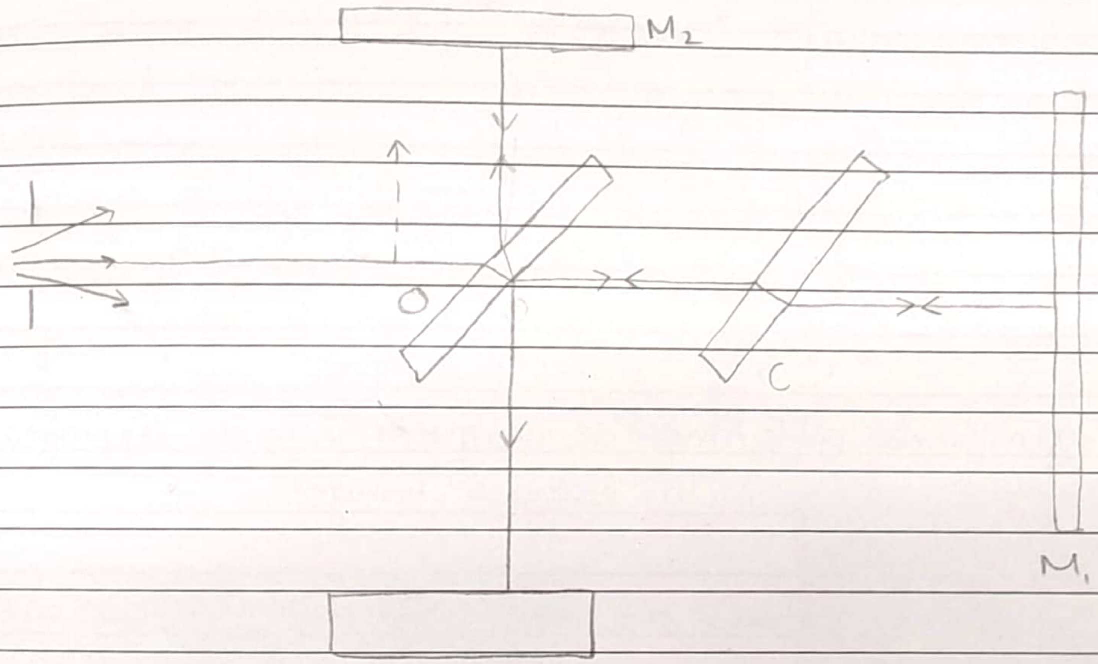
Friday, 17th November 2023

2.9: Michelson-Morley Interferometer



Michelson-Morley Interferometer

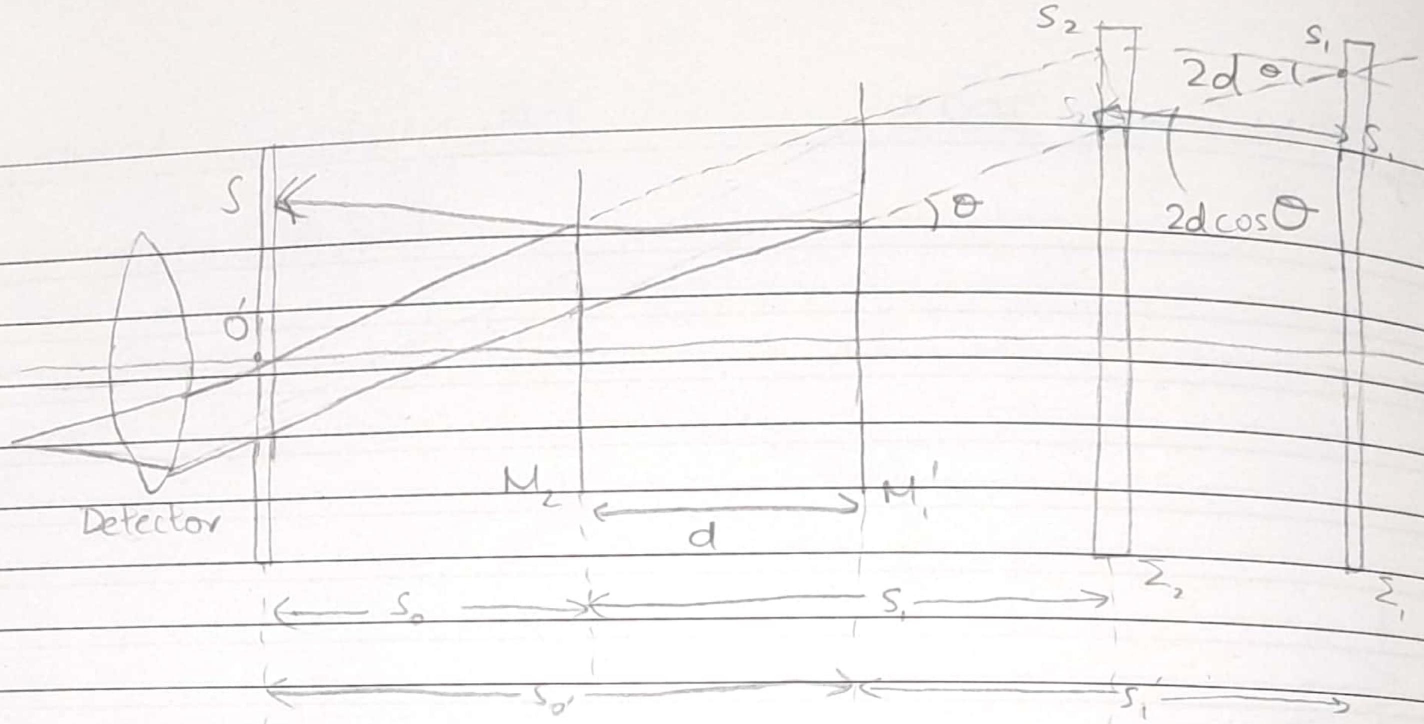
C = compensator plane → negates the effect of dispersion [would not be necessary for ~~monochromatic~~ lasers]

At O, beamsplitter is ~~simply~~ an uncoated glass plate, the relative phase shift resulting from two reflection is π rad.

There will be destructive interference will exist

$$2d\cos\theta_m = m\lambda_0$$

→



Using a laser beam, and aligning the components in an appropriate way, we ~~ne~~ end up with circular ~~fringes~~ fringes.

Each remaining ring broadens as more and more fringes vanish at the center.

→ We mounted the components onto the breadboard and started aligning the laser ~~beams~~ beam to get a consistent one dot onto the screen.

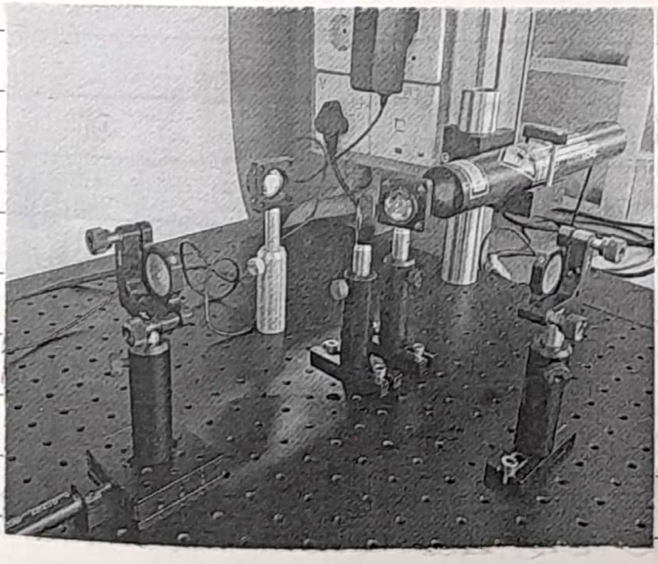
Friday, 1st December 2023

→ We aligned our laser beams and then used ~~two~~ two plano-convex lenses (~~tw~~ one with a focal length of 35mm and placed b/w ~~a~~ the laser and beam splitter, and the other one with a focal length of 25.4mm and placed before target screen) to get the ~~fringe~~ concentric fringe pattern of dark and bright circles.

→ The lens of focal length 25.4mm is there to magnify the fringes on the target screen, so that it is easy to count the fringes ~~w~~ when the servo controlled motor is used.

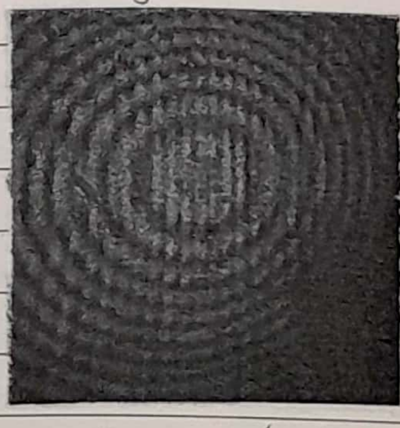
→

Q.1-



Experimental Setup

Fringe Pattern



Fringe Pattern (zoomed in)

Q.2-

→ To measure the wavelength of the laser:

①. ~~use the computer~~ We used 'Kinesis', the computer software which is connected to our Thor lab Apparatus (All the components ~~of our on~~ in our experimental setup ~~is connecte~~ is from Thor Labs).

②. ~~Using this We~~ We then used the servo controlled motor to move M_2 by a fixed distance of 0.010 mm; with ~~the~~ an acceleration of $0.03\,mms^{-2}$ and a max. velocity (when reached) = $0.0003\,mms^{-1}$

③. Moving M_2 causes the fringes to enter / exit the centre of the fringe pattern.

④. We then count the number N of bright fringes that appear during this movement.

⑤. ~~We use the~~ We ~~λ~~ then use the formula,

$$\lambda = \frac{2\Delta d}{N}$$

λ: wavelength of laser

Δd: distance moved by laser (0.01 mm)

to get the wavelength of the ~~laser~~ HeNe laser

⑥. We repeat ④ and ⑤ 5 times to get the data:

Reading	Number of Fringes Moved	Δd/mm	Square Deviation $(\Sigma(\bar{N}-N))$
1	30	0.01	1.96
2	32	0.01	0.36
3	31	0.01	0.16
4	32	0.01	~~0.16~~ 0.36
5	33	0.01	~~0.36~~ 2.56

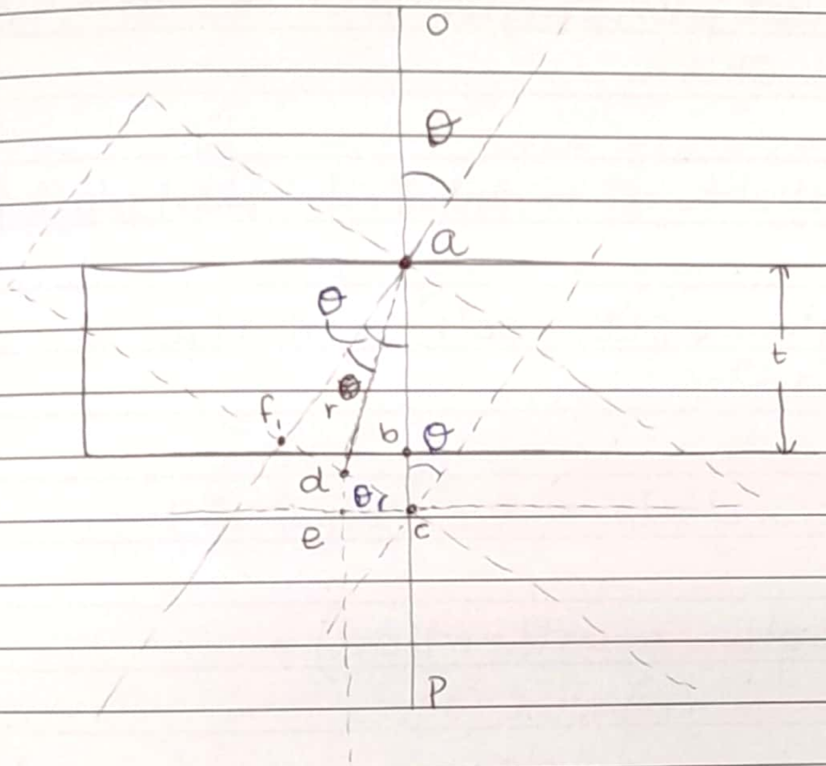
$N_{av.}$ = 31.6.

$$\lambda_{av} = \frac{2\Delta d}{N_{av}} = \frac{2(0.01)}{31.6} = \cancel{6.4} \; 6.37 \times 10^{-4}\,\cancel{mm}$$

$$= 6.37 \times 10^{-7}\,m = 637\,nm$$

→ Uncertainties calculated at the end

Q.3-



- OP is the original direction of light normal to the glass plate
- $ad = \dfrac{t}{\cos r}$
- $de = dc \sin\theta$
  $= (f_1c - f_1d)\sin\theta$
  $= t(\tan\theta - \tan r)\sin\theta$
- $bc = \dfrac{t}{\cos\theta} - t$

Q.4- ~~Increase in Optical path len~~

Increase in Optical Path Length as light is normal to the ~~glane~~ glass plate and traverses ~~twice~~ it twice

$= 2n_g t - 2(1)t$

$= 2(n_g - 1)t = N\lambda$ (later on we use n_g as n)

where n_g is the refractive index of the glass plate and 1 is the refractive index of air.
N is the number of fringes ~~created~~ introduced
~~When~~ when ~~plan~~ ~~pa~~ plate was inserted

~~For the~~ $\rightarrow$

For the change in optical path length after rotations, we must consider ~~two~~ two scenarios. ~~OP is th~~

1) When the light is parallel to the normal of the ~~plan~~ plate (i.e. before rotation):

Total optical path length b/w a and c $= \cancel{a}(n)t + bc(1) = nt + bc$

2) When the light is at an angle θ to the normal of the plate (i.e. after rotation):

Total optical path length b/w a and c $\cancel{= ad(n(n)(ad) +}$

$= (n)ad + (1)de$

$= nad + de$

So, for the total change in path length

$= 2(nad + de) - 2(nt + bc) \quad = 2(nad + de - nt - bc)$

$\therefore \quad 2(nad + de - nt - bc) = N\lambda$

$\Rightarrow \cancel{2}\dfrac{nt}{\cos r} + t\,\cancel{\tan\theta \tan r \sin\theta} - t\tan r\sin\theta - nt - \dfrac{t}{\cos\theta} + t = \dfrac{N\lambda}{2}$

Using Snell's Law: $n\sin r = \sin\theta$

$\longrightarrow$

$$\frac{nt}{\cos r} + \frac{t\sin^2\theta}{\cos\theta} - \frac{t\sin r\sin\theta}{\cos r} - nt - \frac{t}{\cos t} + t = \frac{N\lambda}{2}$$

$$\frac{nt}{\cos r} + \frac{t}{\cos\theta}\left(\sin^2\theta - 1\right) - \frac{t\sin r}{\cos r}\sin\theta - nt - \frac{t}{\cos\theta} + t = \frac{N\lambda}{2}$$

$$\frac{nt}{\cos r} - t\cos\theta - t\frac{\sin r}{\cos r}\sin\theta - nt + t = \frac{N\lambda}{2}$$

$$\frac{t}{\cos r}\left(n - \frac{1}{n}\sin^2\theta\right) - t\cos\theta - nt + t = \frac{N\lambda}{2}$$

$$\frac{t}{\sqrt{1-\sin^2 r}}\left(n - \frac{1}{n}\sin^2\theta\right) - t\cos\theta - nt + \cancel{t}\ t = \frac{N\lambda}{2}$$

$$\frac{tn}{\sqrt{1-\frac{1}{n^2}\sin^2\theta}}\left(1 - \frac{1}{n^2}\sin^2\theta\right) - t\cos\theta - nt + t = \frac{N\lambda}{2}$$

$$nt\left(1 - \frac{1}{n^2}\sin^2\theta\right)^{1/2} = \frac{N\lambda}{2} + t\cos\theta + t(n-1) = \frac{N\lambda}{2} + t\left(\cos\theta + n - 1\right)$$

$$n^2t^2 - t^2\sin^2\theta = \frac{N^2\lambda^2}{4} + N\lambda t\left(\cos\theta + n - 1\right) + t^2\left(\cos\theta + (n-1)\right)^2$$

$$n^2t^2 - t^2 + t^2\cos^2\theta = \frac{N^2\lambda^2}{4} + N\lambda t\cos\theta + Nn\lambda t - N\lambda t$$

$$+ t^2\left(\cos^2\theta + 2\cos\theta(n-1) + (n-1)^2\right)$$

$$\cancel{n^2t^2} - t^2 = N^2\lambda^2 + N\lambda t\cos\theta + Nn\lambda t - N\lambda t + 2t^2 n\cos\theta$$

$$-2t^2\cos\theta + \cancel{n^2t^2} - 2nt^2 + t^2$$

$$= \frac{N^2\lambda^2}{4t} + N\lambda\cos\theta + Nn\lambda - N\lambda + 2tn\cos\theta - 2t\cos\theta - 2tn$$

$$-2t = \frac{N^2\lambda^2}{4t} + N\lambda\cos\theta + Nn\lambda - N\lambda + 2nt\cos\theta - 2t\cos\theta - 2nt$$

$$-\frac{N^2\lambda^2}{4t} - 2t - N\lambda\cos\theta + N\lambda + 2t\cos\theta = Nn\lambda + 2tn\cos\theta - 2nt$$

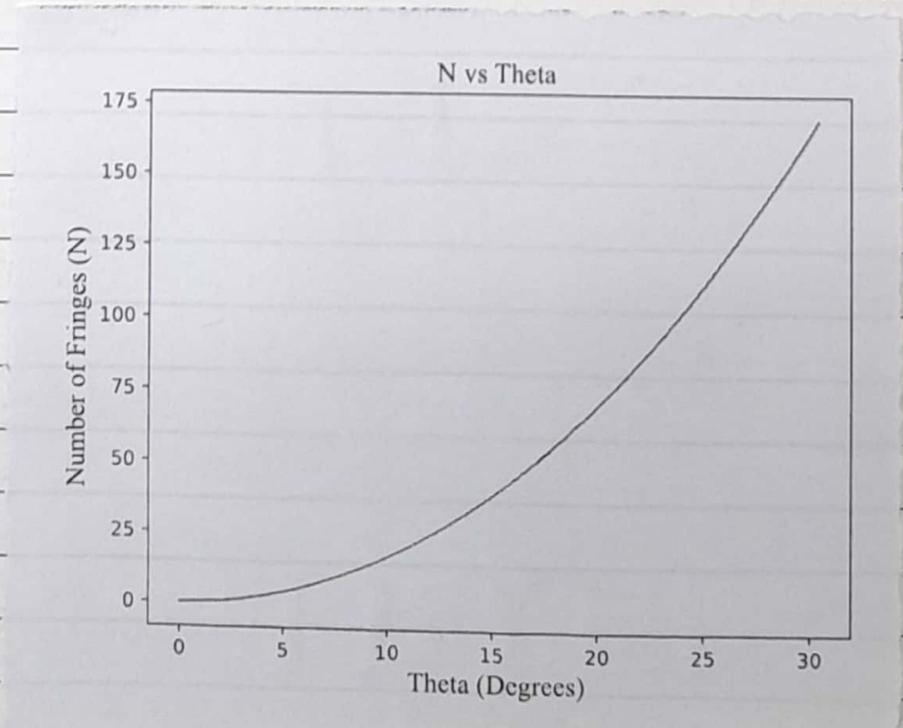
$$-\frac{N^2\lambda^2}{4t} - (2t + N\lambda\cos\theta - N\lambda - 2t\cos\theta) = Nn\lambda - n(1-\cos\theta)2t$$

$$-\frac{N^2\lambda^2}{4t} - (2t - N\lambda)(1-\cos\theta) = n\left[N\lambda - 2t(1-\cos\theta)\right]$$

$$(2t - N\lambda)(1-\cos\theta) + \frac{N^2\lambda^2}{4t} = n\left[2t(1-\cos\theta) - N\lambda\right]$$

$$\therefore\ n = \frac{(2t-N\lambda)(1-\cos\theta)}{\left[2t(1-\cos\theta) - N\lambda\right]}$$

Q.5-



Q.6-

Thickness of glass plate = 1.04 ± 0.01 mm

Rotating the Glass Slide:-

S.No	Rotation/°	Fringes Displaced (N)	Square Deviation ($\Sigma\epsilon$)
1	20°	43	43
2	20°	48	2.56
3	20°	53	11.56
4	20	49	0.36
5	20	55	29.16
	20	(Av. Fringes) 49.6	86.64

$\Rightarrow n = \frac{(2t - N\lambda)(1-\cos\theta)}{2t(1-\cos\theta) - N\lambda} = 1.32$ ~~Average Fringes = 49.6~~

~~(from Q.2)~~

Uncertainty in N: $\sqrt{\frac{\Sigma d}{5}} = 4.16 = s$

~~ΔN~~ $\Delta N = \frac{s}{\sqrt{4}} = 2.08$

Uncertainty in θ : $\Delta\theta = \frac{0.2/2}{\sqrt{6}} = \frac{0.1}{\sqrt{6}} = 0.04$

~~Uncertainty~~ (from Q.2)

~~P Propagation:-~~ $\Delta d = \frac{0.01 \times 10^{-2}/2}{\sqrt{3}} = 0.000289\,mm$

Uncertainty in N $= \Delta N = \sqrt{\frac{1.96 + 2(0.36) + 0.16 + 2.56}{5}} = 1.039$

$\Delta\lambda$ on last page

$\frac{d\lambda}{dN} = -2\Delta d \frac{1}{N^2} \Rightarrow (\Delta\lambda)_N = \left|\frac{2\Delta d}{N^2}\right| \Delta N = 21.098\,nm$ $\quad \frac{d\lambda}{d(\Delta d)} = \frac{2}{N} \Rightarrow (\Delta\lambda)_{\Delta d} = \left(\frac{2}{N}\right)\Delta d = 1.84 \times 10^{-5}\,mm = 1.84\,nm$

Propagation of Uncertainties:-

$$\frac{\partial n}{\partial \theta} = \frac{\partial}{\partial \theta}\left[\frac{(2t-N\lambda)(1-\cos\theta)}{2t\,\cancel{\cos}(1-\cos\theta)-N\lambda}\right]$$

$$\frac{\partial n}{\partial \theta} = (2t-N\lambda)\left[\frac{[2t(1-\cos\theta)-N\lambda][\sin\theta]-[1-\cos\theta][2t\sin\theta]}{(2t(1-\cos\theta)-N\lambda)^2}\right]$$

$$= \sin\theta\,(2t-N\lambda)\left[\cancel{2t}-2t\cancel{\cos\theta}-N\lambda-\cancel{2t}+2t\cancel{\cos\theta}\right]$$

$$= \sin\theta\,(2t-N\lambda)(-N\lambda)$$

$$\Rightarrow (\Delta n)_\theta = \left|\frac{\sin\theta(2t-N\lambda)(N\lambda)}{(2t(1-\cos\theta)-N\lambda)^2}\right| |\Delta\theta|$$

$$\frac{\partial n}{\partial N} = (1-\cos\theta)\left[\frac{(-\lambda)(2t\,(1-\cos\theta)-N\lambda)-(2t-N\lambda)(-\lambda)}{(2t(1-\cos\theta)-N\lambda)^2}\right]$$

$$= (1-\cos\theta)\left[\frac{-\lambda(2t-2t\cos\theta-N\lambda-2t+N\lambda)}{[2t(1-\cos\theta)-N\lambda]^2}\right]$$

$$\Rightarrow (\Delta n)_N = (1-\cos\theta)\left[\frac{\lambda\,2t\cos\theta}{(2t(1-\cos\theta)-N\lambda)^2}\right]\Delta n$$

$$(\Delta n)_n = 0.01764 \;;\; (\Delta n)_\theta = 2.52\times 0.04 = 0.10083$$

(from prev. page) $\Delta\lambda = \sqrt{1.84^2+21^2} = 21.1\,mm$

$$\lambda = 640 \pm 20\,nm$$

$$(\Delta n)_\lambda = (1-\cos\theta)\left[\frac{N2t\cos\theta}{(2t(1-\cos\theta)-N\lambda)^2}\right]\Delta\lambda$$

$$= \cancel{0.0276}\ 0.0279$$

$$\Rightarrow \Delta n = \sqrt{0.0176^2 + 0.10083^2 + 0.0279^2}$$
$$= 0.106$$

$$\therefore n = 1.3 \pm 0.1$$