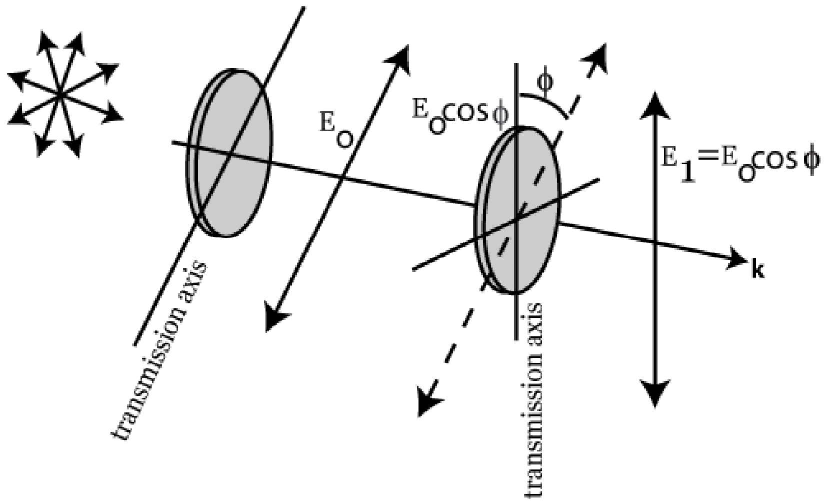
(B) He-Ne gas laser

(C) AlGaN diode laser

Schematic of the experiment (cont…)



Linear polarizer



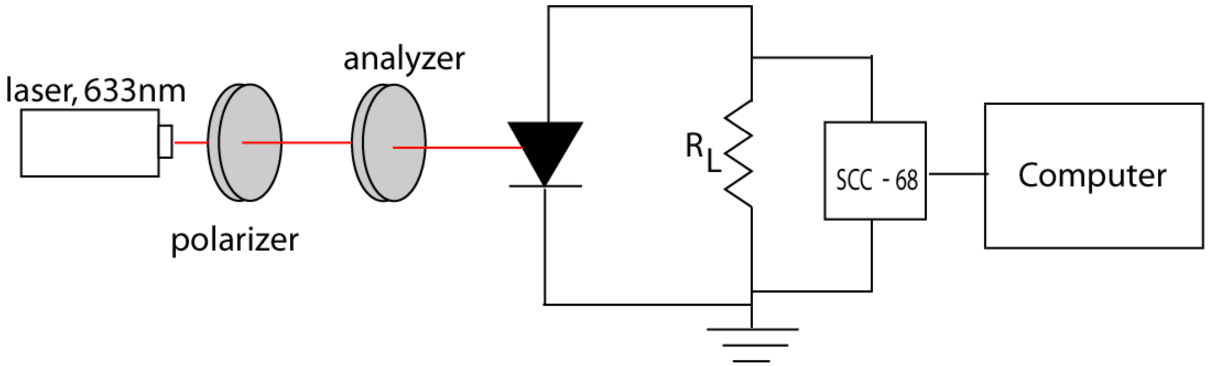
Malus's law

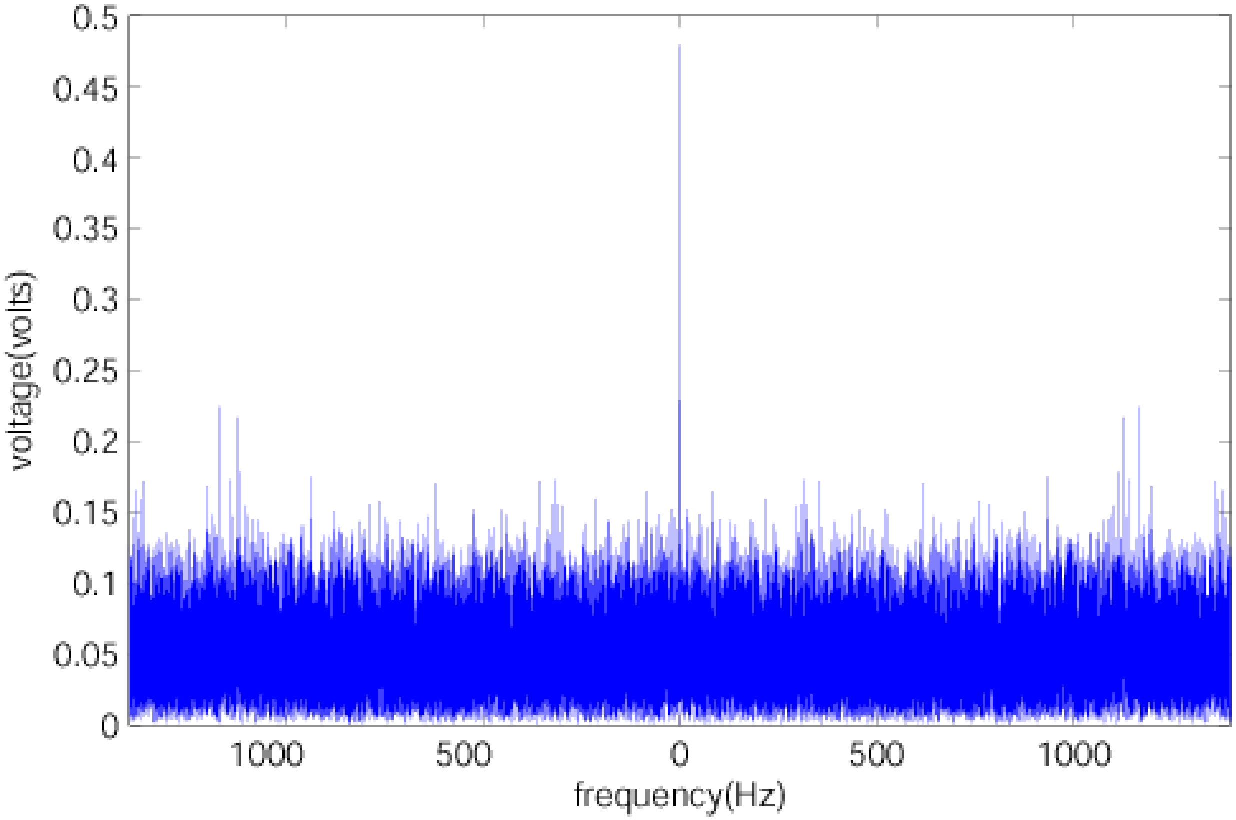
Source of magnetic field

- Dc source

large, expansive, bulky water cooled electromagnets.

Schematic of the experiment (cont…)





Schematic of the experiment (cont…)

- Ac source

Low magnetic field is required.

Signal of interest can be extracted successfully through PSD.

Solenoid, Helmholtz coil.

Helmholtz coil

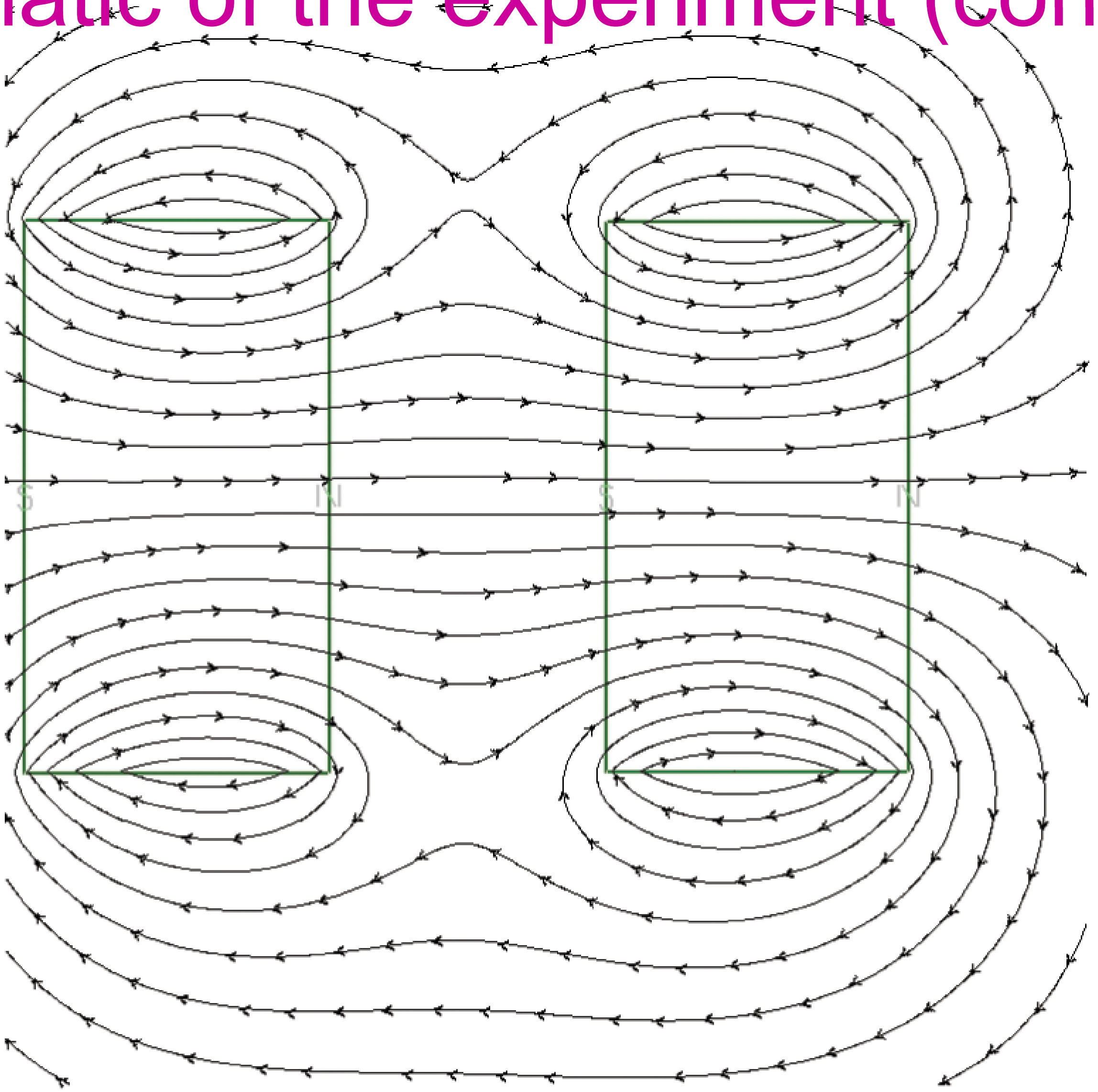
Two identical coils with separation equal to their common radius.

$$B = B_o(1 + c_4 x^4 + c_6 x^6 + ...)$$

In superposition

$$B = (\frac{4}{5})^{3/2} \mu_o \frac{N}{a} i$$

Schematic of the experiment (cont…)



Magnetic lines of force of our Helmholtz coil drawn in Vizimag

Schematic of the experiment (cont…)

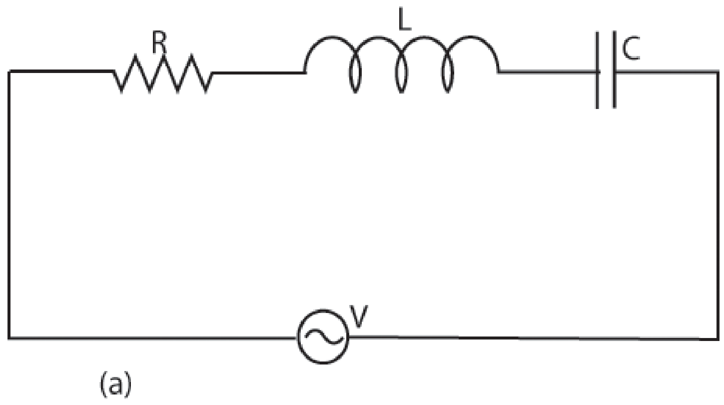
Helmholtz coil

Schematic of the experiment (cont…)

- Parameters of coil
  - 18 gauge copper wire, d=1.2 mm
  - N=324 , l=2.7 cm , R=1.5Ω
  - D1=6.5 cm, D2=10.2 cm , a=4.8 cm
  - L=7 mH

- Connected a capacitor of 0.97μF to resonate the coil at 1.22 kHz to maximize the current and hence B

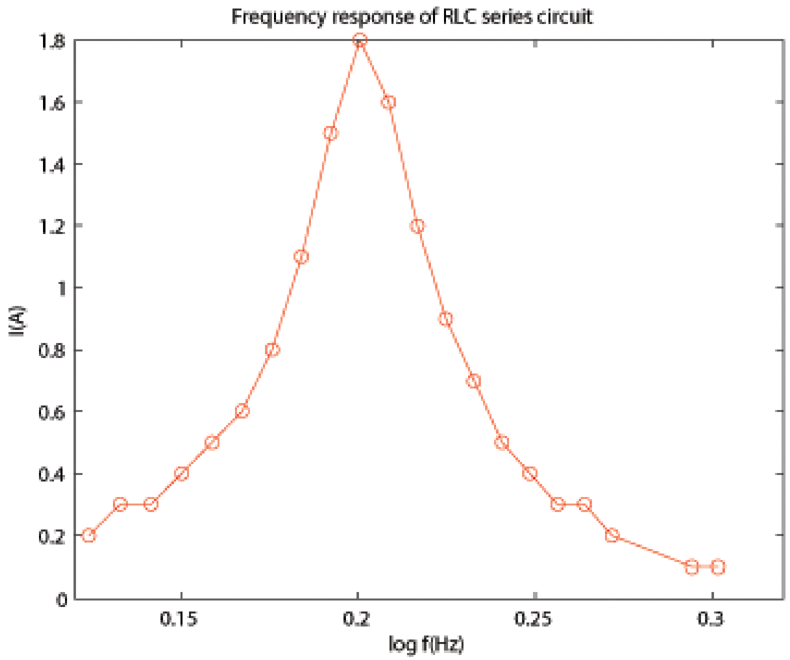
$$f = \frac{1}{2\pi}\sqrt{LC}$$



Schematic of the experiment (cont…)

Gauss meter

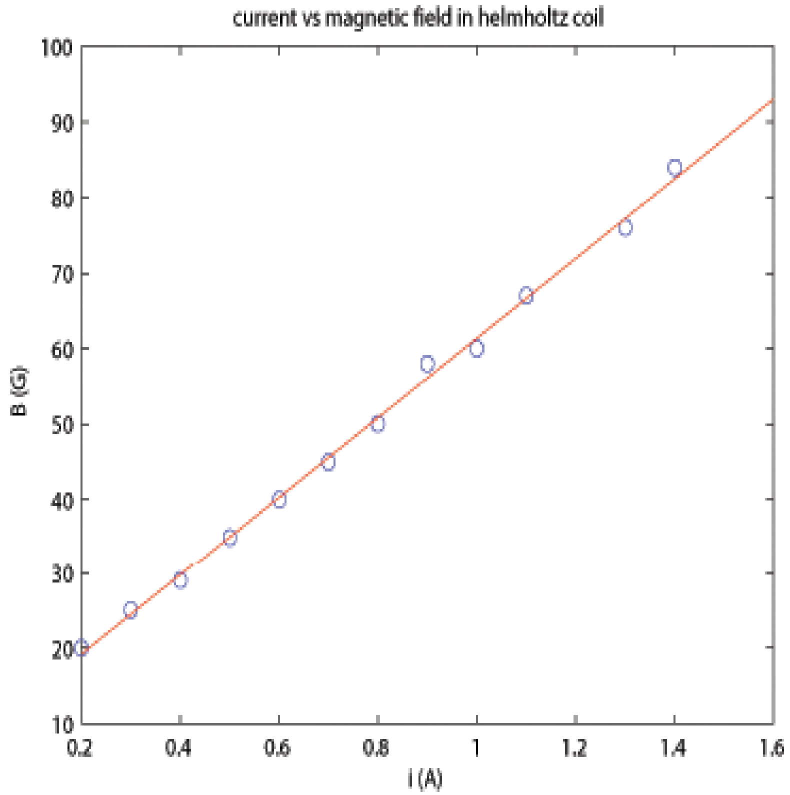
Resonance in Helmholtz coil

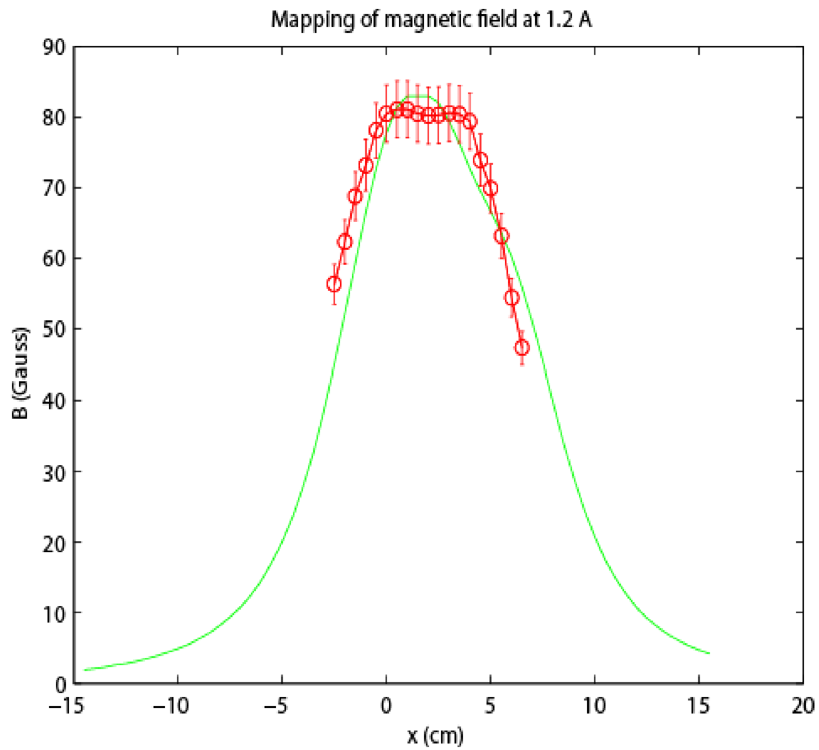


$$B = 6.7 \times 10^{-3} i \ \text{T}$$

$$\text{Q} = 38$$

Schematic of the experiment (cont…)





Magnetic field varies linearly with the current applied.

Schematic of the experiment (cont…)

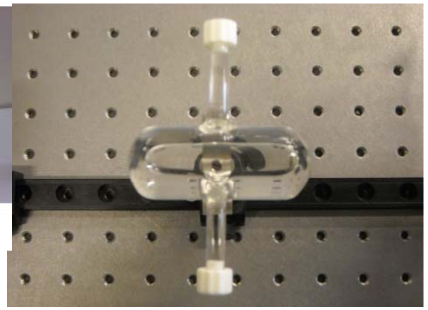
(a)

(b)

(c)

(d)

(e)



Setup for the experiment

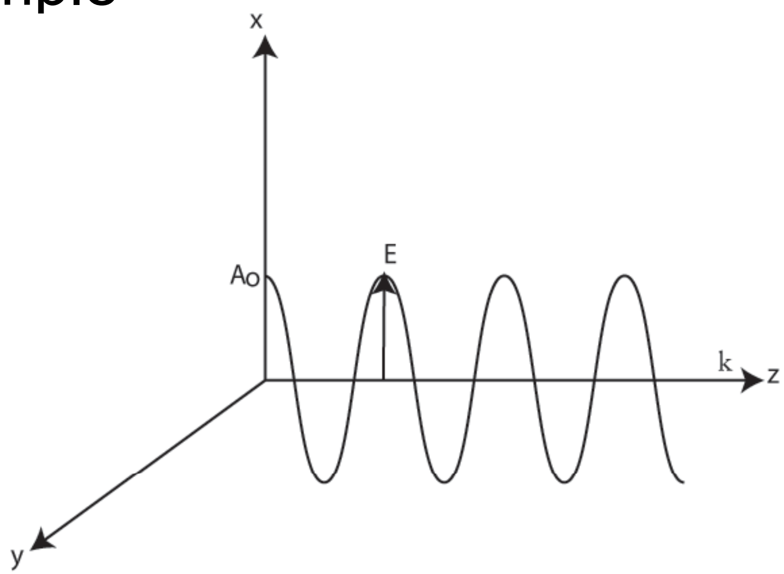
Working principle

Jones vector of horizontally polarized light traveling in z direction

$$E_o = \begin{pmatrix} 1 \\ 0 \end{pmatrix} A_o \exp\, i(kz - \omega t)$$

For rotated polarized light, after sample

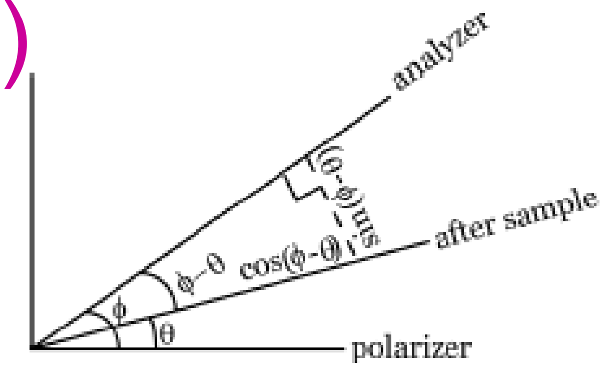
$$E_o = \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix} A_o \exp i(kz - \omega t)$$



Working principle (cont…)

After analyzer

$$E_o = \begin{pmatrix} \cos(\phi-\theta)\cos(\phi) \\ \cos(\phi-\theta)\sin(\phi) \end{pmatrix} A_o \exp i(kz-\omega t)$$



Intensity measured by photodiode

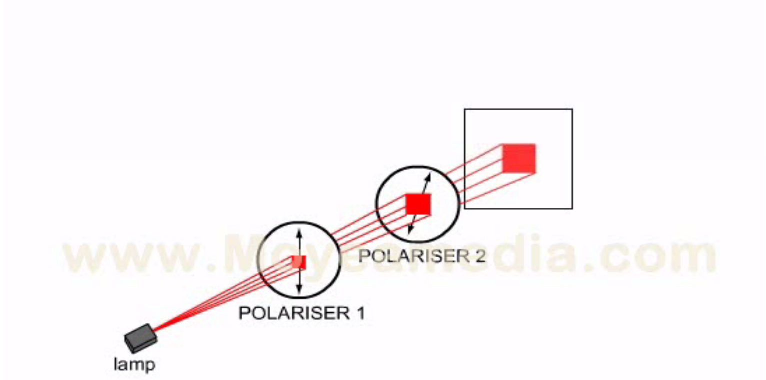
$$I = E^*.E$$

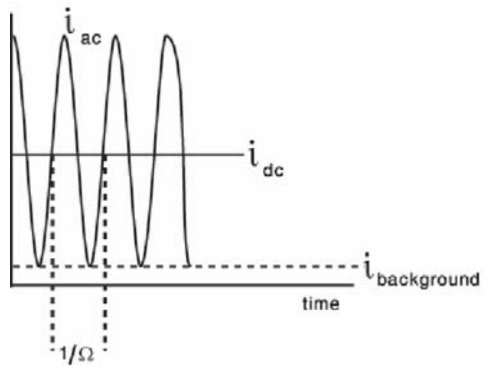
$$= A_o^2 \cos^2(\phi-\theta)$$

$$I \cong \frac{A_o^2}{2}(1+2\theta)$$

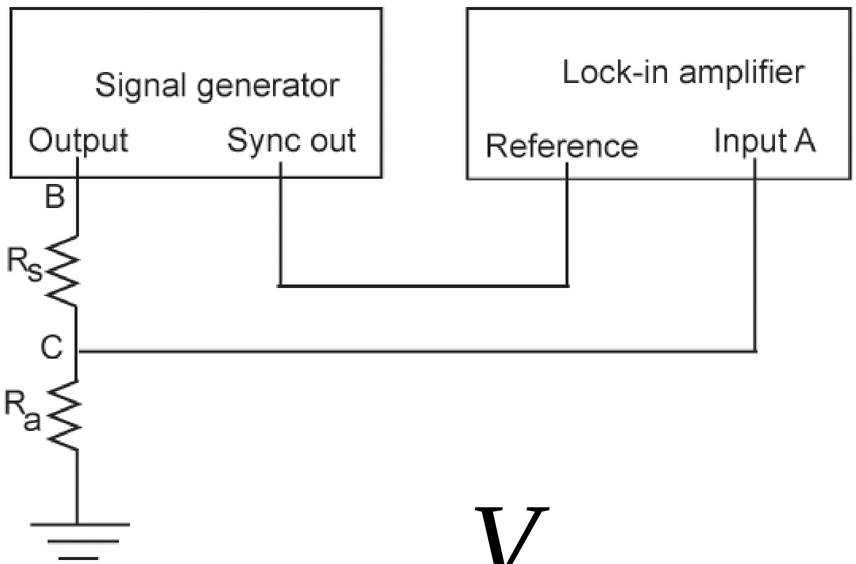
$$\cong I_0 + \Delta I \sin(\Omega t)$$

$$\theta = i_{ac}/2i_{dc}$$



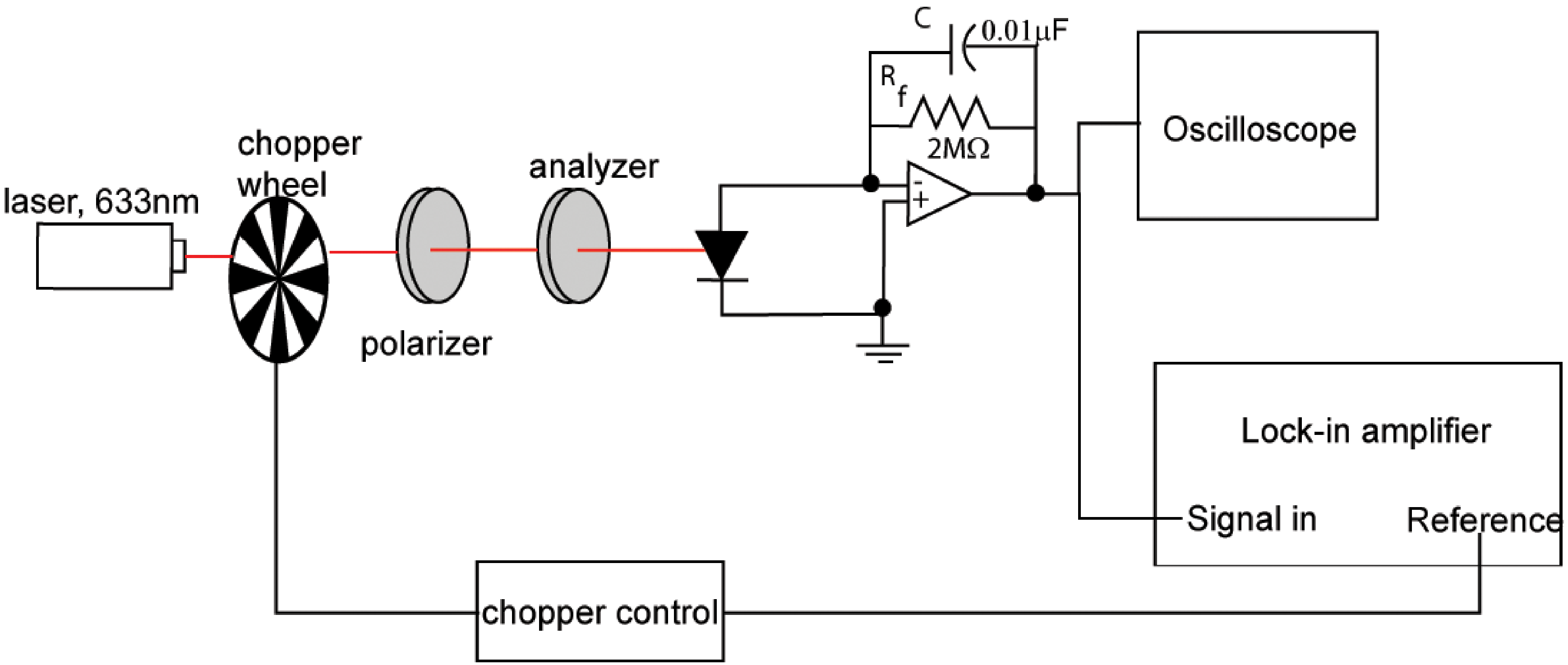


Resistance measurement through Lock-in Amplifier

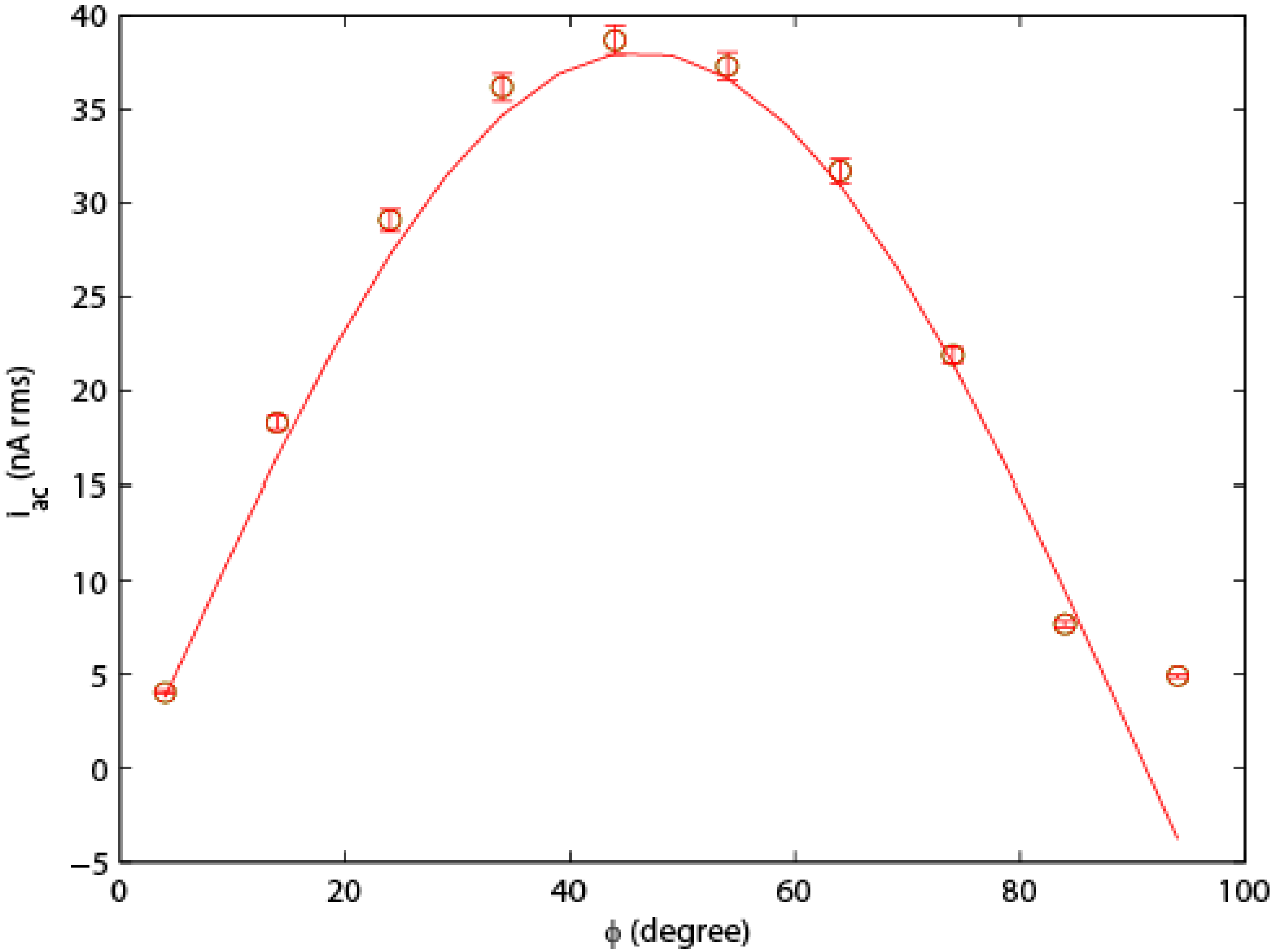


$$R_a = \frac{V_c}{V_B} R_s$$

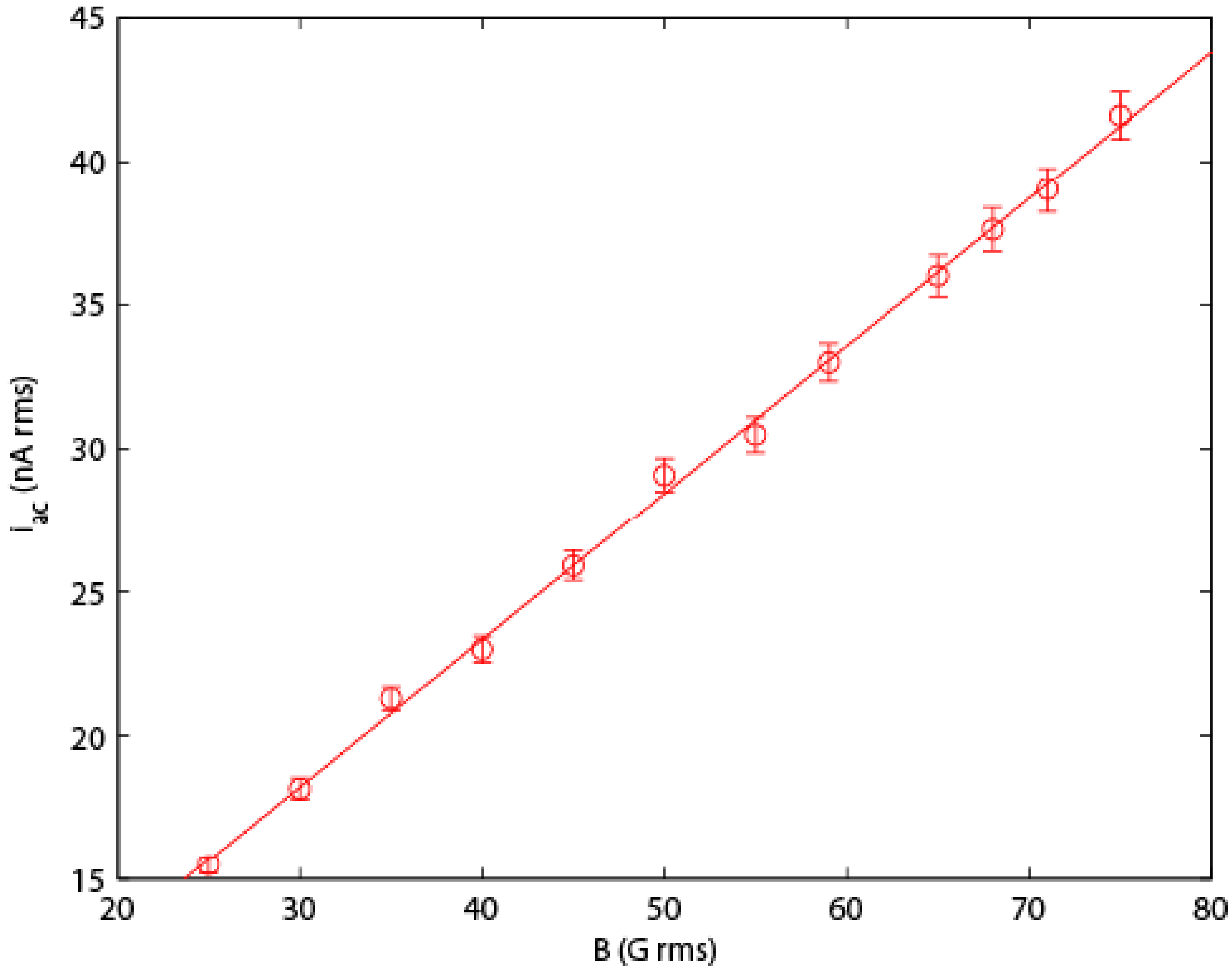
Using optical chopper with Lock-in Amplifier



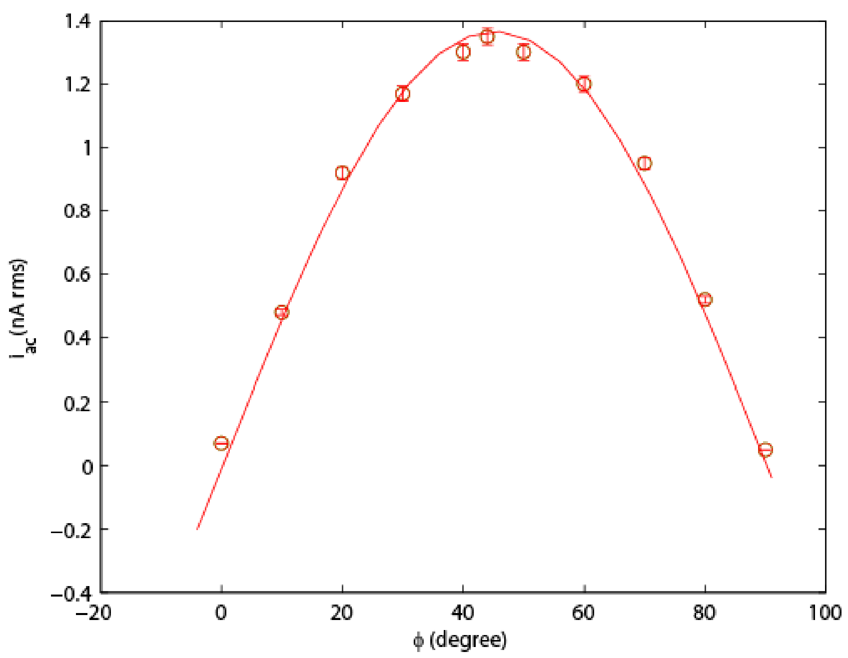
Results
TGG at 405 nm

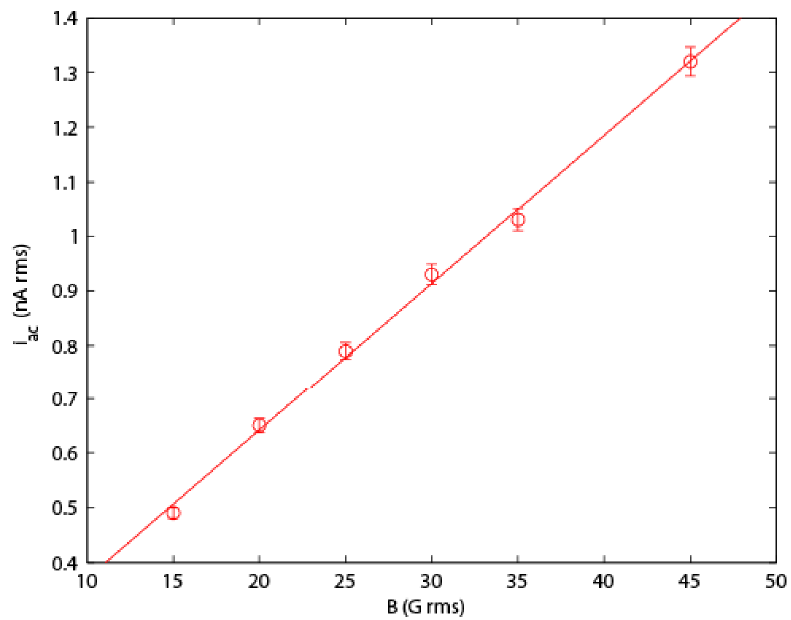


TGG at 405 nm

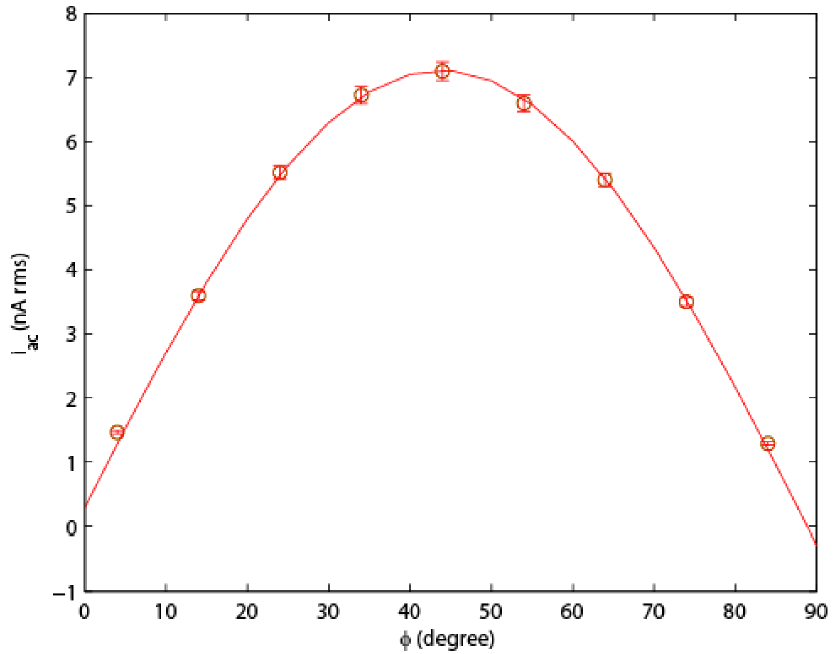


TGG at 633 nm

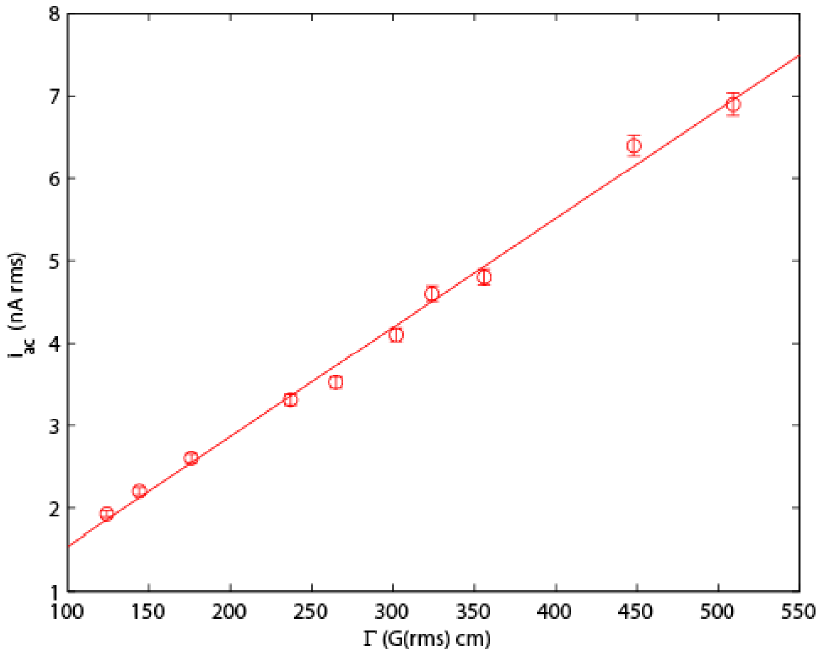




CS_2 at 405 nm

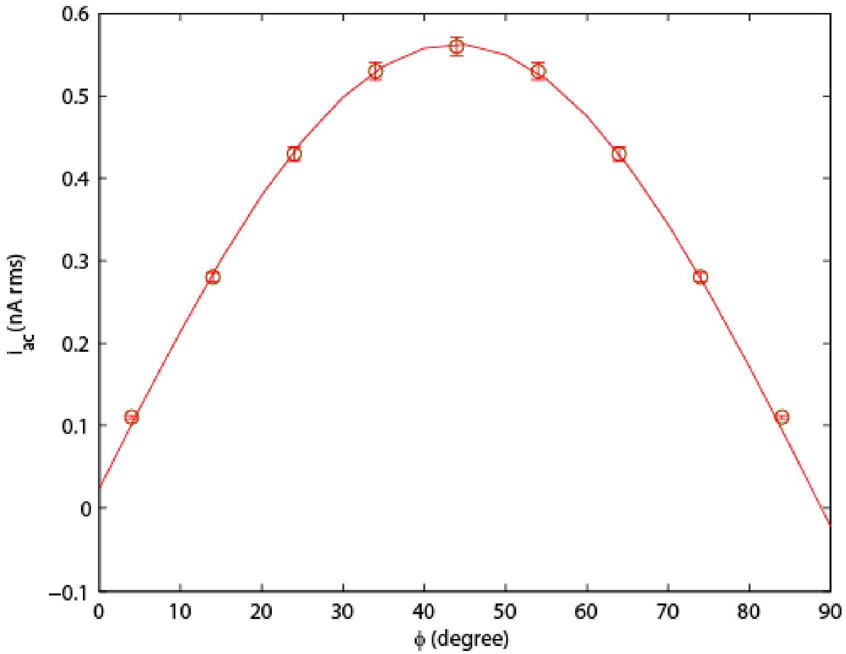


8
7
6
5
4
3
2
1
0
−1
0
10
20
30
40
50
60
70
80
90
i_ac (nA rms)
ϕ (degree)

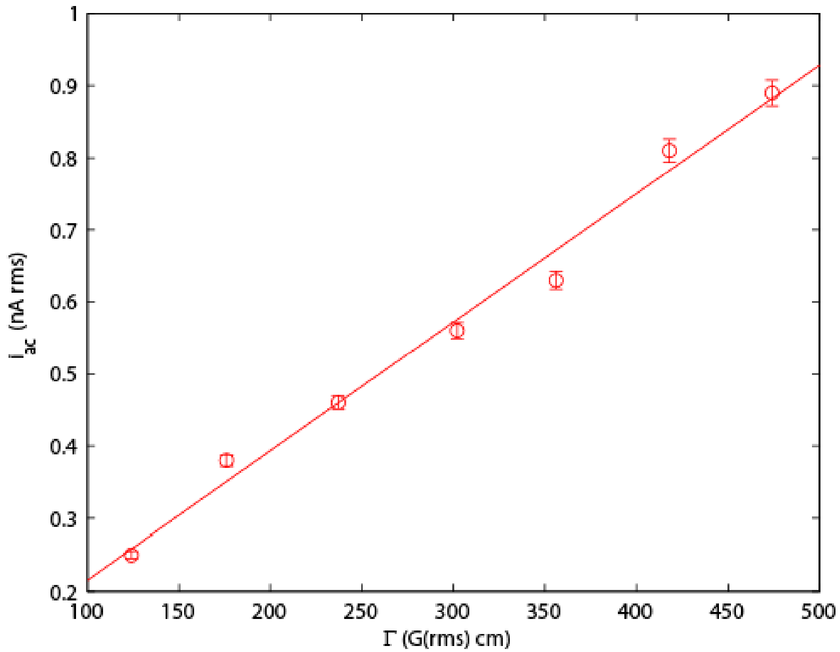


8
7
6
5
4
3
2
1
100
150
200
250
300
350
400
450
500
550
i_ac (nA rms)
Γ (G(rms) cm)

CS_2 at 633 nm

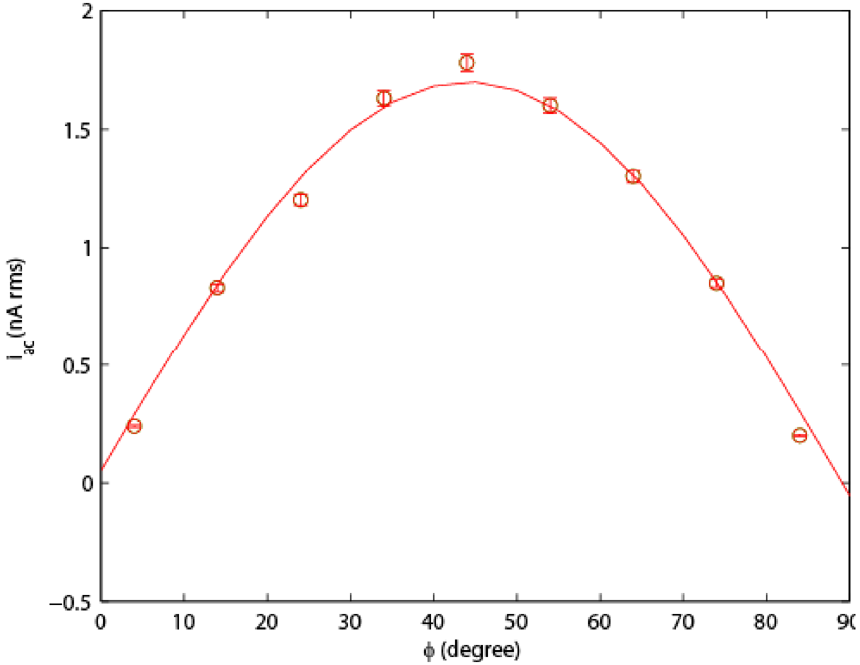


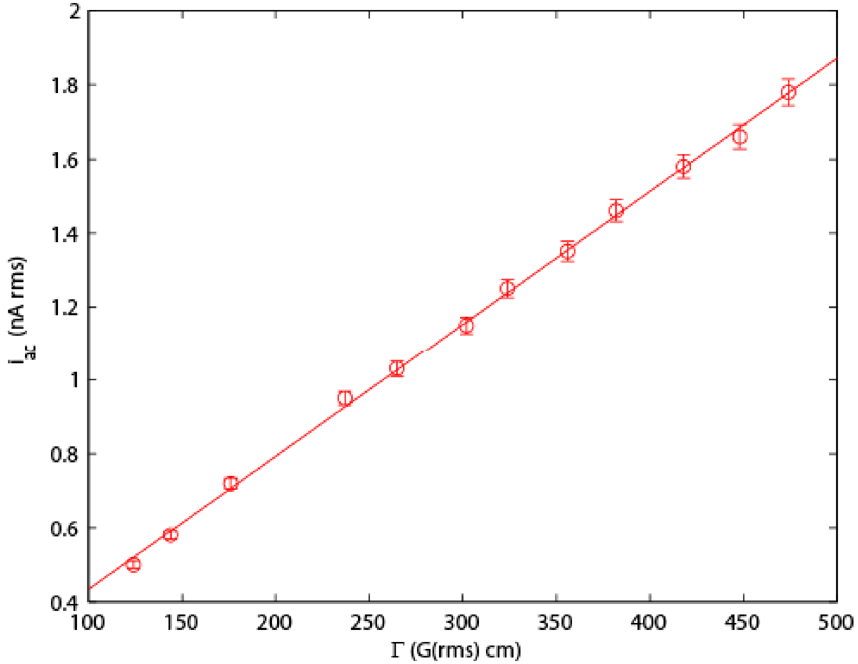
0.6
0.5
0.4
0.3
0.2
0.1
0
−0.1
0
10
20
30
40
50
60
70
80
90
ϕ (degree)
i_{ac} (nA rms)



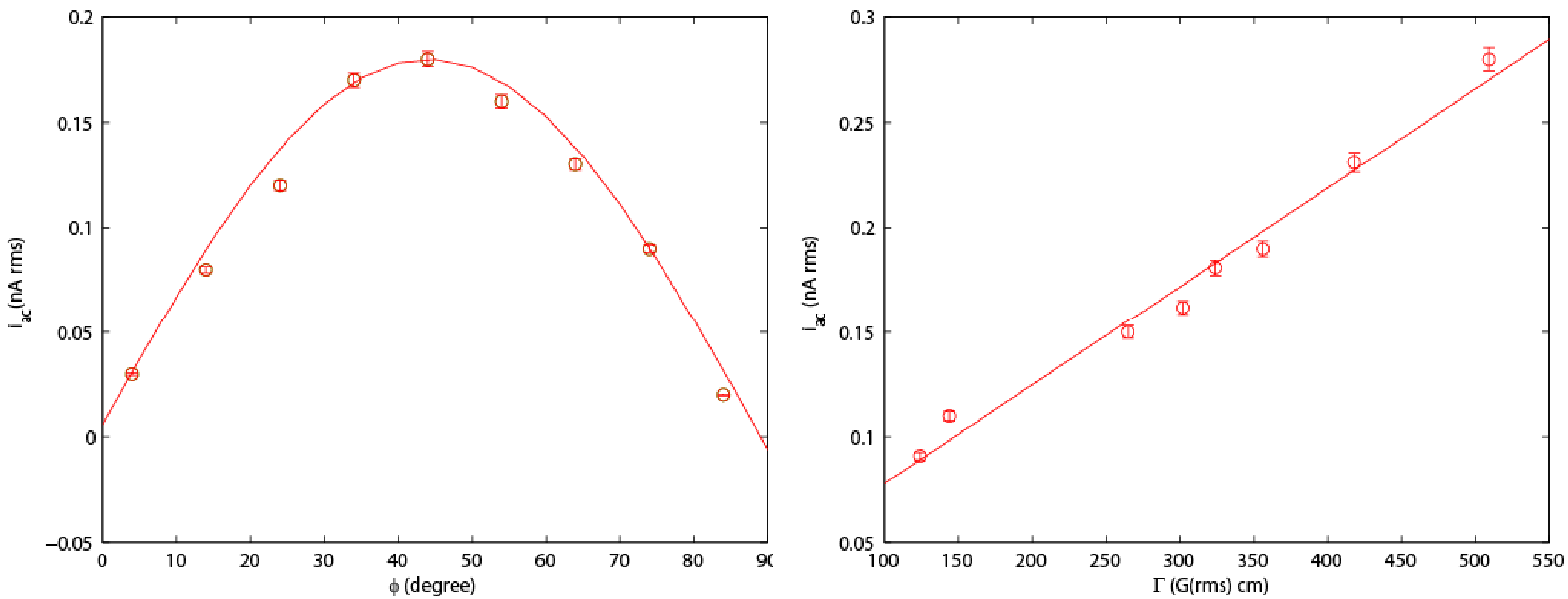
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
100
150
200
250
300
350
400
450
500
Γ (G(rms) cm)
i_{ac} (nA rms)

Methanol at 405 nm

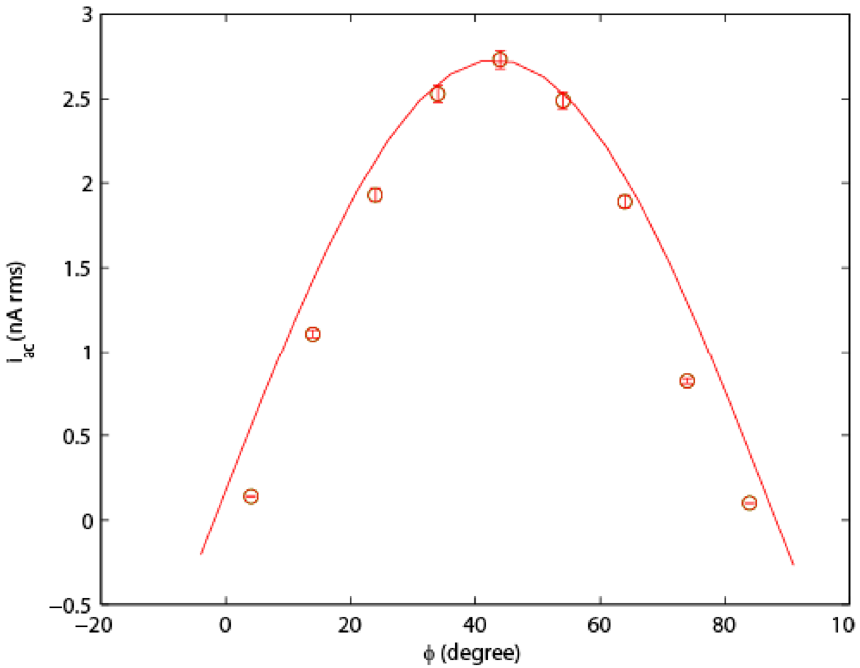


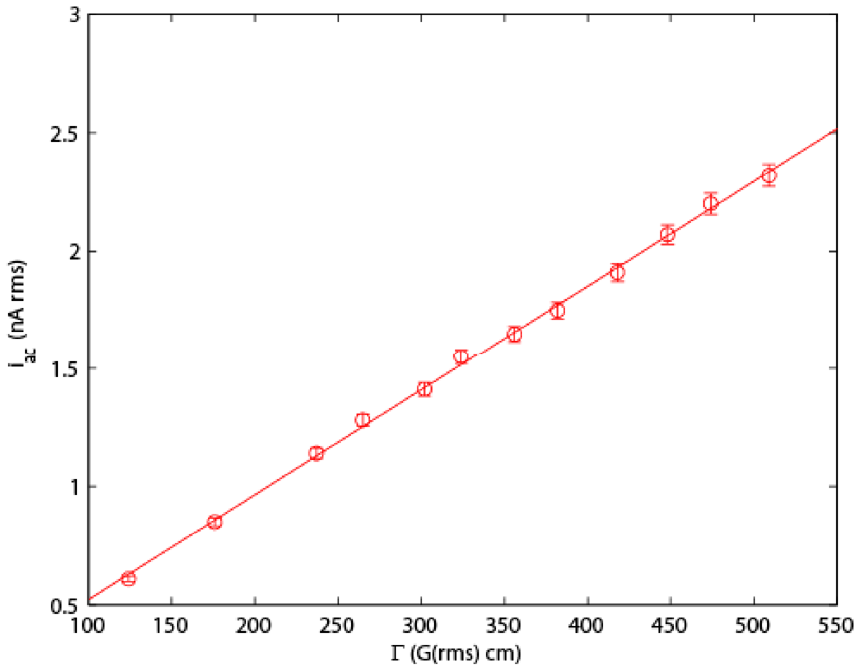


Methanol at 633 nm

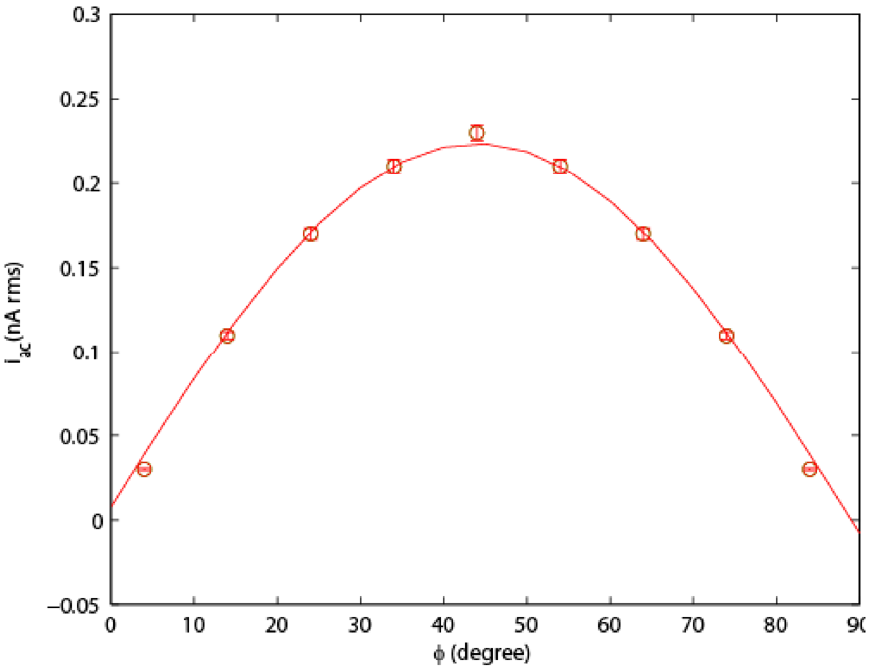


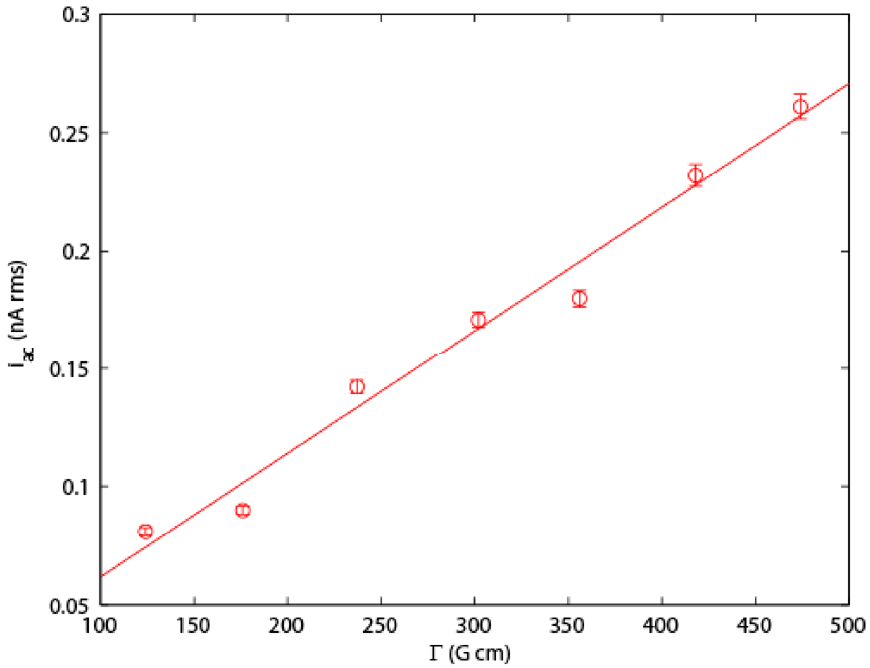
Water at 405 nm



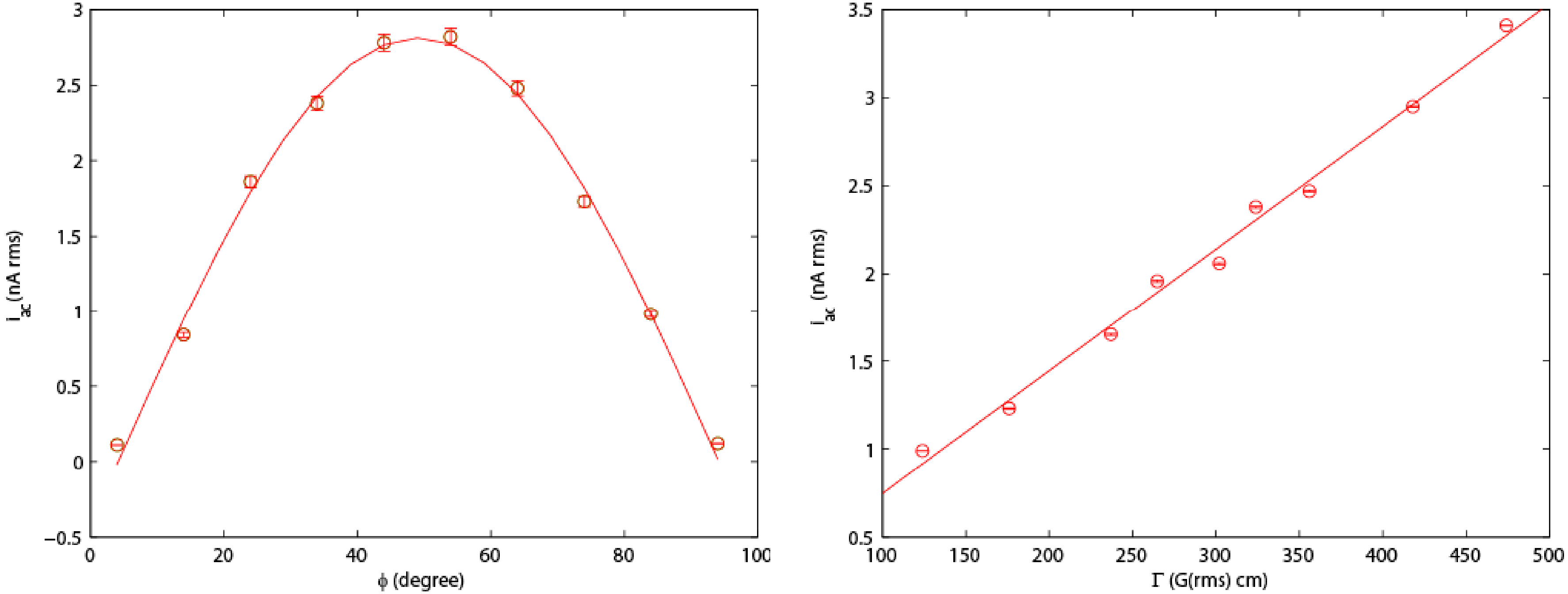


Water at 633 nm

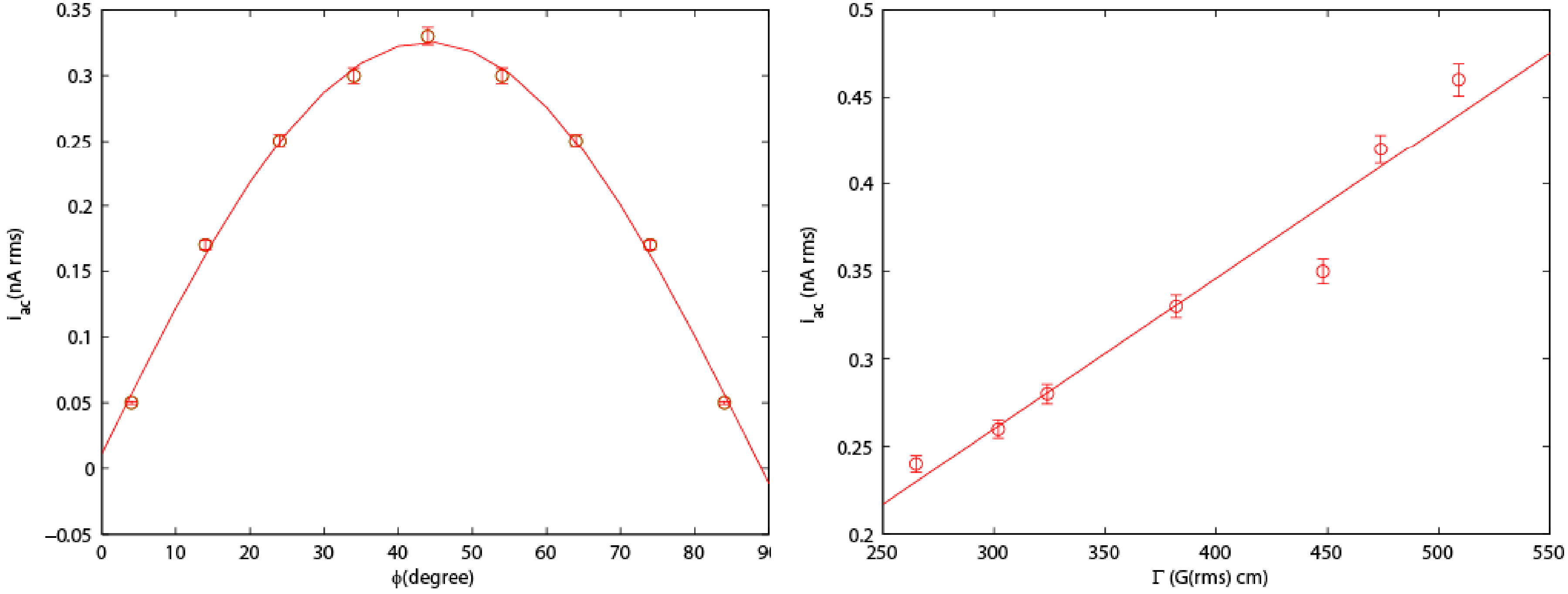




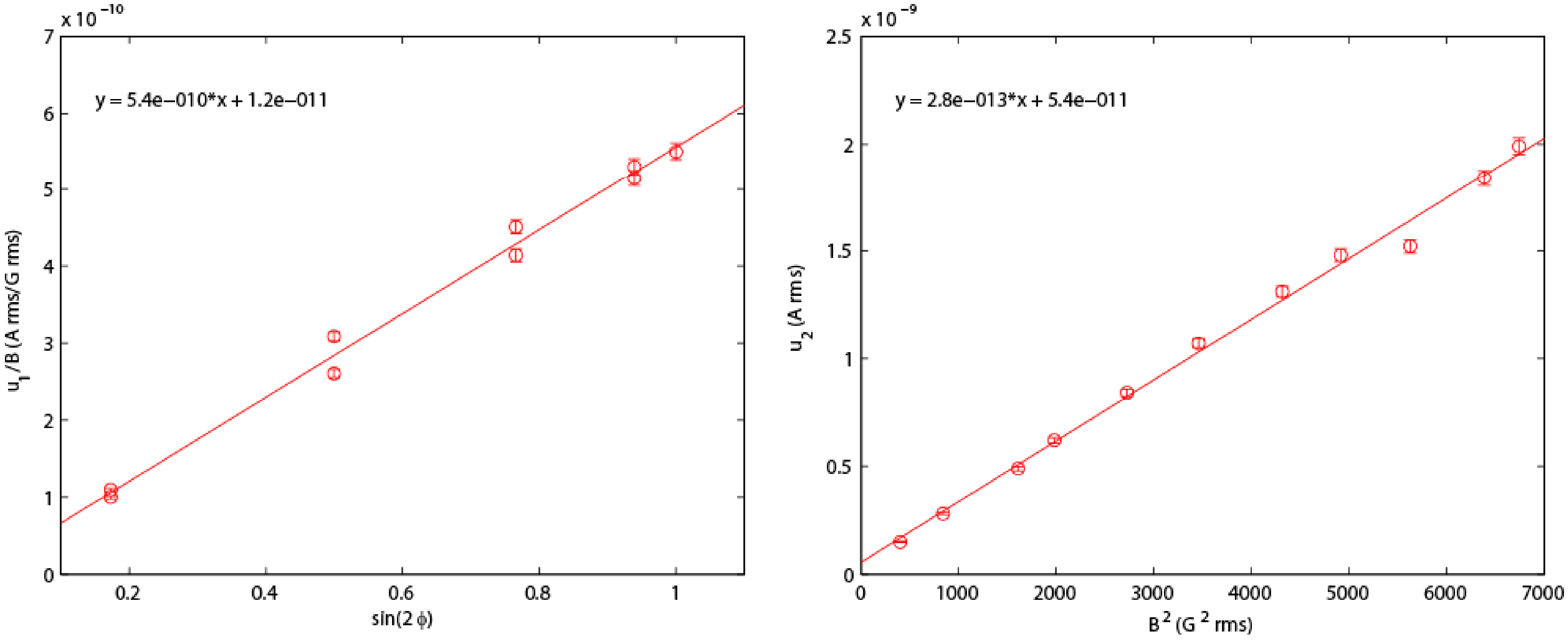
Isopropanol at 405 nm



Isopropanol at 633 nm



Determination of V using higher harmonics



Summary

sample	Wave length (nm)	Verdet constant (rad/G cm)
TGG	633	$(0.146\pm0.003)\times10^{-3}$
TGG	405	$(1.70\pm0.02)\times10^{-3}$
CS_2	633	$(0.97\pm0.07)\times10^{-5}$
CS_2	405	$(0.50\pm0.01)\times10^{-4}$
Methanol	633	$(0.24\pm0.02)\times10^{-5}$
Methanol	405	$(1.20\pm0.05)\times10^{-5}$

Summary (cont…)

sample	Wave length (nm)	Verdet constant (rad/G cm)
Water	633	$(0.30 \pm 0.02) \times 10^{-5}$
Water	405	$(1.50 \pm 0.08) \times 10^{-5}$
Isopropanol	633	$(3.55 \pm 0.02) \times 10^{-6}$
Isopropanol	405	$(2.2 \pm 0.06) \times 10^{-5}$

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THANK YOU