Resistivity and Hall Effect Measurements

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Resistivity and Hall Effect Measurements

- Resistivity Measurements
  - General Formula
  - Van Der Pauw for arbitrary shape
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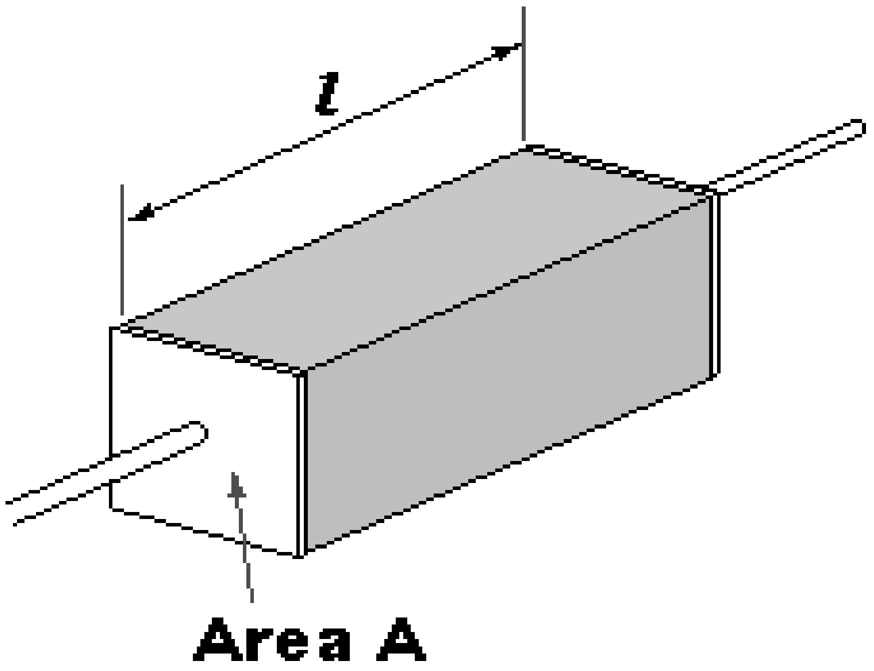
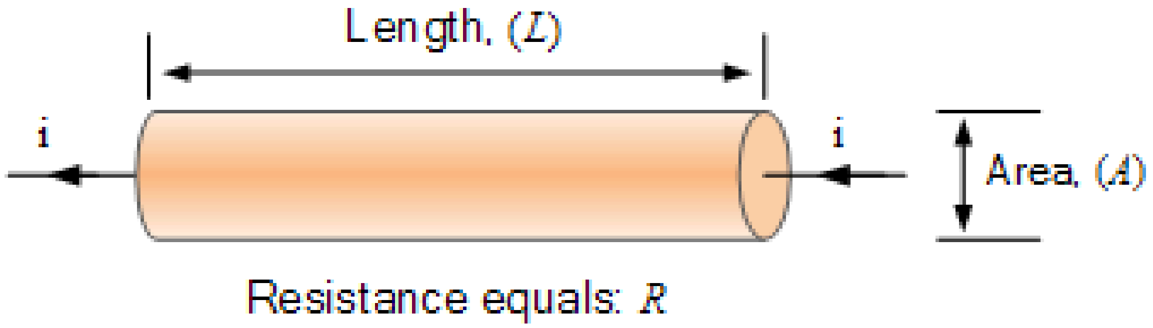
References
1. https://www.nist.gov/pml/engineering-physics-division/popular-links/hall-effect
2. https://en.wikipedia.org/wiki/Van_der_Pauw_method

Resistivity Measurements by General Formula

General Formula for the resistivity measurements

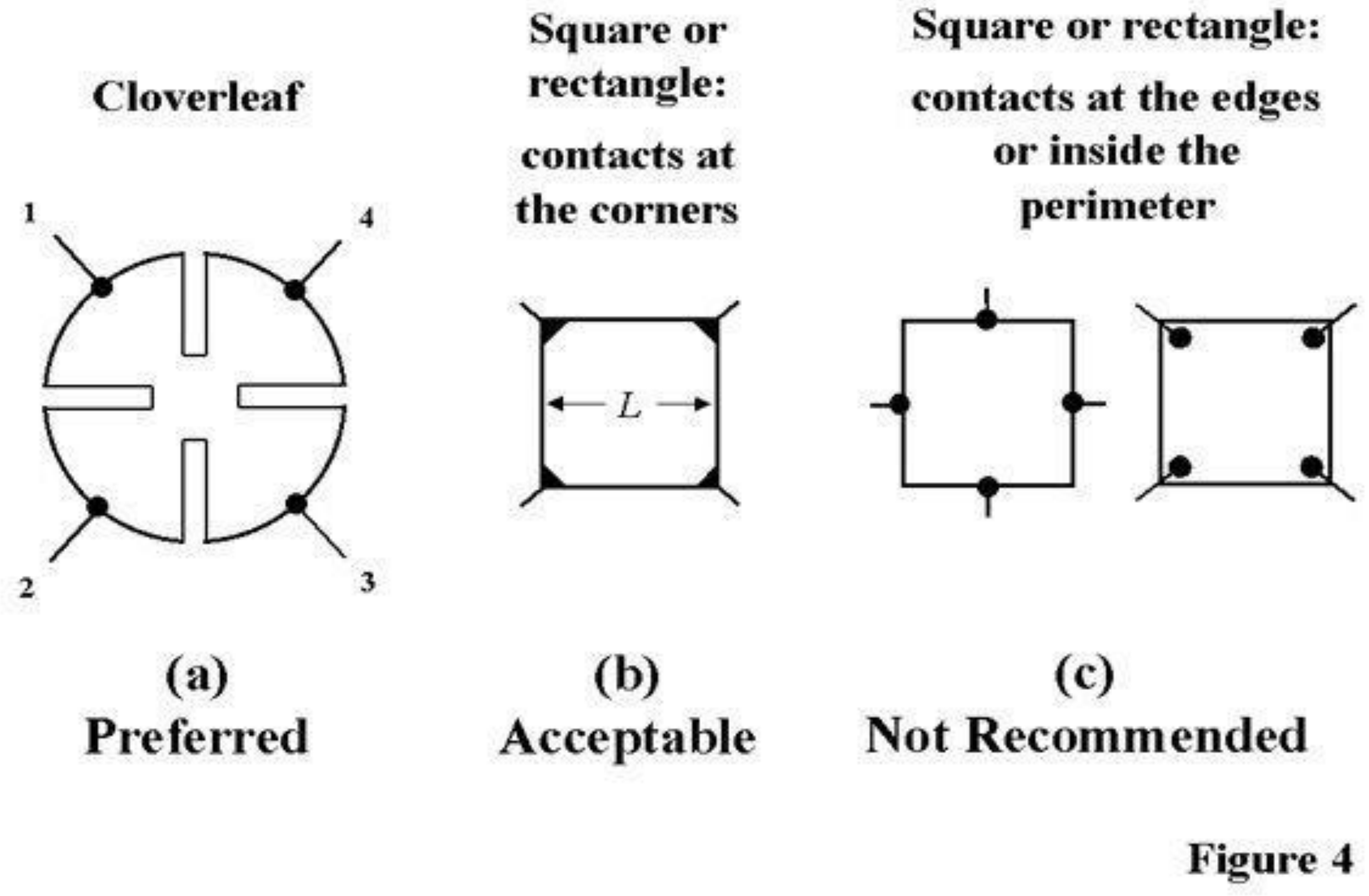
$$\rho = R\frac{A}{l}$$





Sample Geometry for Van Der Pauw

- The average diameters (*D*) of the contacts, and sample thickness (*d*) must be much smaller than the distance between the contacts (*L*). Relative errors caused by non-zero values of *D* are of the order of *D*/*L*.
- The cloverleaf design will have the lowest error due to its smaller effective contact size, but it is more difficult to fabricate than a square or rectangle.



Sample geometries for van der Pauw resistivity and Hall effect measurements

Resistivity Measurements by Van Der Pauw

Van Der Pauw formula for an arbitrary shape of sample

$$\exp(-\pi R_A/R_S) + \exp(-\pi R_B/R_S) = 1$$

which can be solved numerically for R_S

The bulk electrical resistivity ρ can be calculated

$$\rho = R_S d$$

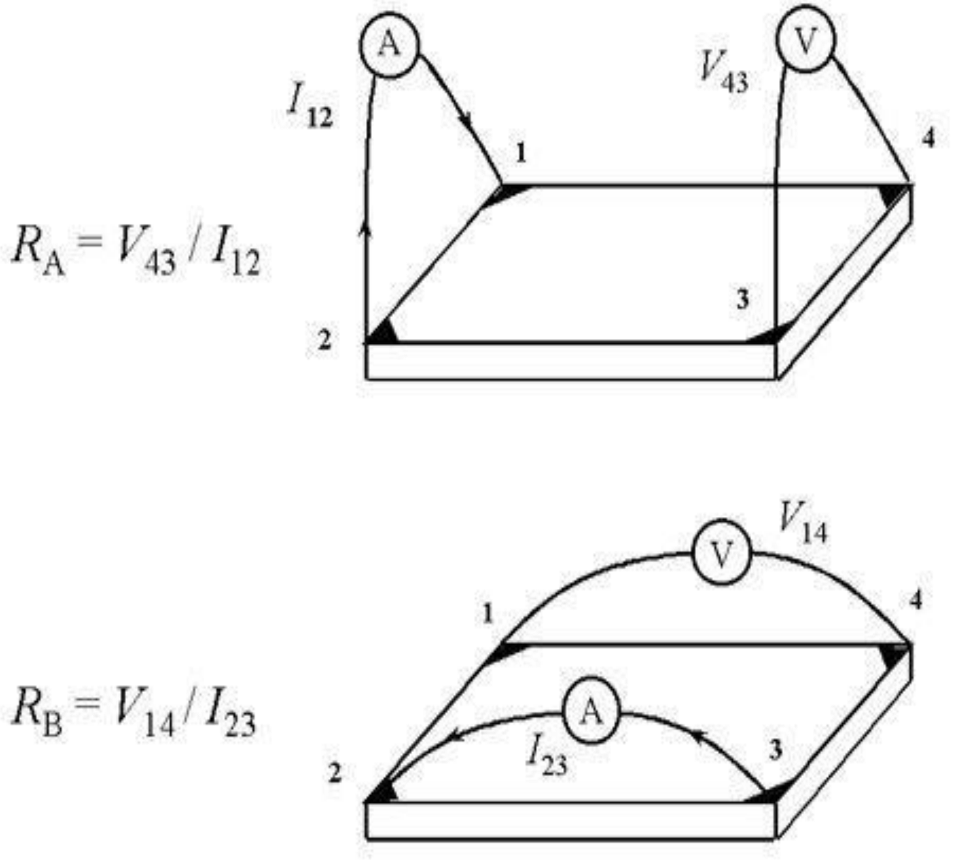


Figure 2

Sheet resistance R_S Measurements Algorithm

The sheet resistance R_S can be obtained from the two measured characteristic resistances R_A and R_B by numerically solving the van der Pauw equation by iteration

$$\exp(-\pi R_A/R_S) + \exp(-\pi R_B/R_S) = 1$$

Set the error limit $\delta = 0.0005$, corresponding to 0.05 %

Calculate the initial value of z_i, or $z_o = 2\,ln(2)/[\pi(R_A + R_B)]$

Calculate the i[th] iteration of $y_i = 1/\exp(\pi z_{i\text{-}1} R_A) + 1/\exp(\pi z_{i\text{-}1} R_B)$

Calculate the i[th] iteration of z_i where

$$z_i = z_{i\text{-}1} - [(1\text{-}y_i)/\pi] / [R_A/\exp(\pi z_{i\text{-}1} R_A) + R_B/\exp(\pi z_{i\text{-}1} R_B)]$$

When $(z_i - z_{i\text{-}1})/z_i$ is less than δ, stop and calculate the sheet resistance $R_S = 1/z_i$

The resistivity ρ is given by $\rho = R_S d$, where d is the thickness of the conducting layer

Resistivity Measurements by Van Der Pauw

Set up a dc current *I* such that when applied to the sample the power dissipation does not exceed 5 mW (preferably 1 mW) by measuring the resistance *R* between any two opposing leads (1 to 3 or 2 to 4).

$I < (200R)^{-0.5}$

Eight measurements of voltage yield the following eight values of resistance, all of which must be positive:

$R_{21,34} = V_{34}/I_{21}$, $R_{12,43} = V_{43}/I_{12}$,
$R_{32,41} = V_{41}/I_{32}$, $R_{23,14} = V_{14}/I_{23}$,
$R_{43,12} = V_{12}/I_{43}$, $R_{34,21} = V_{21}/I_{34}$,
$R_{14,23} = V_{23}/I_{14}$, $R_{41,32} = V_{32}/I_{41}$.

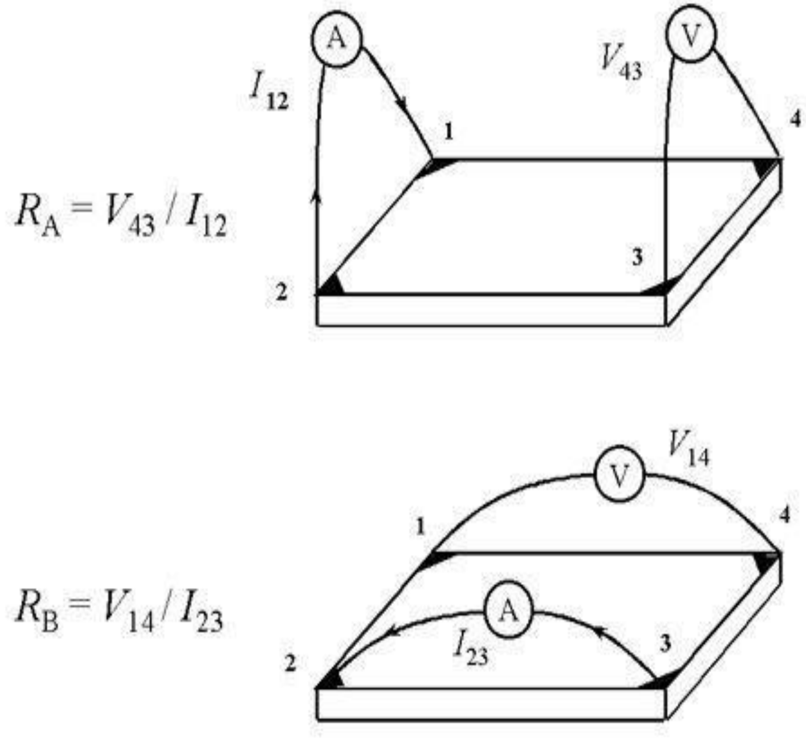


Figure 2

Resistivity Measurements by Van Der Pauw

Measurement consistency following current reversal requires that

$R_{21,34} = R_{12,43}$

$R_{32,41} = R_{23,14}$

and

$R_{43,12} = R_{34,21}$

$R_{14,23} = R_{41,32}$

The reciprocity theorem requires that

$R_A = (R_{21,34} + R_{12,43} + R_{43,12} + R_{34,21})/4$

and

$R_B = (R_{32,41} + R_{23,14} + R_{14,23} + R_{41,32})/4$

Hall Measurements

Hall Calculations

Measure and calculate the difference of the voltages for positive (P) and negative (N) magnetic fields.

$V_{13} = V_{13,\,P} - V_{13,\,N}$
$V_{24} = V_{24,\,P} - V_{24,\,N}$
$V_{31} = V_{31,\,P} - V_{31,\,N}$
$V_{42} = V_{42,\,P} - V_{42,\,N}$

Calculate carefully, to maintain the signs of measured voltages to correct for the offset voltage.

The overall Hall voltage is then

$$V_H = \frac{V_{13} + V_{24} + V_{31} + V42}{8}$$

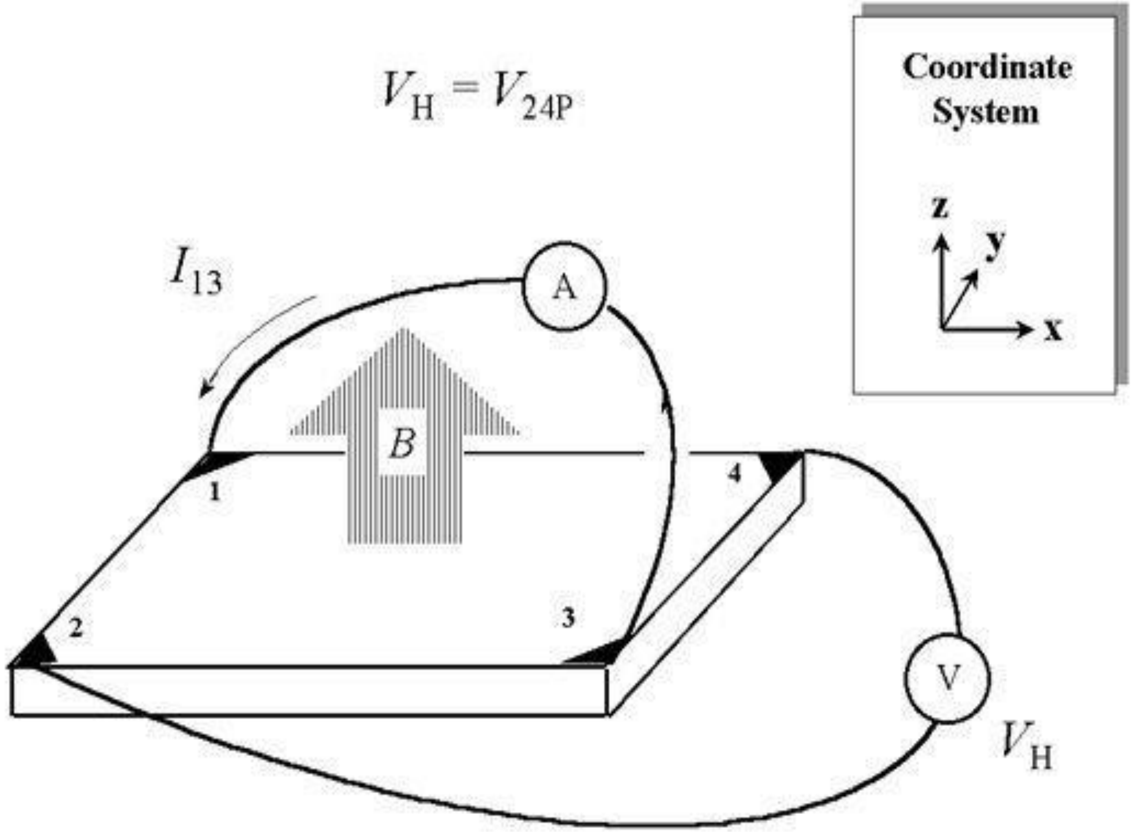


Figure 3

Hall Measurements

The sheet density ($n_s = nd$) can be calculated by

$n_s = IB/q|V_H|$

After calculating R_S from the van der Pauw equation, the Hall mobility

$\mu = |V_H|/R_S IB = 1/(q n_S R_S)$

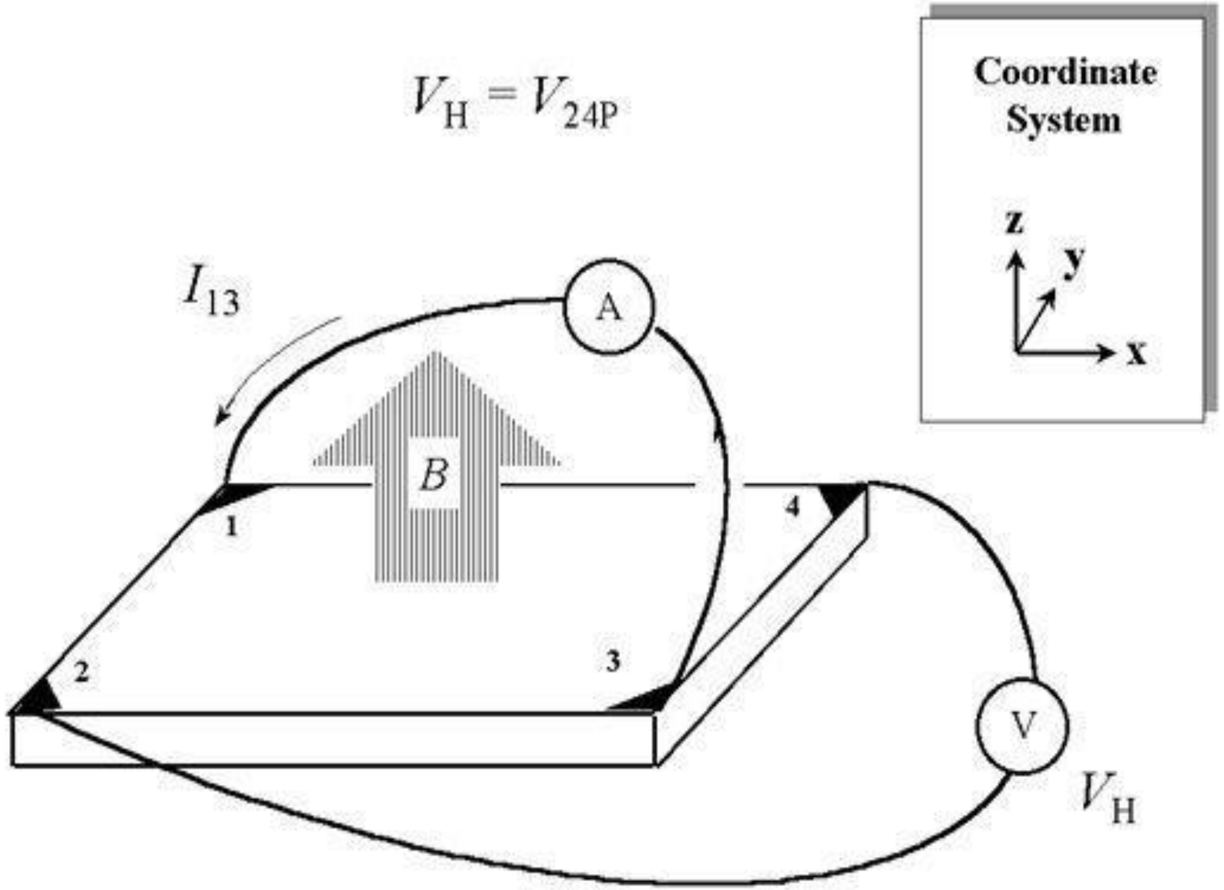


Figure 3

Resistivity and Hall Effect Measurements Sources of Error

Check the followings:

- Is the paste, epoxy or paint used for connections is compatible with the sample?
- Is the paste, epoxy or paint used for connections is good for cryogenic applications?
- Are the probes or wires making good contact to the sample?
- Are the contact *I-V* characteristics linear?
- Is any contact much higher in resistance than the others?
- Do the voltages reach equilibrium quickly after current reversal (there will be some delay for semi-insulating sample)?
- Is there visible damage (cracks, especially around the contacts)?
- Is the sample in the dark?
- Is the sample temperature uniform?
- Is there are large temperature gradients across the wiring, or dissimilar wiring materials being used?

Worksheet for Resistivity Measurements by Van Der Pauw

	$V_{diagonal}$	$I_{diagonal}$	$R_{diagonal}$	$I<(200R)^{-0.5}$	P_{diss}	
1	1.55E-05	1.0E-06	1.5E+01	0.017977937	1.5E-11	
2						
3						
I_V	$I_{[7,8]}$	$I_{[8,7]}$	$I_{[11,12]}$	$I_{[12,11]}$	I_S =	1.00E-06
	1.00E-06	1.00E-06	1.00E-06	1.00E-06		
I_H	$I_{[8,12]}$	$I_{[12,8]}$	$I_{[7,11]}$	$I_{[11,7]}$		
	1.00E-06	1.00E-06	1.00E-06	1.00E-06		
V_V	$V_{[11,12]}$	$V_{[12,11]}$	$V_{[7,8]}$	$V_{[8,7]}$		
	1.55E-05	1.54E-05	1.59E-05	1.54E-05		
V_H	$V_{[7,11]}$	$V_{[11,7]}$	$V_{[8,12]}$	$V_{[12,8]}$		
	3.08E-05	3.08E-05	3.09E-05	3.10E-05		
R_V	$R_{[7,8],[11,12]}$	$R_{[8,7],[12,11]}$	$R_{[11,12],[7,8]}$	$R_{[12,11],[8,7]}$	∑	∑/4
	15.47	15.4	15.85	15.35	62.07	15.5175
R_H	$R_{[8,12],[7,11]}$	$R_{[12,8],[11,7]}$	R[7,11],[8,12]	$R_{[11,7],[12,8]}$	∑	∑/4
	30.84	30.8	30.85	30.98	123.47	30.8675

No.	$R_{[7,8],[11,12]}$	$R_{[8,12],[7,11]}$	Z_o	Y_i	Z_i	Error	R_S	van der pau
1	15.5175	30.8675	0.0095132306	1.026423232	0.009895029	0.038584898	101.0608431	1.000377385
2			0.009895029	1.000377385	0.009900642	0.000566877	101.0035539	1.00000008
3			0.009900642	1.00000008	0.009900643	1.19962E-07	101.0035418	1
4			0.009900643	1	0.009900643	5.2564E-15	101.0035418	1
5			0.009900643	1	0.009900643	0	101.0035418	1
6			0.009900643	1	0.009900643	0	101.0035418	1
7			0.009900643	1	0.009900643	0	101.0035418	1
8			0.009900643	1	0.009900643	0	101.0035418	1
9			0.009900643	1	0.009900643	0	101.0035418	1
10			0.009900643	1	0.009900643	0	101.0035418	1
11			0.009900643	1	0.009900643	0	101.0035418	1
12			0.009900643	1	0.009900643	0	101.0035418	1
13			0.009900643	1	0.009900643	0	101.0035418	1
14			0.009900643	1	0.009900643	0	101.0035418	1
15			0.009900643	1	0.009900643	0	101.0035418	1
16			0.009900643	1	0.009900643	0	101.0035418	1
17			0.009900643	1	0.009900643	0	101.0035418	1
18			0.009900643	1	0.009900643	0	101.0035418	1
19			0.009900643	1	0.009900643	0	101.0035418	1
	Thickness t (cm)		**0.1**					
	Rersistivity P(Ω-cm)		**10.10035**					

Worksheet for Hall Measurements

Measurements

I_S	9.00E-07								
B= +5T	$V_{[7,12]P}$	$I_{[8,11]P}$	$V_{[12,7]P}$	$I_{[11,8]P}$	$V_{[8,11]P}$	$I_{[7,12]P}$	$V_{[11,8]P}$	$I_{[12,7]P}$	V_H
	1.45E-05	0.0000009	0.00001478	0.0000009	0.00001441	0.0000009	0.0000149	0.0000009	
B= -5T	$V_{[7,12]N}$	$I_{[8,11]N}$	$V_{[12,7]N}$	$I_{[11,8]N}$	$V_{[8,11]N}$	$I_{[7,12]N}$	$V_{[11,8]N}$	$I_{[12,7]N}$	
	0.00001435	0.0000009	0.00001459	0.0000009	0.00001437	0.0000009	0.00001467	0.0000009	
Diff	1.90E-07		1.9E-07		4E-08		2.3E-07		8.125E-08

Calculations

I_S	V_H	B	q	n_S	n_S	n	R_S	μ
A	V	T=web/m^2	C	m^2	cm^{-2}	cm^{-3}	Ω/squar	$cm^{-2}V^{-1}s^{-1}$
9.00E-07	8.125E-08	5	1.60E-19	3.46E+20	3.46E+16	3.46E+17	92.7342728	1.95E-04