Michelson Interferometry

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In this experiment we explore Michelson Interferometry by investigating few of its many applications in optics. The procedure was used to calculate the wavelength of a Helium-Neon laser using the linear relationship between change in number of fringes and the distance moved by a mirror. The resultant wavelength was calculated to be 640 ± 21 nm. Further, using notions of optical path length and Snell's law, a relation between refractive index, angle of the glass, and the change in number of fringes emerged, which was then used to calculate the value of the refractive index as 1.65 ± 0.33 .

I. INTRODUCTION

Interferometry is a technique that uses the interference of waves to extract information. It makes use of the principle of superposition to combine waves in a way such that the resulting wave has properties characteristic of the original state of the waves [1]. The principle of superposition arises from the linearity of the wave equation, hence a sum of solutions still yields a resultant wave.

Michelson interferometer in particular, came into existence to measure the Earth's motion through ether, and since then it has played important roles in the modern day in Fourier spectroscopy, laser-beam interferometers, and much more [2]. The procedure uses symmetry in its setup and exploits the wave nature of light to produce interference fringes, which are studied to understand a lot of physical phenomena and calculate different properties of materials. In this experiment, we explore a few of its applications- calculating the refractive index of glass and measuring the wavelength of a Helium-Neon laser.

II. THEORY

A. What are interferometers?

Interferometers work on the principle of optical interference, which corresponds to the interference of two or more light waves, yielding a resultant irradiance that deviates from the sum of component irradiances. These devices in general can be divided into two: i) Wavefront splitting, where primary waves are used to emit secondary waves, which then interfere, and ii) Amplitude splitting, where a primary wave itself is divided into two segments and travel different paths to interfere[3].

An important condition for two waves to interfere is that they must have nearly the same frequency, which leads to the concept of coherence; when waves are coherent, the phase relation between the waves is constant. Two types of coherences relate to waves: i)Spatial coherence, where the waves are traveling more or less in the same direction, ii)Temporal coherence, where the waves have more or less the same frequency [3].

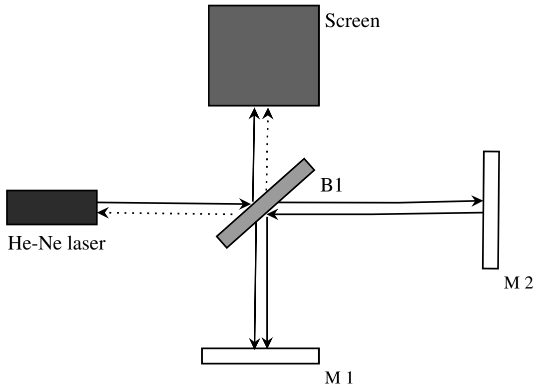

FIG. 1. Schematic of Michelson Interferometer

Michelson interferometer is an amplitude-splitting interferometer, that utilizes temporal coherence of waves for interference patterns. Figure 1 shows its basic setup, which consists of two mirrors, a beam source, a beam splitter, and a screen. The beam is divided into two through the beam splitter and is then reflected from the mirrors. The resultant beams then interfere to create interference patterns on the screen.

B. Measuring wavelength using Michelson interferometry

The technique uses very simple principles of path length difference to measure the wavelength of a beam. When the beams are initially split, due to a difference in lengths traveled by the two beams, a phase difference arises. Moving one of the mirrors would vary the path difference, by whatever distance the mirror has moved multiplied by two. If the mirror moves a distance of $\frac{\lambda}{4}$, then a resultant path difference of $\frac{\lambda}{2}$ arises, and the maxima and minima in the interference pattern switch places. Similarly, if the mirror is moved by a $\frac{\lambda}{4}$, the original interference pattern is restored. This leads to a simple, yet important formula for measuring the wavelength of a beam:

$$\lambda = \frac{2d_m}{m}, \tag{1}$$

where d_m is the total distance moved by the mirror, and m is the number of times the fringe pattern is restored.

C. Measuring the refractive index of a glass slide

Measuring refractive index utilizes the concept of optical path length, which is the length that light needs to travel through the air to create the same phase difference as it would have when traveling through some homogenous material [4]. Through simple geometrical arguments, one can arrive at the following expression (detailed derivation is done in the appendix):

$$n_g = \frac{(2t - N\lambda)(1 - \cos\theta)}{2t(1 - \cos\theta) - N\lambda}, \qquad (2)$$

where n_g is the refractive index, t is the thickness of the glass slide, N is the number of times the fringe pattern was restored, and θ is the angle through which the glass slide was rotated.

III. EXPERIMENTAL PROCEDURE

The experimental setup consisted of a Helium-Neon laser, HR020, from Thor Labs, a beam splitter, two silver mirrors, a magnifying glass, a screen, and lastly a breadboard to assemble the interferometer. The crucial part of this experiment was alignment, so before any measurements were carried out, it was important to ensure that the beams were aligned and they interfered to produce circular fringes. The result was the following pattern:

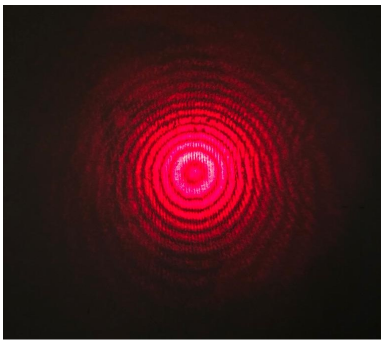

FIG. 2. Fringe patterns

Once a stable interference pattern was produced, we could move on to measure the wavelength. For this, we made use of the DC Servo Motor Controller from Thor Labs. One of the two mirrors (M2 in Fig 1.) was attached to an actuator, which was connected to the motor. The software used to control the motor was Kinesis by Thor Labs. We set a constant velocity of 0.3×10^{-6} m/s for the actuator and varied the distance moved by the mirror. The screen was marked to help count the number of times the pattern was restored. With several readings, we were able to plot m against d_m to get the wavelength, as can be seen in Fig. 3.

To measure the refractive index of the glass slide, a precision rotation platform was used, and the slide was placed on the platform through a stand. The stand was placed between the beamsplitter and M1 (in Fig.1). It was rotated through particular angle increments, and the number of times the fringe pattern was restored, was noted. The procedure was repeated three times, and consequently, the refractive index was found through Eq. 2.

IV. RESULTS AND DISCUSSION

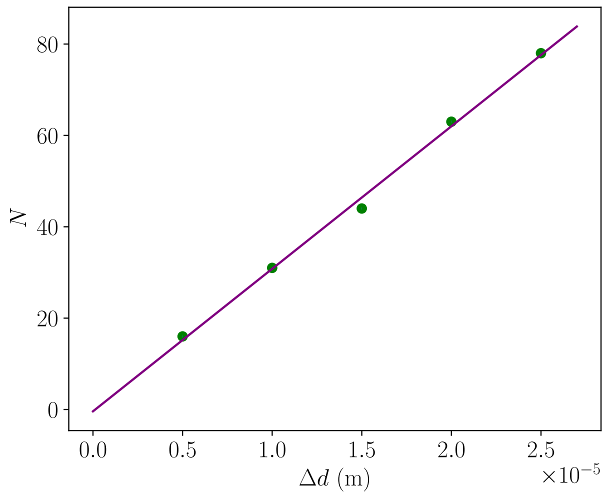

FIG. 3. Number of fringes restored against the distance moved by the mirror.

The gradient of Fig. 3 is given by $\frac{2}{\lambda}$, as can be seen through Eq. 1. Through linear curve fitting, the value of the gradient was found to be 3.12×10^6 , and then accounting for the uncertainties through the covariance matrix, the final wavelength was calculated to be 641 ± 21 nm. According to Thorlabs[5], the actual output of the He-Ne laser is 632.8 nm, which lead to a percentage error of only 1.14 %, hence, the measured value was in good agreement with the actual value.

The main sources of error in this wavelength calculation came from the precision of the motor, and the inherent error in curve fitting.

TABLE I. Measurement of the angle θ of the glass slide, the number of fringes restored, and the corresponding refractive index.

$\theta(^\circ)$	N1	N2	N3	N4	N5	n
4.0 ± 0.2	1	2	2	1	1	1.36 ± 0.28
8.0 ± 0.2	13	9	9	10	10	1.92 ± 0.06
12.0 ± 0.2	22	15	19	21	20	1.68 ± 0.04

Three values of the refractive index were calculated using the results in Table 1, through Eq. 2. The final averaged refractive index was then calculated to be 1.65 ± 0.33 . The actual value for glass is 1.52 , and so the percentage error in the measured value was 8.55 %. Given the sensitivity of the apparatus, the measured value fell within the accepted range of the actual value.

An uncertainty that could not be accounted for in both measurements was the human error involved when the number of fringe patterns being restored was counted by eye. They deviated too quickly at times to be noted precisely. The experiment was repeated several times to eliminate this error. Another major hurdle in the experiment was the sensitivity of the apparatus to minor external disturbances. This could not be tackled with in this experiment, but it could be improved by carrying the experiment out in a more sensitive environment, for example conducting it in a vacuum.

V. CONCLUSION

Michelson interferometry is a technique that has allowed people to probe into a lot of rich physics over the course of history. This simple experiment serves a glimpse into its wide range of applications, and also encourages one to explore them further.

Appendix

Deriving refractive index in terms of slide angle

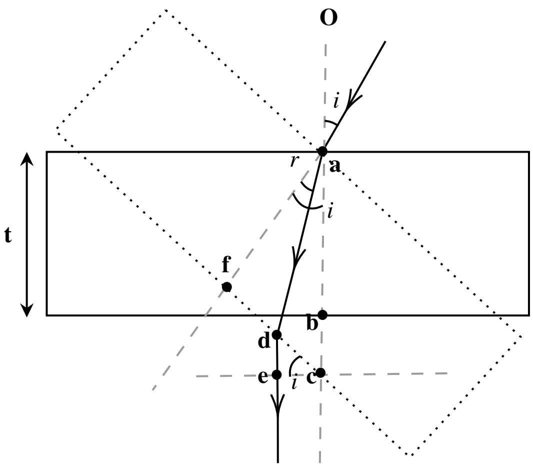

FIG. 4. Schematic displaying the geometry of a beam entering an inverted glass slide.

The relation between refractive index and the angle that the glass is rotated arises from the the concept of optical path length [6]. In general optical path difference is defined as:

$$OPD = d_1 n_1 - d_2 n_2,$$

where d_1 and d_2 are the distances travelled by light in the mediums with the respective refractive indexes n_1 and n_2.

In this case, the optical path difference is defined as:

$$\begin{aligned} OPD &= 2tn - 2t(1) \\ &= 2t(n-1), \end{aligned}$$

where t is the thickness of the glass slide, and the factor of two arises because light traverses twice. Also note that the refractive index of air is taken to be one.

For a monochromatic light of wavelength λ, the difference of paths introduced is $N\lambda$, where N is the number of times the fringes are restored when the glass slide is inserted. Hence, when the glass slide is rotated, there is a difference in path length, and consequently N can be measured, and the following relation can be established:

$$2t(n-1) = N\lambda.$$

The difference in path lengths can now be calculated using the geometry shown in figure 3.

Optical distance travelled by light when glass is parallel: $nt + bc$.

Optical distance travelled when glass is at an angle: $adn + de$.

So the total increase in optical path length can be rewritten as:

$$2(adn + de - nt - bc) = N\lambda. \tag{A.1}$$

Further relations can be developed from the geometry of the figure:

$$\begin{aligned} ad\cos r &= t \\ \implies ad &= \frac{t}{\cos r} \\ de &= dc \sin i \\ dc &= fc - fd \\ \implies de &= (fc - fd)\sin i. \end{aligned}$$

From the triangles in the setup, the following relations can also be derived, which can then be substituted in the last equation:

$$\begin{aligned} \tan i &= \frac{fc}{t} \\ \tan r &= \frac{fd}{t} \\ \implies de &= t\tan i \sin i - t \tan r \sin i. \end{aligned}$$

Another relation can be seen in the figure:

$$\begin{aligned} bc + t &= \frac{t}{\cos i} \\ \implies bc &= \frac{t}{\cos i} - t. \end{aligned}$$

Substituting the above relations into Eq. A.1 we get:

$$\frac{nt}{\cos r} + t\tan i \sin i - t\tan r \sin i - nt - \frac{t}{\cos i} + t = \frac{N\lambda}{2}.$$

From here one can use Snell's law to arrive at the required equation:

$$n_g = \frac{(2t - N\lambda)(1-\cos\theta)}{2t(1-\cos\theta) - N\lambda}, \tag{A.2}$$

[1] Wikipedia, Interferometry (2023).

[2] R. S. Shankland, Michelson and his interferometer, Physics Today **27** (1974).

[3] E. Hecht, *Optics 5th Edition* (Pearson, 2015).

[4] Wikipedia, Optical path length (2023).

[5] ThorLabs, Hene lasers (2023).

[6] G. S. Monk, *Light Principles and Experiments* (McGraw-Hill book company, 1937).