A simple determination of Heat Capacity Measurements at Low Temperatures

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Outline

- Idea behind designing this experiment
- Extension of Tompson and White Experiment
- Modification in the technique and design
- Interfacing with computer
- Statistical Analysis of errors
- Heat capacity & Einstein curves for Cu and Al
- Verwey transition of FeO.Fe2O3

Extention of C. W Tompson and H. W. White’s Work… (1982)

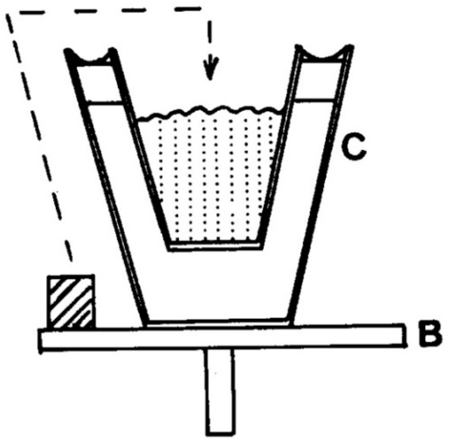
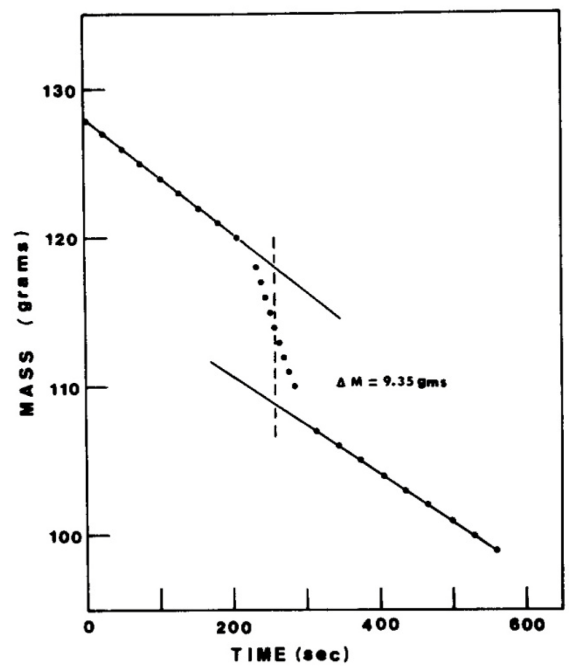


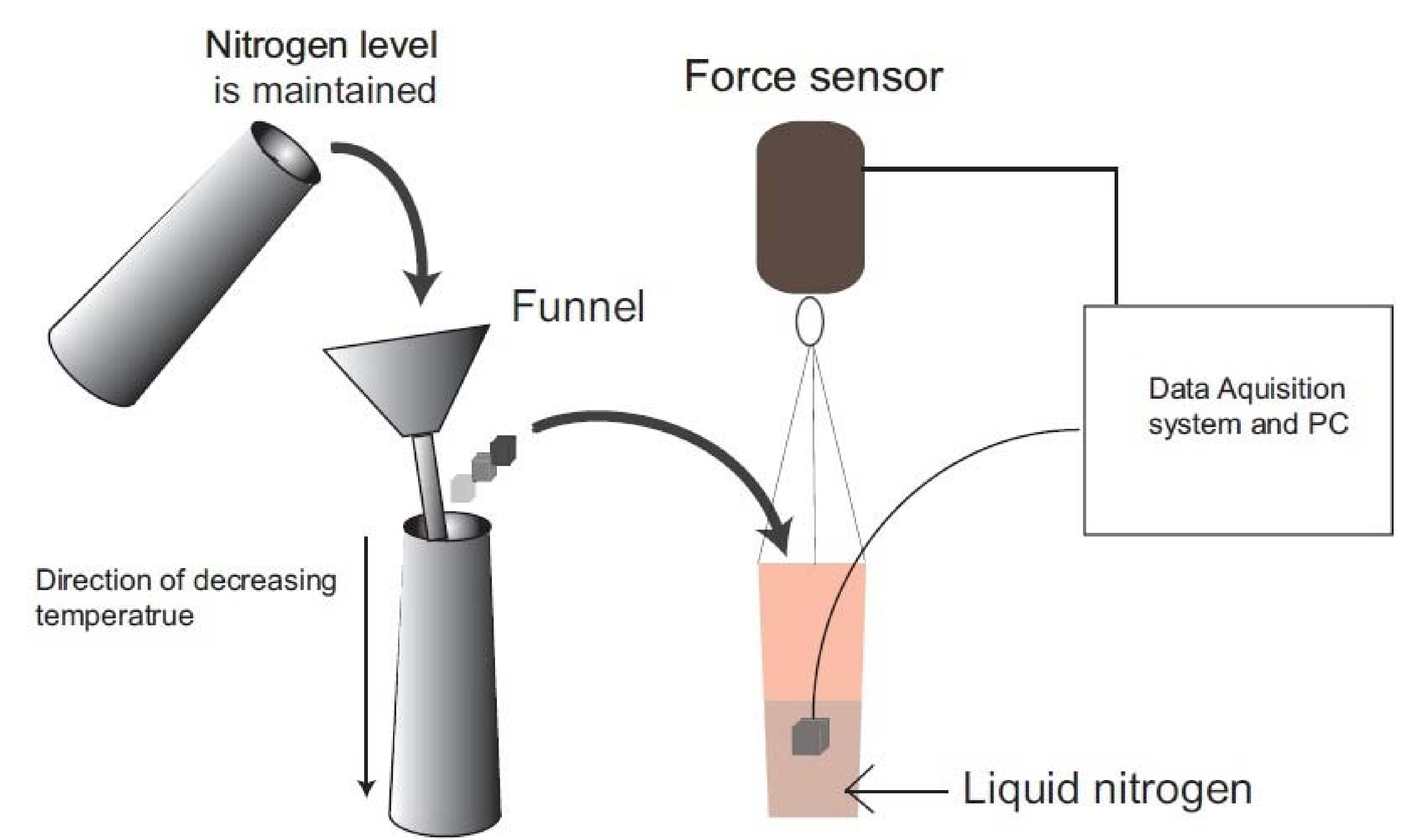
Fig. 3. Schematic diagram illustrating that the metal sample (lead or beryllium) is tranferred from the balance pan into the liquid nitrogen during the heat capacity measurements. The transfer is made after about 200 s, as shown by the data in Fig. 4. *B* is the balance pan and *C* is the cup assembly.



Un desired errors

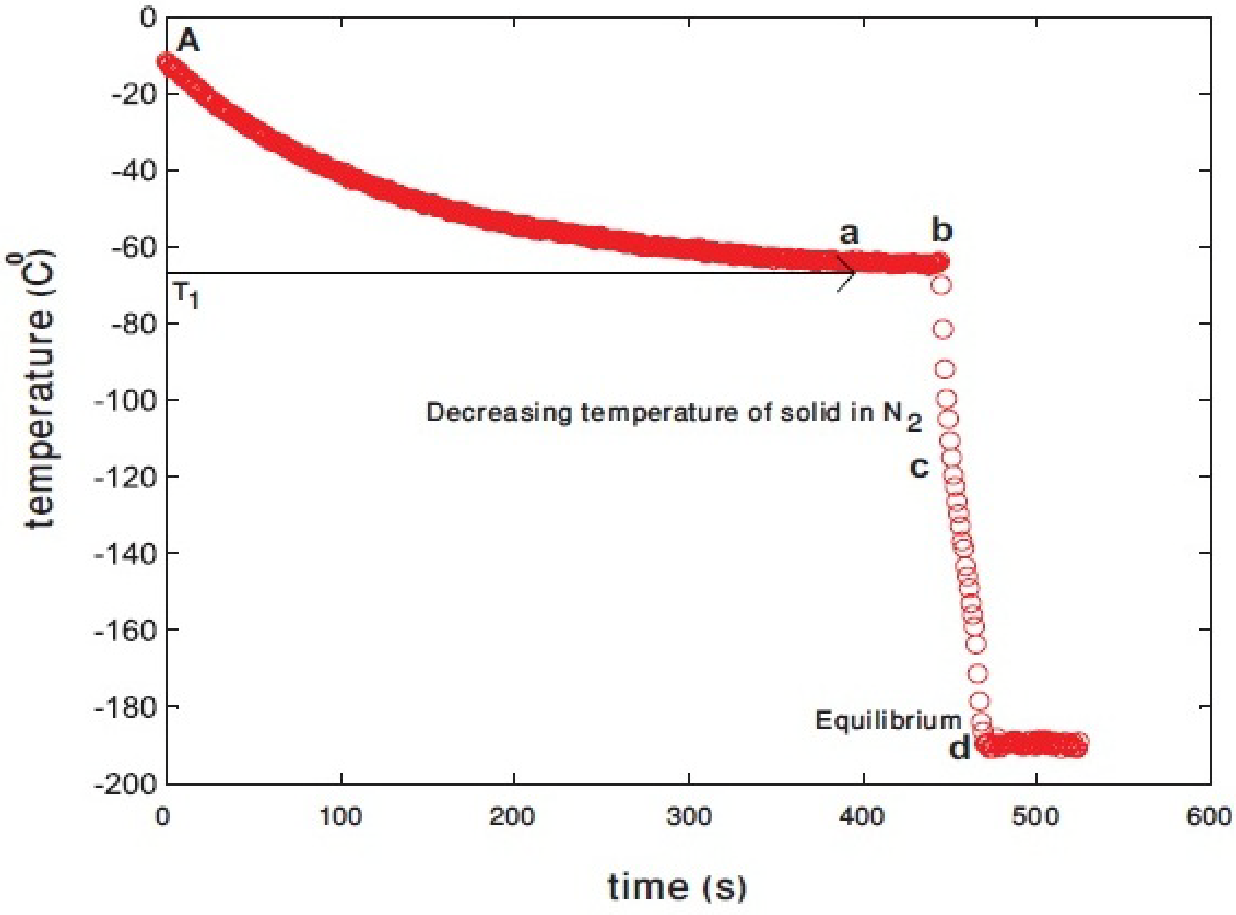
- Fluctuations in the mass readings when cold smoke moves across the balance.
- When solid is dropped; its pressure produce an error.
- Tongs boil off extra mass of liquid nitrogen. (Inevitable)
- Extra mass (ice etc.) got attached to the solid when cooling baths are used.

Modification in the technique and design



- The accuracy of this DFS-BTA (Vernier Instruments) force sensor is 0.1 N.

Temperature of solid versus time



- Leiden frost effect is observed from b-d.

Silicon diode attached to Copper

Diode Equations

The forward bias current is related to reverse bias saturated current as follows,

$$I_f = I_s e^{\frac{qV_f}{k_B T}}$$

The saturation current is given by,

$$I_s = \left(\frac{A k_B}{\tau E} \right) n$$

The density of the carriers is given by,

$$n = \frac{1}{4} \left(\frac{2 m k_B T}{\pi \hbar^2} \right)^{3/2} \exp\left(\frac{-(E_c - \mu)}{k_B T} \right) \approx \frac{1}{4} \left(\frac{2 m k_B T}{\pi \hbar^2} \right)^{3/2} \exp\left(\frac{-E_g}{2 k_B T} \right)$$

Solving the above equations for $V_{f,}$ we have,

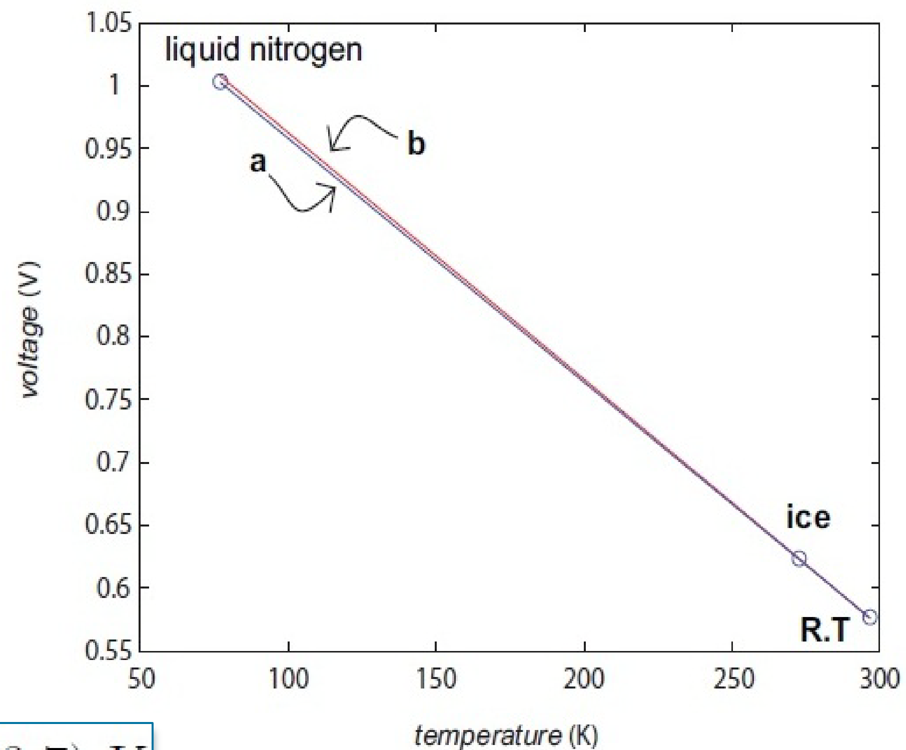
$$V_f(T) = \frac{E_g}{2q} - \left(\log(\alpha) + \frac{3}{2}\log(T) - \log(I_f)\right)\frac{k_B T}{q},$$

where,

$$\alpha = \frac{1}{4}\left(\frac{2mk_B}{\pi\hbar^2}\right)^{\frac{3}{2}}\frac{Ak_B}{\tau E_g},$$

We provide a constant current of 10 micro Ampere using Keithle's 6221 current source.

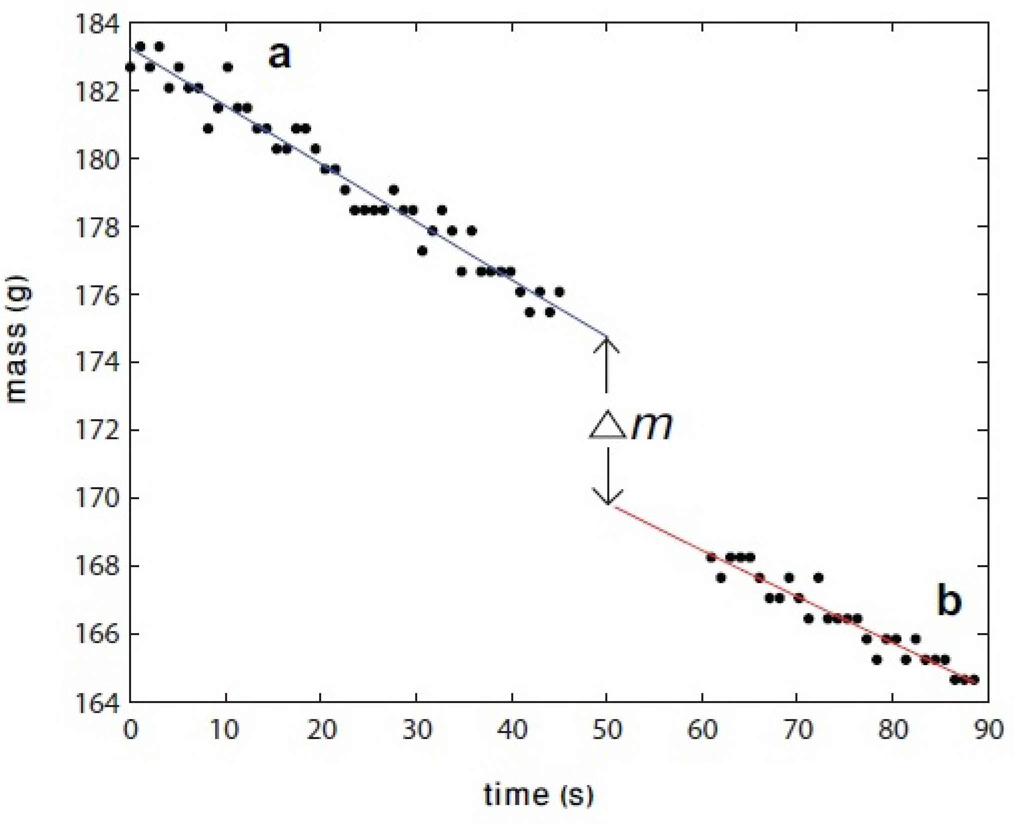
Calibration curve



$$\boxed{T = (-514.8V + 593.7)\ \mathrm{K}}$$

FIG. 4: a is the simple straight line fit to the data points. b is a straight line fit using Squid's $V_f(T)$ vs T relationship.

Statistical analysis of errors



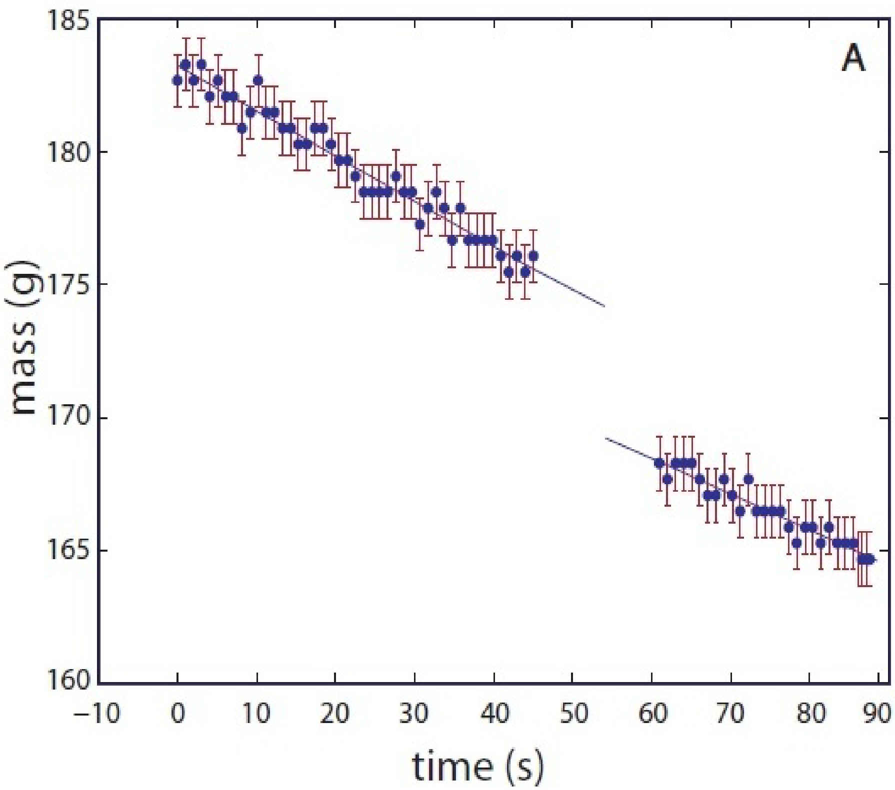
$$(u_s)^2 \approx \frac{1}{D}\frac{\sum d_i^2}{n-2}, \quad \text{and}$$

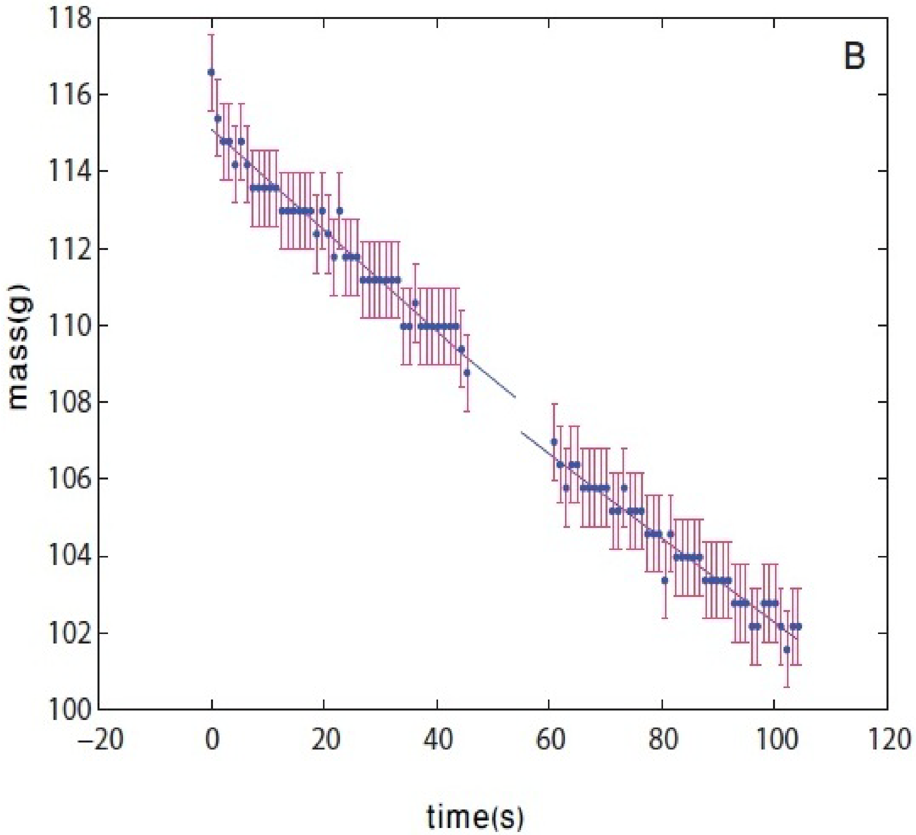
$$(u_i)^2 \approx \left(\frac{1}{n} + \frac{\bar{m}^2}{D}\right)\frac{\sum d_i^2}{n-2},$$

At high temperature change in mass is large

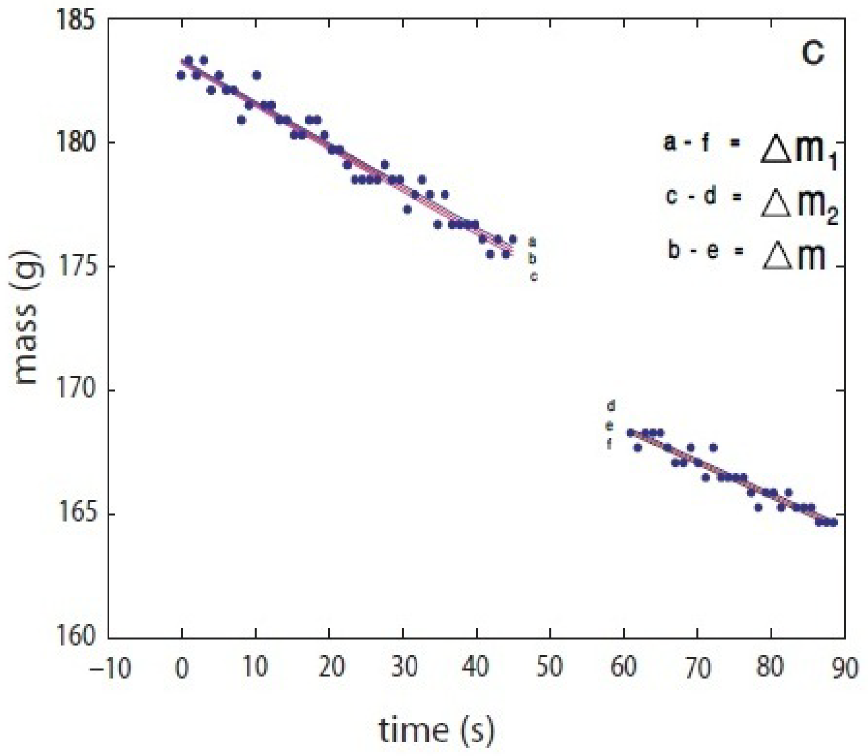
At low temperature change in mass is small

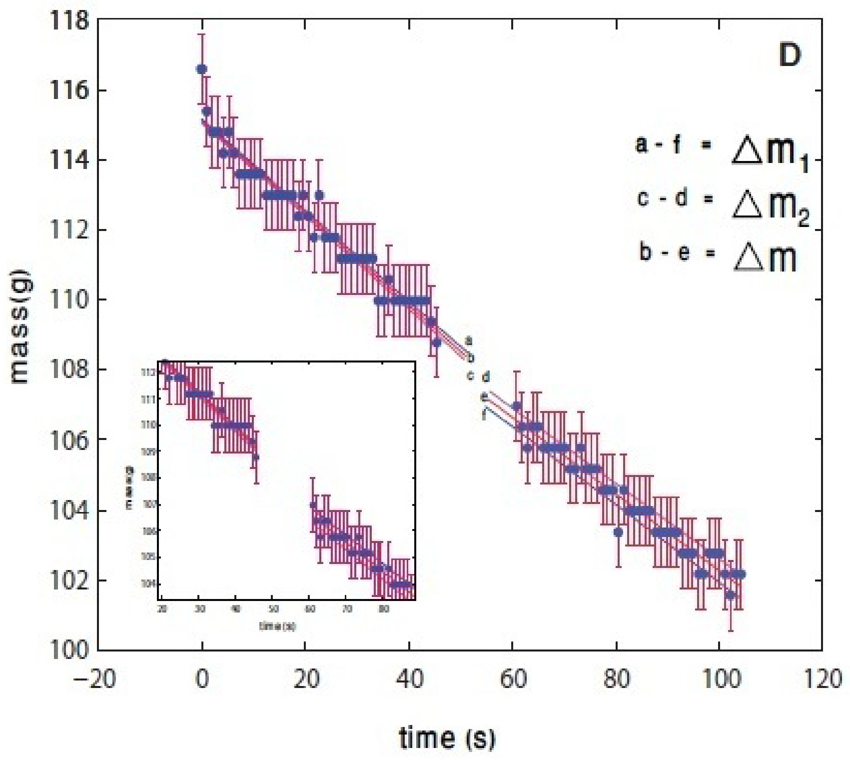
A: 200 K and B: 120 K





C: Error bands for 200 K & D: Error bands for 120 K





Change in mass and Cp

$$(u_{\Delta m})^2 = (\Delta m_1 - \Delta m)^2 + (\Delta m - \Delta m_2)^2$$

$$C_p = (\frac{L_v \Delta m}{n_{moles}(T_1 - 77)} \pm u_{C_p}) \quad \text{J/mol K}$$

$$u_{C_p} = \frac{L_v u_{\Delta_m}}{n(T_0 - 77)}$$

For 200 K the results are,

$\Delta m_1 = 5.10\ g,\ \Delta m_2 = 4.50\ g,\ \Delta m = 4.80\ g,\ u_{\Delta m} = 0.45$ g and $u_{C_p} = 0.44$ J/mol K **D**

For 120 K we have the following results,

$\Delta m_1 = 1.10\ g,\ \Delta m_2 = 0.34\ g,\ \Delta m = 0.72\ g,\ u_{\Delta m} = 0.43$ g $u_{C_p} = 1.4$ J/mol K

Dulong Petit Law & Einstein Model

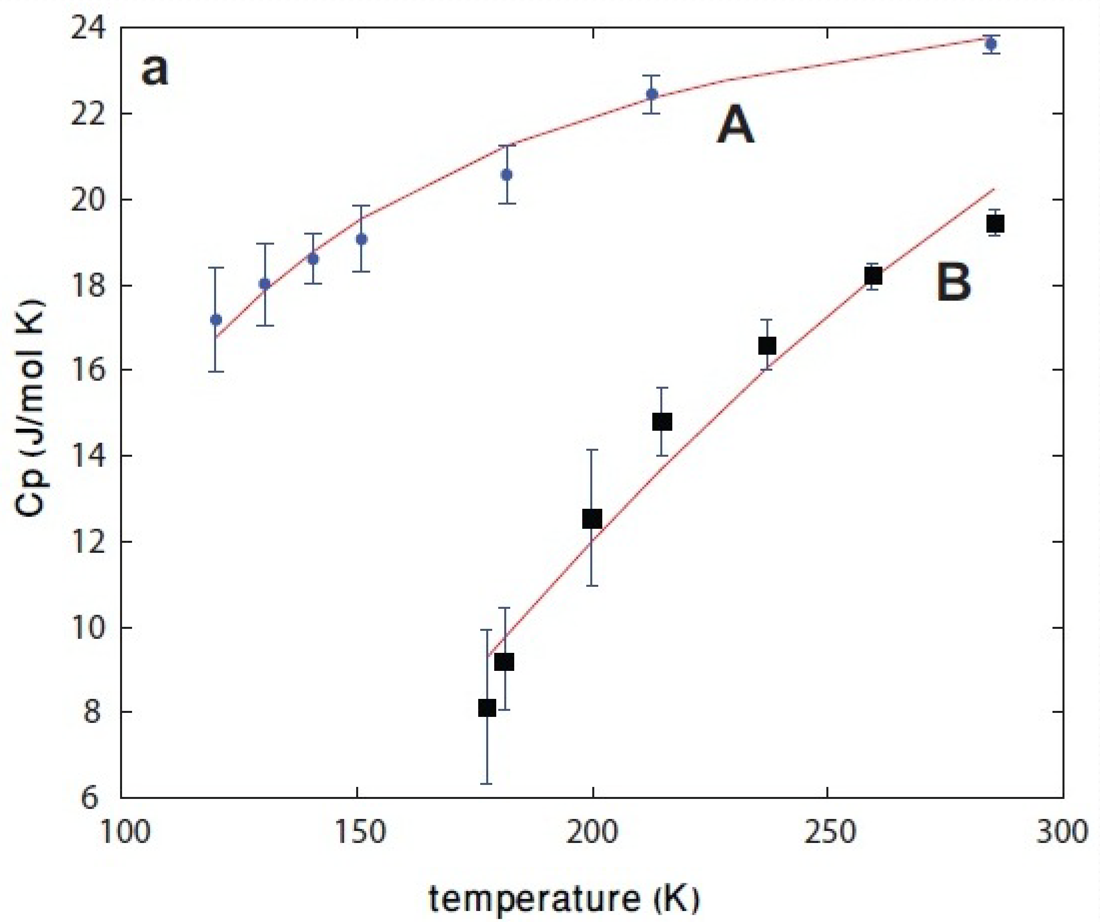
- Dulong Petit (1819)law states that when expressed in same units, heat capacity has a constant value 3R.
- Equipartition theorem: The molecules in thermal equilibrium have the same average energy associated with each independent degree of freedom of their motion and that the energy is
- Dulong & Petit used Maxwell-Boltzman statistics

Einstein introduced

- First quantum explanation of the underlying concept and showed the temperature dependence of heat capacity.
- He used Bose-Einstein statistics.
- The Einstein's relation for heat capacity is,

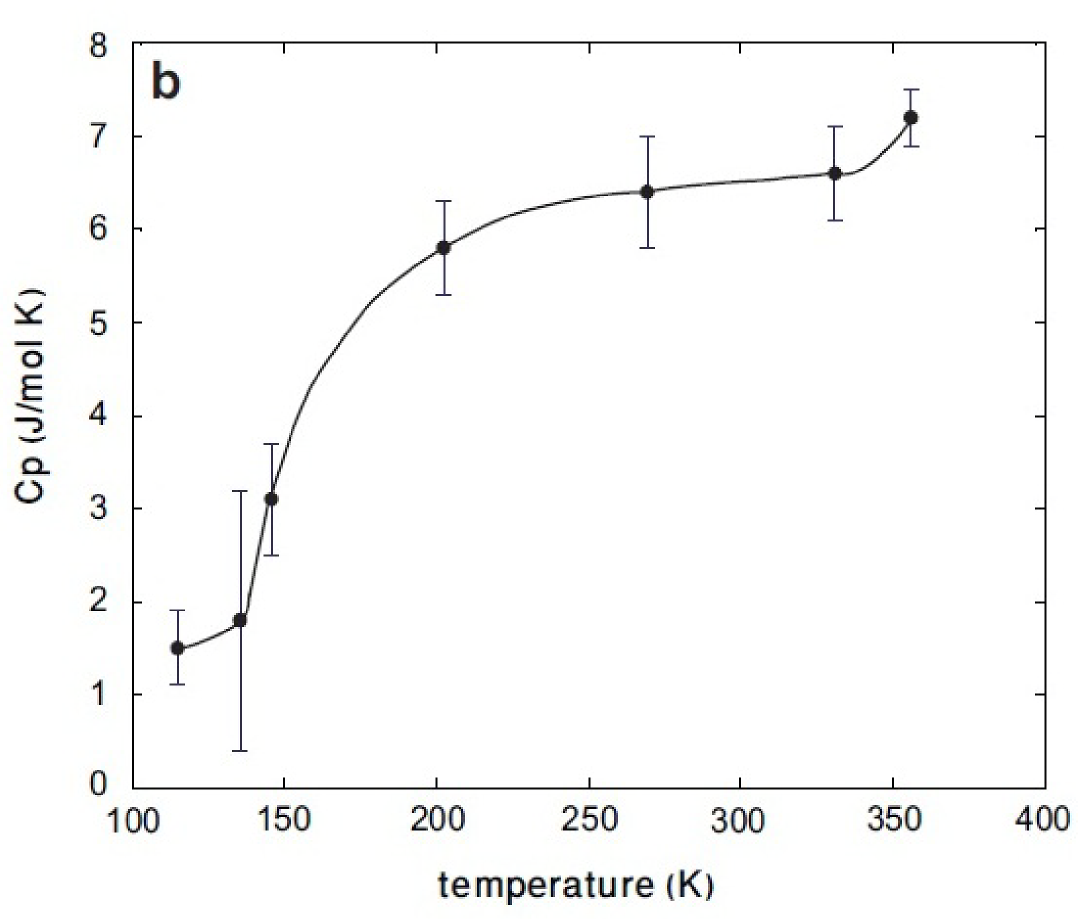
$$C_V = 3N k_B \left(\frac{\theta_E}{T}\right)^2 \frac{e^{(\theta_E/T)}}{(e^{(\theta_E/T)} - 1)^2}$$

Einstein fit on Cu and Al data

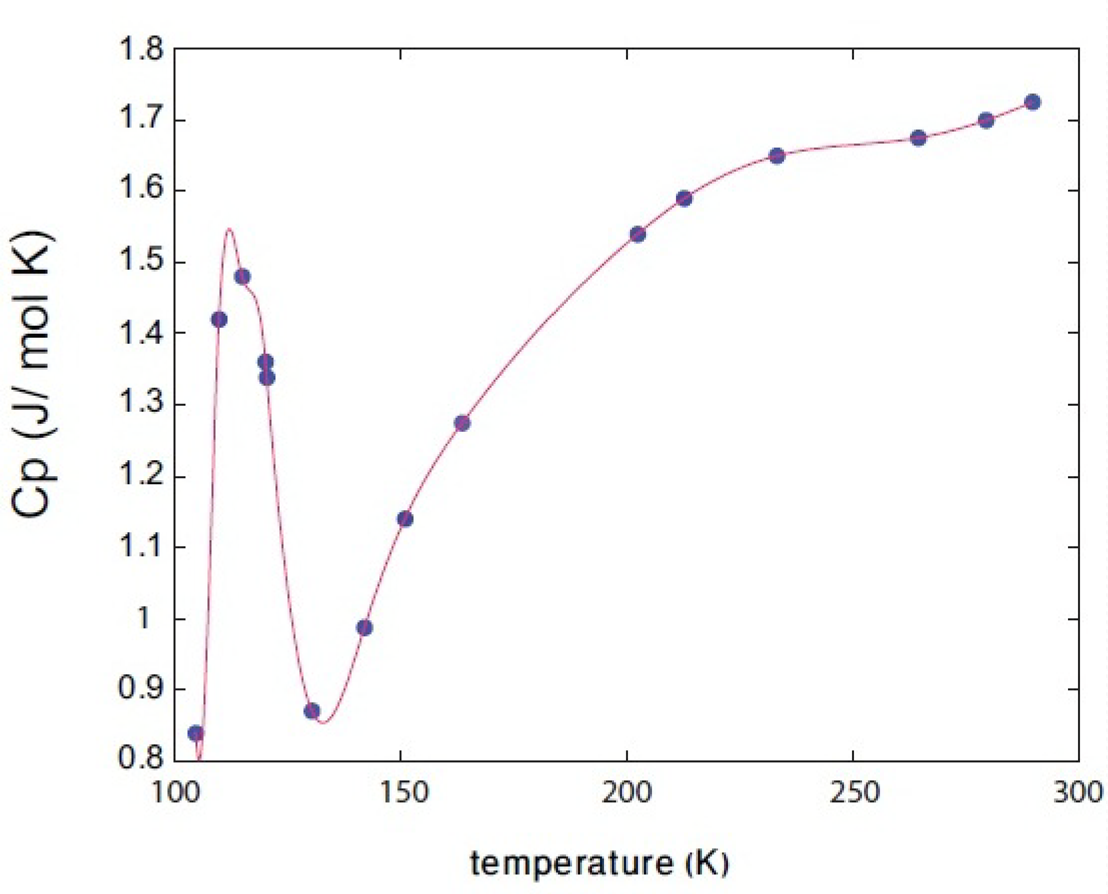


- The published value of Einstein temperature of Cu is from 210-280 K and 269 K for Al.
- We obtained 278 K for Cu and 222 K for Al.

Graphite


b
Cp (J/mol K)
0
1
2
3
4
5
6
7
8
100
150
200
250
300
350
400
temperature (K)

Verwey transition of $FeO.Fe_2O_3$



Verwey transition is observed around 120-130 K.

Conclusion

- Deviation from Dulong Petit Law
- Quantization of Harmonic Oscillators
- Observe Metal-Insulator transitions
- Understanding to keep temperature stable
- Superconductors can also be studied

Thanks