Analyzing the Polarization State of Light through the Fourier Series

Alamdar Hussain and Muhammad Sabieh Anwar
Syed Babar Ali School of Science and Engineering, LUMS

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One can completely determine the polarization of light simply by using a polarizer and a quarter wave plate (QWP). In this experiment you will develop a method to generate as well as analyze different polarization states of light. Furthermore you will learn to utilize matrix manipulations for solving systems of equations.

Essential pre-lab reading: "*Introduction to Optics*" by F. L. Pedrotti, L. S. Pedrotti and L. M. Pedrotti, Pearson Education, 2008; (Chapter 14).

"*Linear Algebra and its Applications*" by Gilbert Strang, Cengage Learning, 2006; (Section 3.3, Page 160 to 168).

1 Test Your Understanding

1. Formulate a method to generate circular and elliptical polarizations.

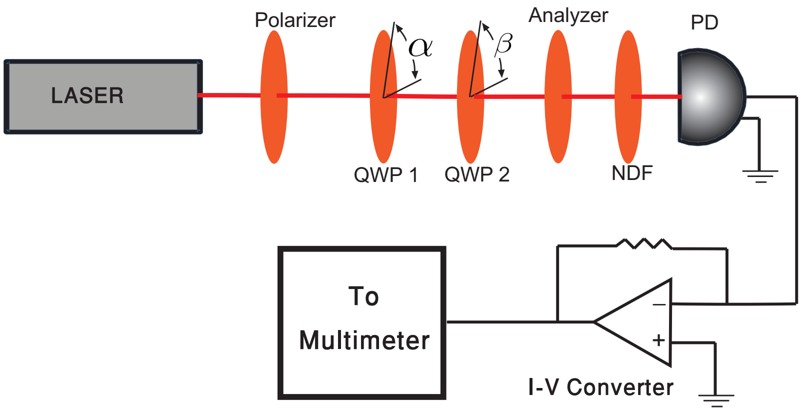

Figure 1: The polarizer and analyzer are oriented along the same direction. The retarders QWP 1 and QWP 2 are at α and β with respect to the first polarizer respectively. The red line shows the perceived path of laser.

2. Using Jones calculus, calculate the intensity output from the setup shown in Figure 1.
3. Simplify and write the answer in the form of a Fourier series of β. The Fourier series will have the form
$$f(\beta) = C_0 + \sum_{n=1}^{\infty} C_n \cos(n\beta) + \sum_{n=1}^{\infty} S_n \sin(n\beta).$$
What are the Fourier coefficients? Your answer should show that only four of the Fourier coefficients are non-zero.
4. Using matrices devise a method for solving a system of equations with more number of equations than the number of unknowns.
5. Devise a strategy for finding α using β. Finding α is equivalent to finding the polarization state of light generated by the first set of QWP and the polarizer.

2 The Experiment

A HeNe laser beam is incident on a polarizer. The polarized light then passes through two QWPs, a neutral density filter and an analyzer. Finally it is received by a photodiode.

Orient the polarizer such that it produces a horizontal polarization. All angles will then be measured with respect to this polarizer. Generate a polarization by rotating the first QWP to some angle α between 0° to 45° (try choosing angles away from these limits). Then change the angle β of the second QWP from 0° to 360° in steps of 20° and record the photodiode output for each step. If β is increased in clockwise direction then all choices of α in the clockwise direction are positive and anticlockwise ones are negative.

For finding α, use the expression for output intensity in the form of Fourier series of β and find the coefficients of this series using the pseudo inverse matrix technique (e.g. refer to the appendix and the pre-lab reading). All four coefficients are functions of α only. Use them to find α and hence the polarization state of light.

Q 1. Which coefficients can give information about the handedness (*right* or *left*) of the polarization?

Q 2. Does the calculated value for α match with the experimentally selected one?

Q 3. Repeat the experiment for three different choices of α.

Appendix

Suppose we are given equal number of equations as there are unknowns then the solution is simply given by
$$A_{m\times m} X_{m\times 1} = B_{m\times 1}$$
$$X_{m\times 1} = A^{-1}_{m\times m} B_{m\times 1}$$
where A is the matrix of coefficients, X is a column vector of unknowns or variables and B represents the right hand sides of the equations.

However if there are more number of equations than unknowns then A becomes a rectangular matrix and A^{-1} becomes non trivial. Suppose there are m equations with n unknowns such that $m > n$ then we have

$$\begin{aligned} A_{m\times n}X_{n\times 1} &= B_{m\times 1} \\ A^T_{n\times m}A_{m\times n}X_{n\times 1} &= A^T_{n\times m}B_{m\times 1} \\ X_{n\times 1} &= (A^T_{n\times m}A_{m\times n})^{-1}A^T_{n\times m}B_{m\times 1}, \end{aligned}$$

where $(A^TA)^{-1}A^T$ is called pseudo inverse of A.