Differential detection of Faraday Rotation and Generation and Detection of Elliptically Polarized Light

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Outlines

Part I

- Polarization of light
- Jones Calculus
- Magneto Optics
- Differential detection
- Schematic of the experiment
- Results

Part II

- Birefringence
- Schematic of Experiment
- Results
- References

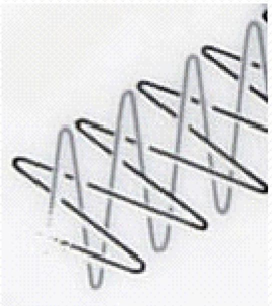
Polarization of light

▪Linear Polarization

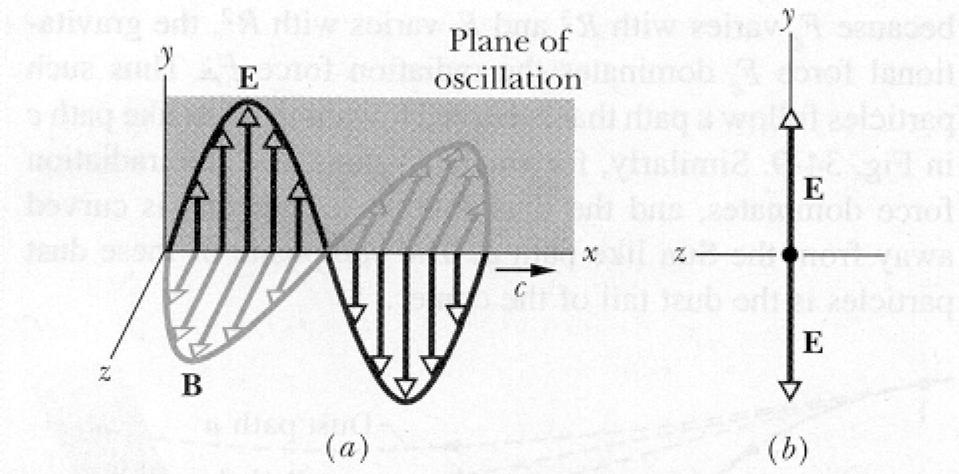
• Electric field vector oscillates along a fixed direction.

• Plane of oscillation contains **E** and **k** vectors.

• Electromagnetic wave with **E** field oscillating parallel to y axis.



Unpolarized light

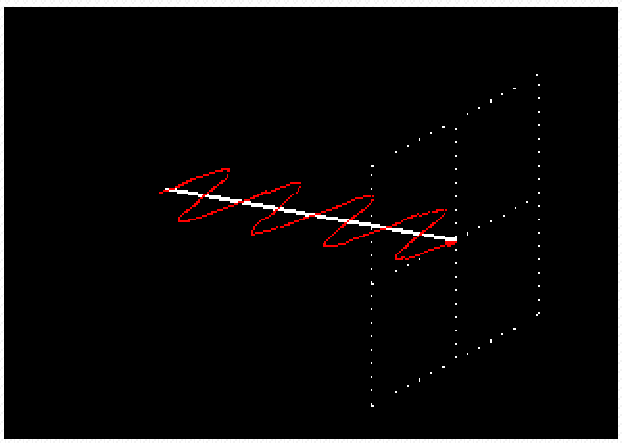


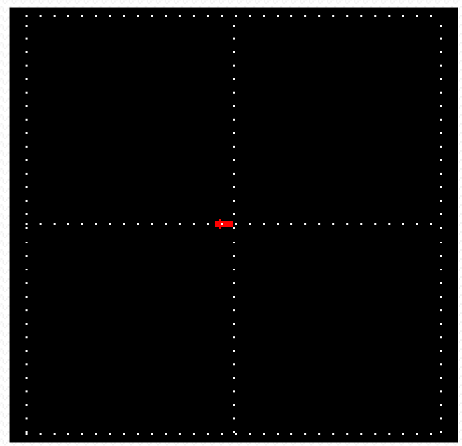
Linearly polarized

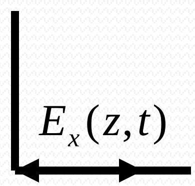
Linear polarization (cont…)

Horizontally polarized light propagating in z-direction

$$E_x(z,t) = \hat{i}\, E_{ox} \cos(kz - wt)$$



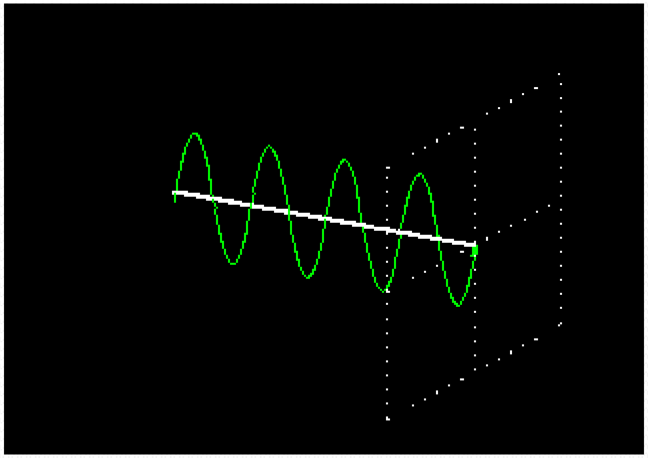


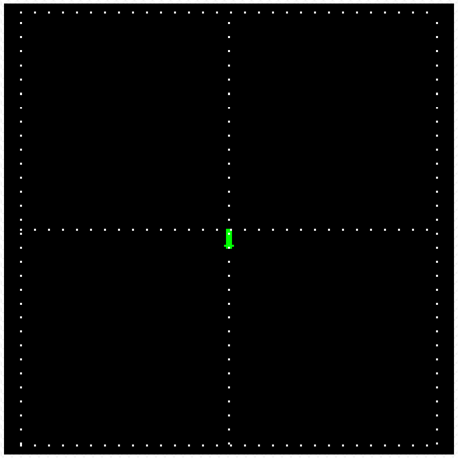


Vertically Polarized light propagating along z-direction at phase difference ε

$$E_x(z,t) = \hat{j}\, E_{0x} \cos(kz - wt + \varepsilon)$$

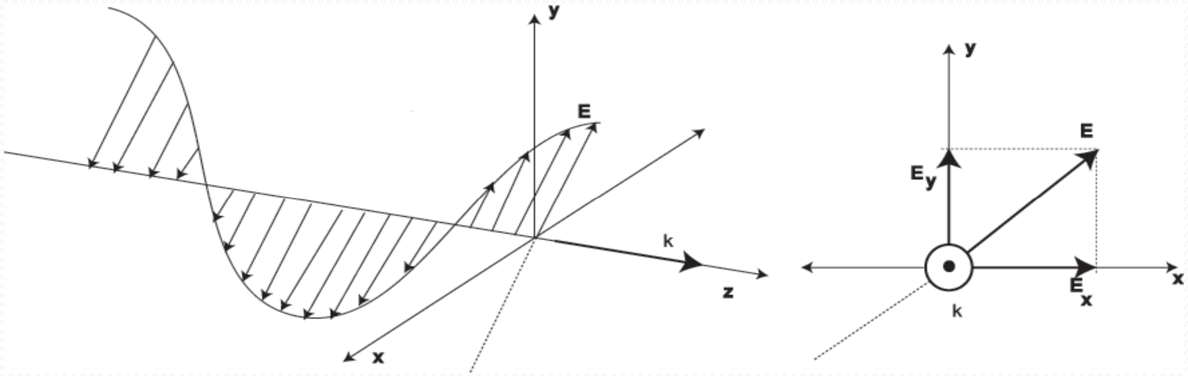
Linear polarization (cont…)





•Superposition of the two waves

$$E(z,t) = \hat{i}\, E_{0x} \cos(kz - wt) + \hat{j}\, E_{0y} \cos(kz - wt + \varepsilon)$$



Circular polarization

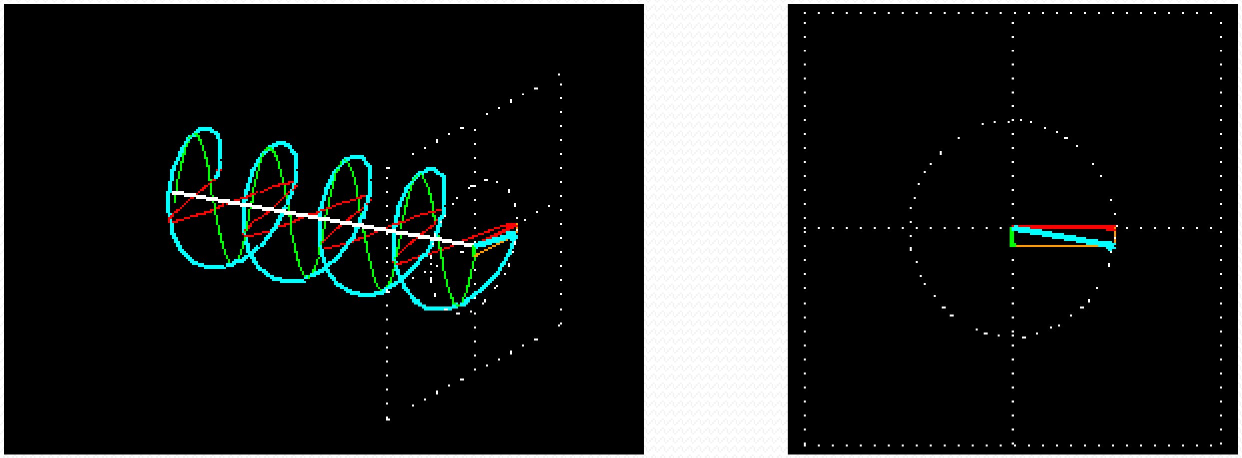
- Two orthogonal components of electric field are equal, i.e. $E_{ox} = E_{oy}$
- Phase difference of 90^o
- Electric field is time varying and magnitude remains constant

a) Right Circular Polarization

- Phase difference of $-\pi/2+2m\pi$
- Rotating clock wise

$$E = E_o[\hat{i}\cos(kz - wt) - \hat{j}\sin(kz - wt)]$$

Circular polarization (cont…)



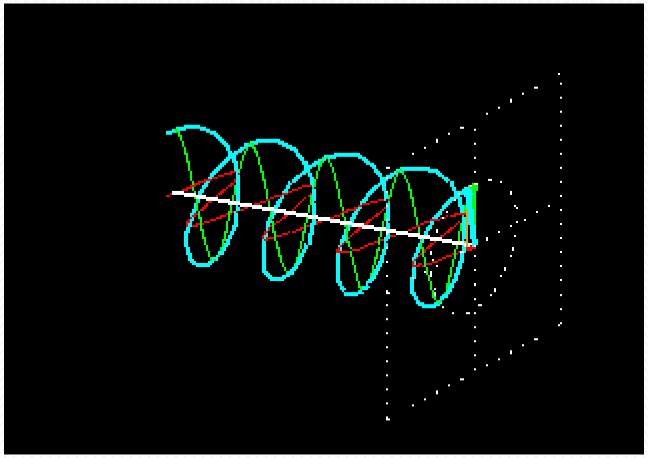
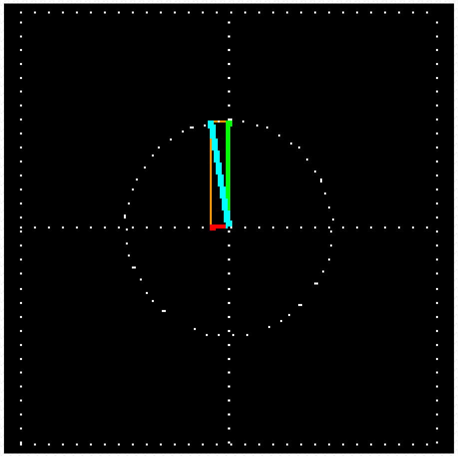
Right circular

b) Left Circularly Polarized light

- Phase shift of $\pi / 2 + 2m\pi$
- Rotates anticlockwise

$$E = E_o[\hat{i}\cos(kz - \omega t) - \hat{j}\sin(kz - \omega t)]$$

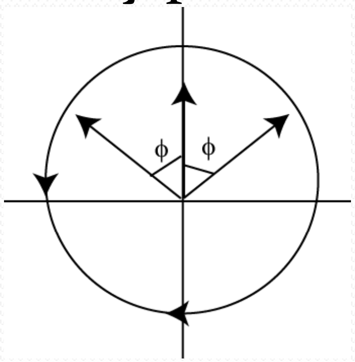
Circular polarization (cont…)

left circular

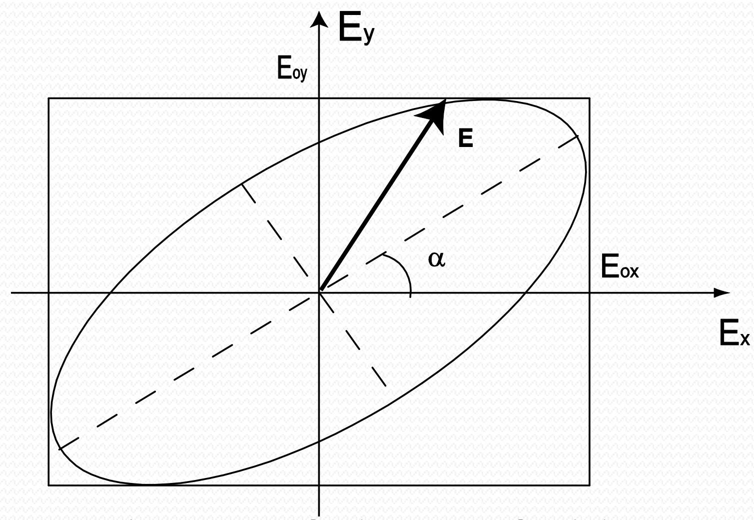
Linearly polarized light is the sum of left circularly polarized and right circularly polarized light.

$$E(z,t) = 2E_o \hat{i} \cos(kz - \omega t)$$



Elliptical Polarization

- Two orthogonal components are not equal in amplitude
- General case of linear and circularly polarized light
- Resultant field vector rotates as well as changes the magnitude



The endpoint of electric field vector sweeps out an ellipse as it rotates once around.

▪Jones Calculus

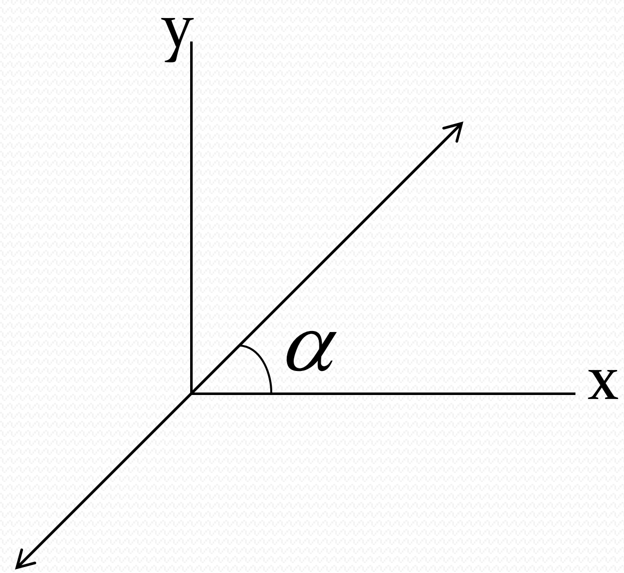
- R. Clark Jones in 1941
- For perfectly polarized light

$$E(z,t) = \hat{i}\, E_{ox} \cos(kz - \omega t) + \hat{j}\, E_{oy} \cos(kz - \omega t + \varepsilon)$$

$$E(z,t) = \begin{pmatrix} E_{ox} \\ E_{oy} e^{i\varepsilon} \end{pmatrix}$$

- For linearly polarized light

$$E(z,t) = \begin{pmatrix} \cos\alpha \\ \sin\alpha \end{pmatrix}$$



▪Jones Calculus (cont…)

- Incident Jones vector E_i is related to transmitted jones vector E_t through matrix, M

$$E_t = ME_i$$

- For beam passing through a series of optical elements

$$E_t = M_n \ldots M_3 M_2 M_1 E_i$$

▪Magneto Optics

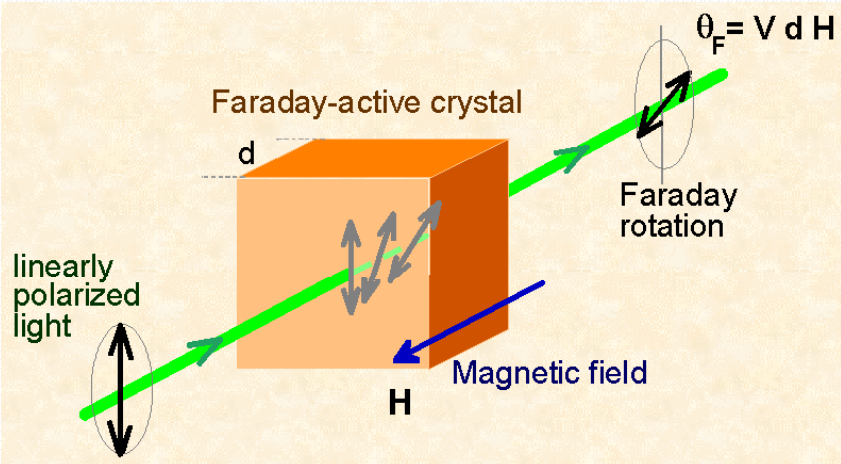
•Interaction of light and magnetism

▪History

•1813	Morrichini
•1814	Faraday
•1826	S. H. Christie
•1834	Faraday
•1844	Faraday

•Faraday Effect
•Kerr Effect

▪Faraday Effect



Faraday rotation

•Magnetically induced birefringence
•Plane of polarization of light is rotated when passes through an optically inactive medium placed in magnetic field
•Non reciprocal

Faraday Effect (cont…)

- For uniform magnetic field

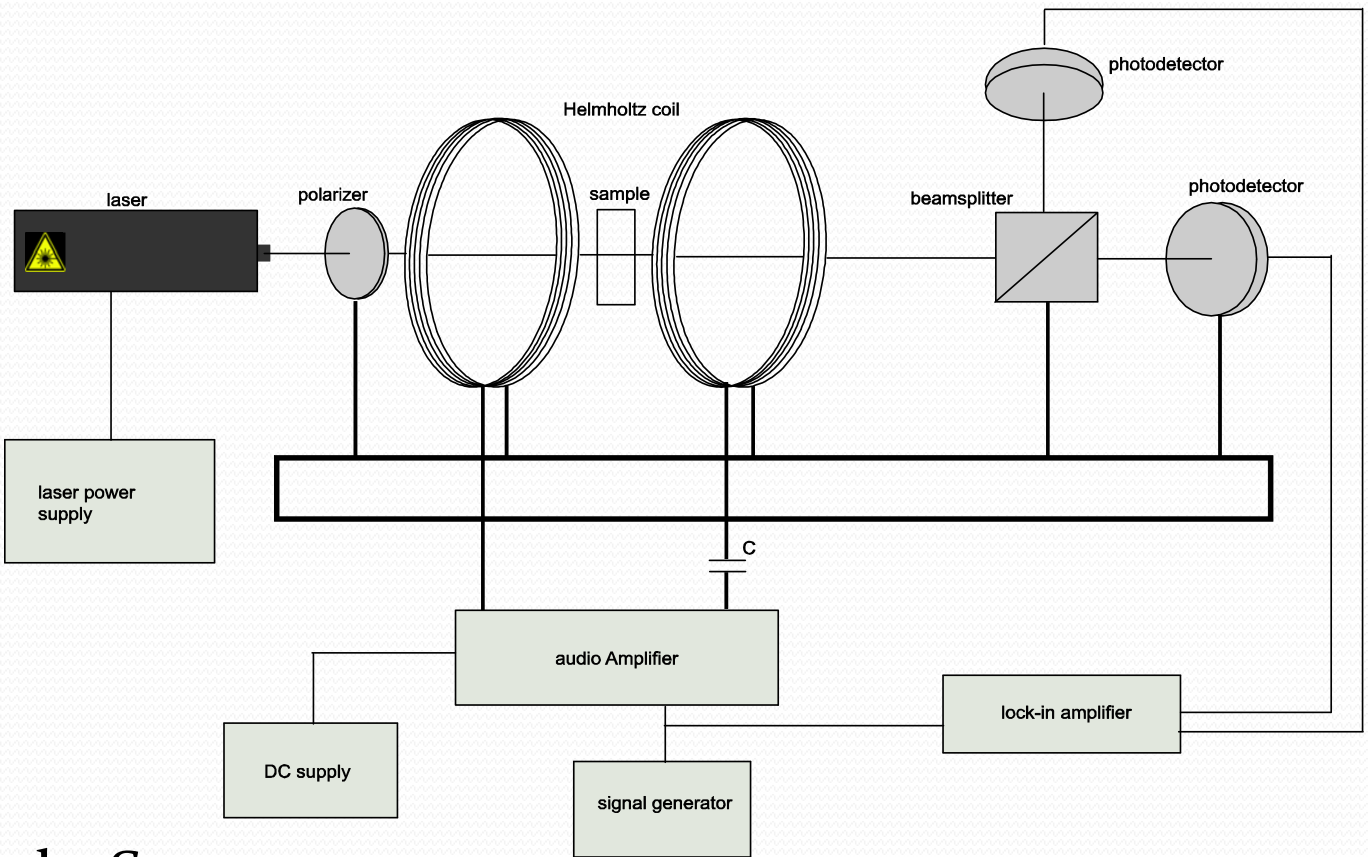
$$\theta = VBd$$

- For non-uniform magnetic field

$$\theta = V\int_0^d B(z)dz$$

- V=Verdet constant
- B=Magnetic field
- d= length of the medium
- V is function of wavelength and temperature
- Very small, of the order of micro radians per gauss

Schematic of the experiment

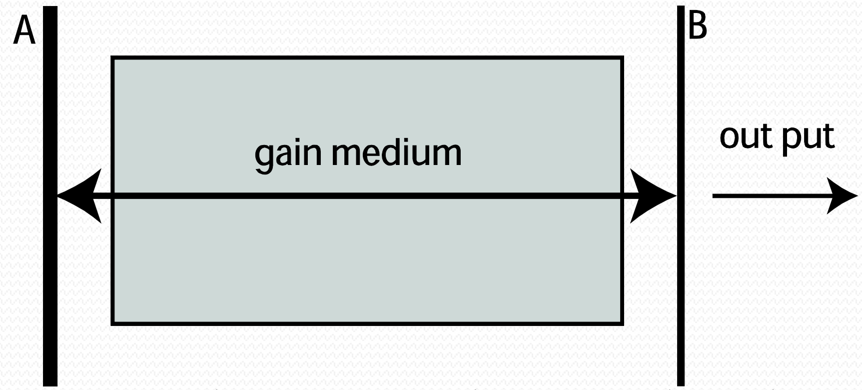


- Light Source
- Polarization rotation elements
- Detecting mechanism

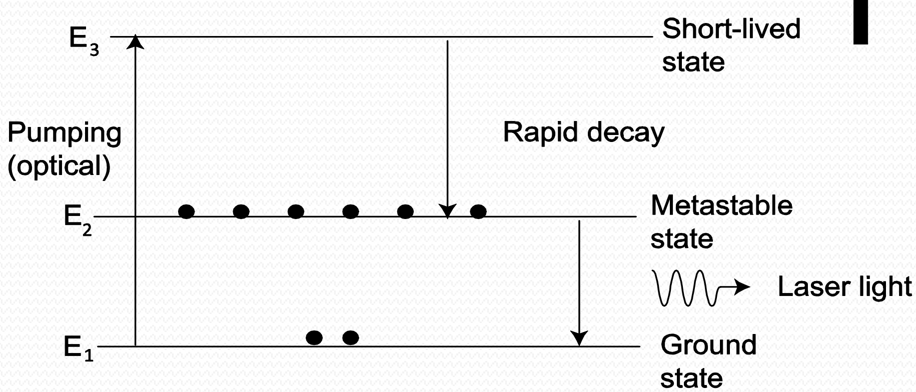
Schematic of the experiment (cont…)

Light Source

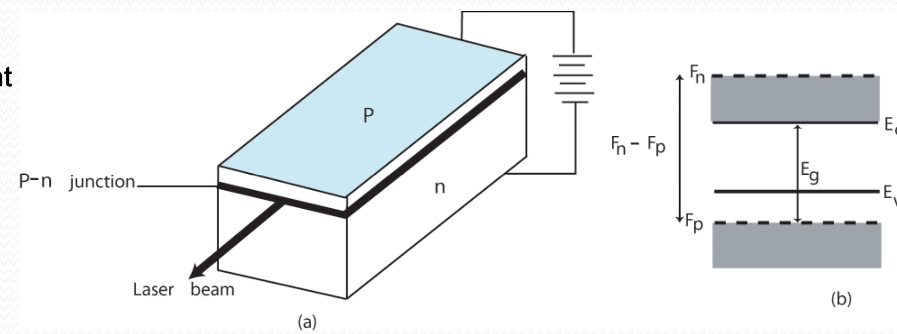
- Laser



The gain medium is placed inside a Febry-Perot resonator



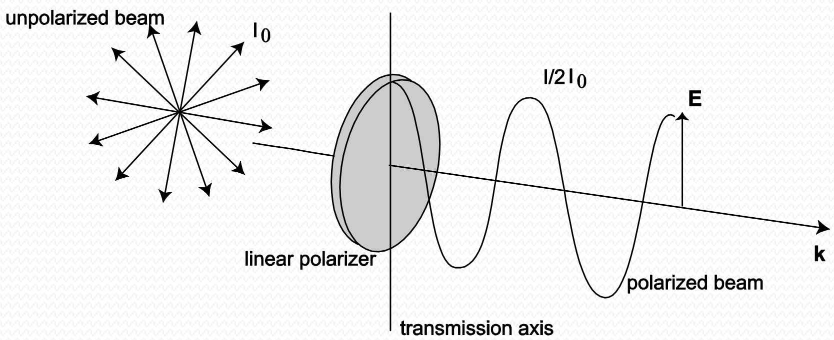
The basic three level scheme for lasers

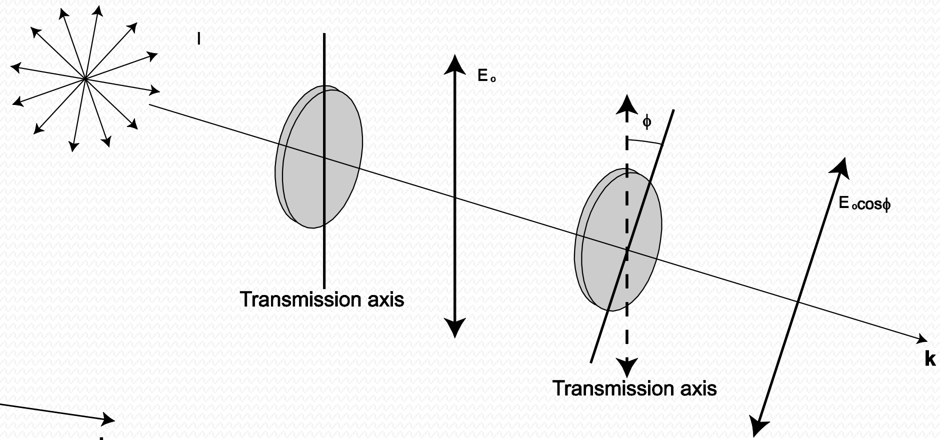


AlGaN diode laser

Schematic of the experiment (cont…)

- Linear polarizer





Mauls' law

Un polarized light is polarized by means of a linear polarizer

Schematic of the experiment (cont…)

▪Source of magnetic field

•DC Source

Large ,bulky, expensive magnets required

•AC Source

Helmholtz coil, Solenoid

to replace DC magnetic source

▪Helmholtz Coil

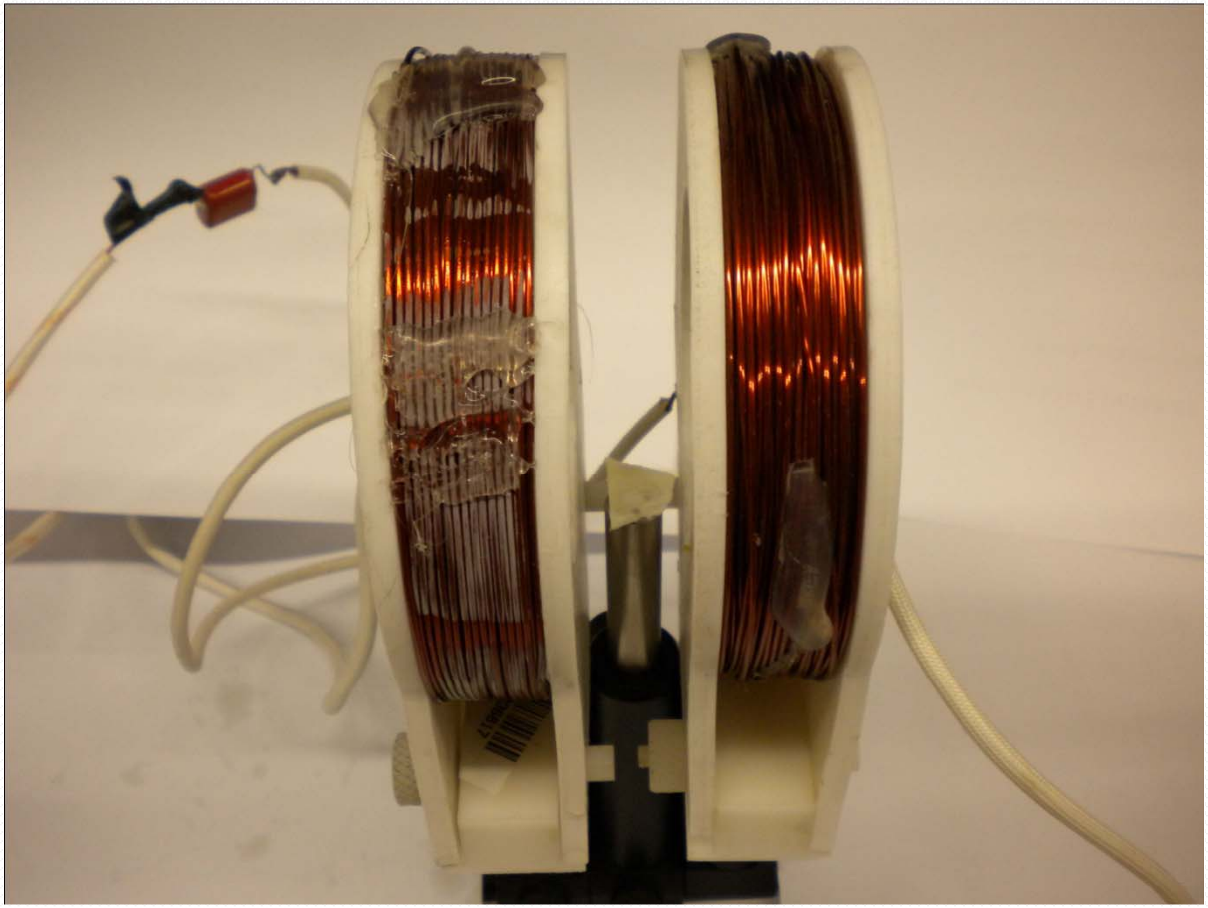
Two identical coils with separation equal
to their common radius.

$$B = B_o(1 + c_4 x^4 + c_6 x^6 + \ldots)$$

In superposition

$$B = (\frac{4}{5})^{3/2} \mu_o \frac{N}{a} i$$

Schematic of the experiment (cont…)



Helmholtz Coil

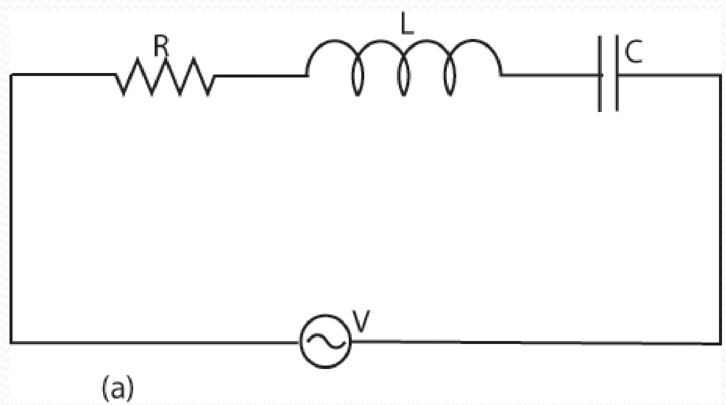
Schematic of the experiment (cont…)

- Parameters of coil

- 18 gauge copper wire, d=1.2 mm
- N=324 , l=2.7 cm , R=1.5Ω
- D1=6.5 cm, D2=10.2 cm , a=4.8 cm
- L=7 mH

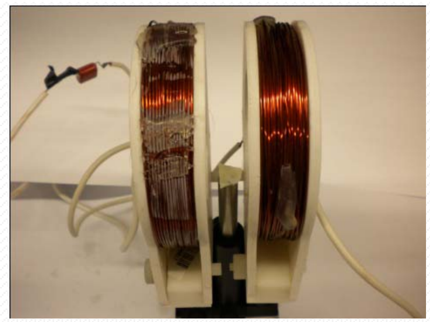
Connected a capacitor of 0.97μF to resonate the coil at 1.22 kHz to maximize the current and hence B

$$f = \frac{1}{2\pi}\sqrt{LC}$$



Schematic of the experiment (cont…)

(a)



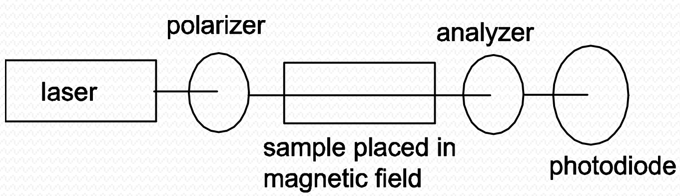
(b)

(c)

(d)

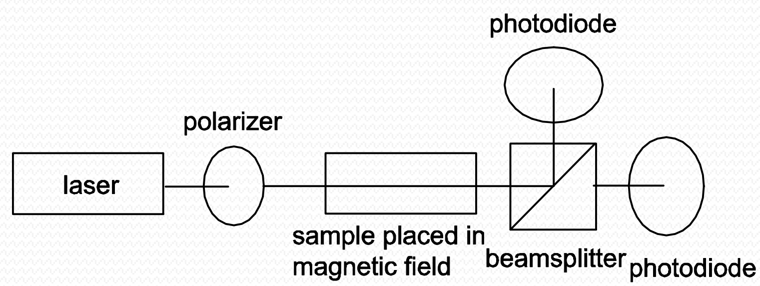
(e)

▪Working Principle



(a)

One sided detection

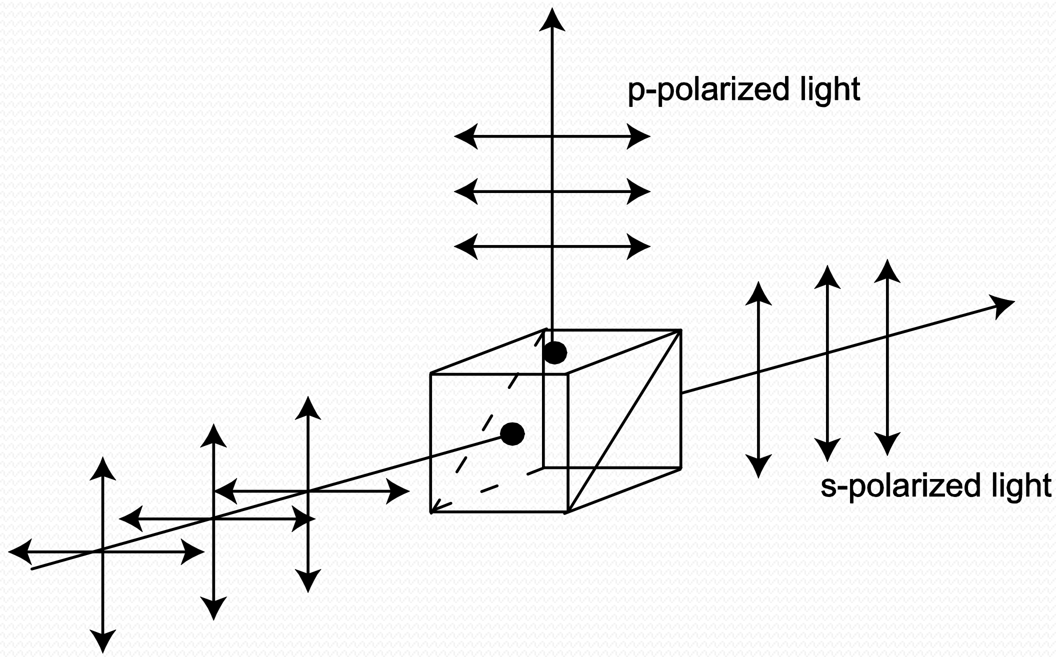


(b)

Differential detection

▪Working Principle (cont…)

•Beam splitter



▪Working Principle (cont…)

Jones vector of a linearly polarized

$$E(z,t)=\begin{pmatrix}1\\0\end{pmatrix}A_o e^{i(kz-wt)}$$

After passing through the sample

$$E(z,t)=\begin{pmatrix}\cos\varphi\\ \sin\varphi\end{pmatrix}A_o e^{i(kz-wt)}$$

After the beam splitter

$$E(z,t)=\begin{pmatrix}\cos\alpha\cos\varphi-\sin\alpha\sin\varphi\\ \sin\alpha\cos\varphi+\cos\alpha\sin\varphi\end{pmatrix}A_o e^{i}(kz-wt)$$

Beam splitter is at 45^o

$$E(z,t)=\frac{1}{\sqrt{2}}\begin{pmatrix}\cos\varphi-\sin\varphi\\ \cos\varphi+\sin\varphi\end{pmatrix}A_o e^{i(kz-wt)}$$

▪Working Principle (cont…)

Intensity after the beam splitter

$$I = E.E^*$$

$$\begin{pmatrix} I_A \\ I_B \end{pmatrix} = \frac{A_o^2}{2}\begin{pmatrix} 1-2\varphi \\ 1+2\varphi \end{pmatrix}$$

$$I_A = V_{dc,A} + V_{ac,A}\cos(\Omega t)$$

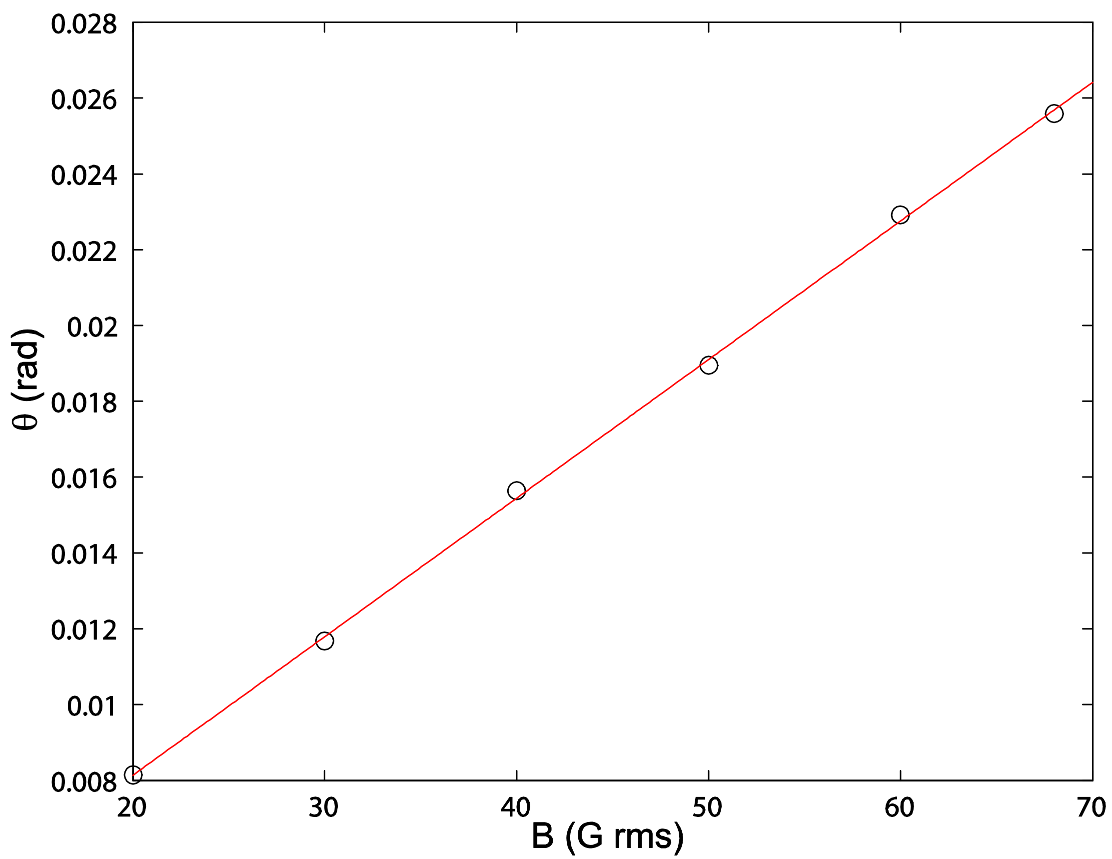
$$\varphi_o = \frac{V_{ac,A}}{2V_{dc,A}}$$

Using differential detection

$$I_A - I_B = 2A_o^2\varphi_o\cos(\Omega t)$$

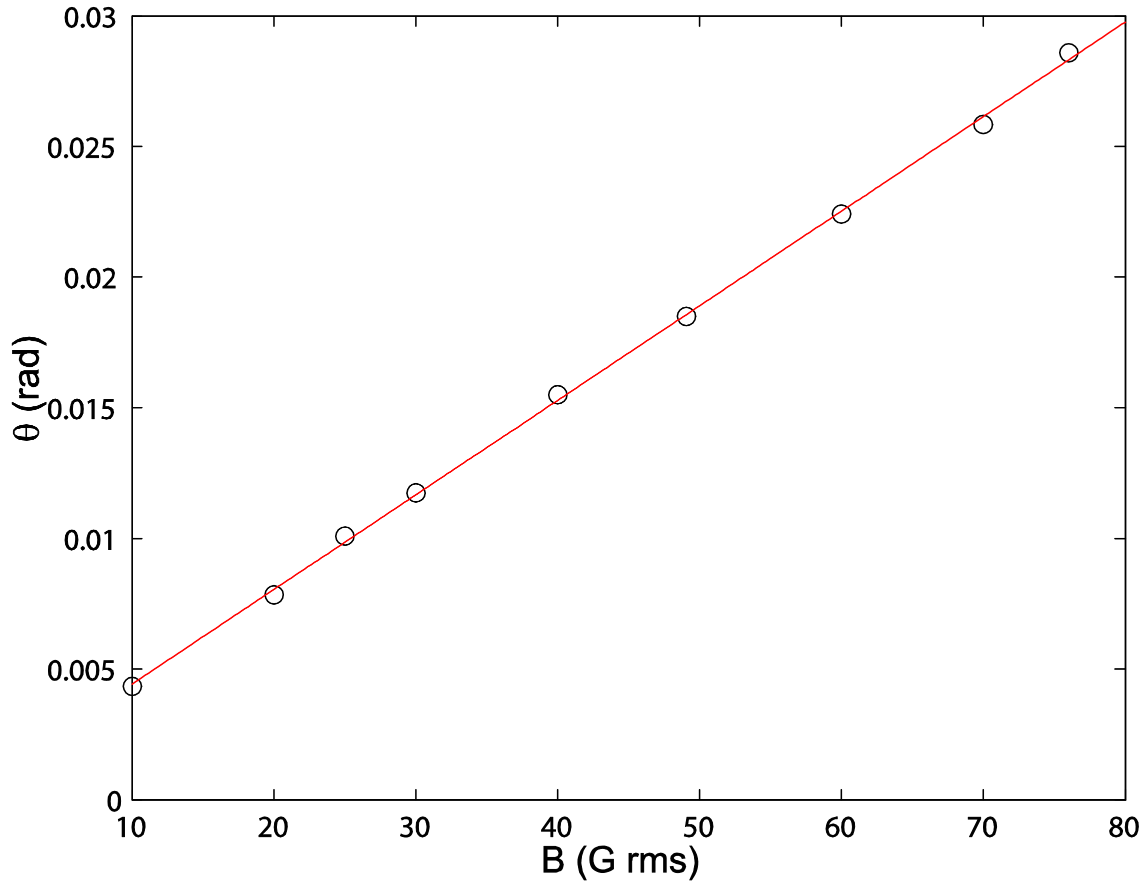
$$\varphi_o = \frac{V_{ac}}{4V_{dc,A}} = \frac{V_{ac}}{4V_{dc,B}}$$

▪Results



Single ended

▪Results



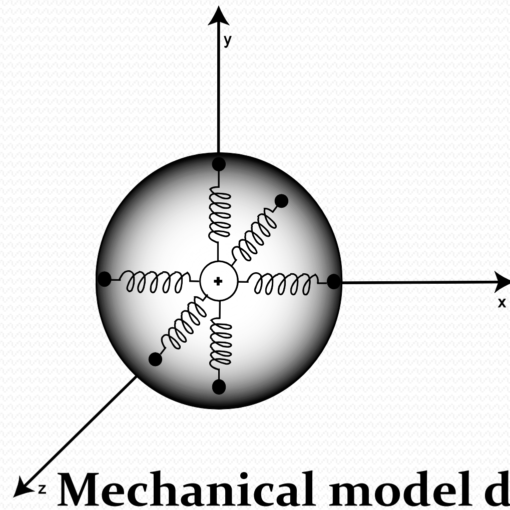
Differential detection

▪Results

Sample	Method	Verdet Constant
TGG	Single ended detection	$3.80 \pm 0.02 \times 10^{-4}$ rad/G-cm
TGG	Differential detection	$3.892 \pm 0.024 \times 10^{-4}$ rad/G-cm

▪Birefringence

- Anisotropic crystal
- Different refractive indices in different directions



Mechanical model depicting a negatively charged shell bound to a positive nucleus by pairs of spring having different stiffness

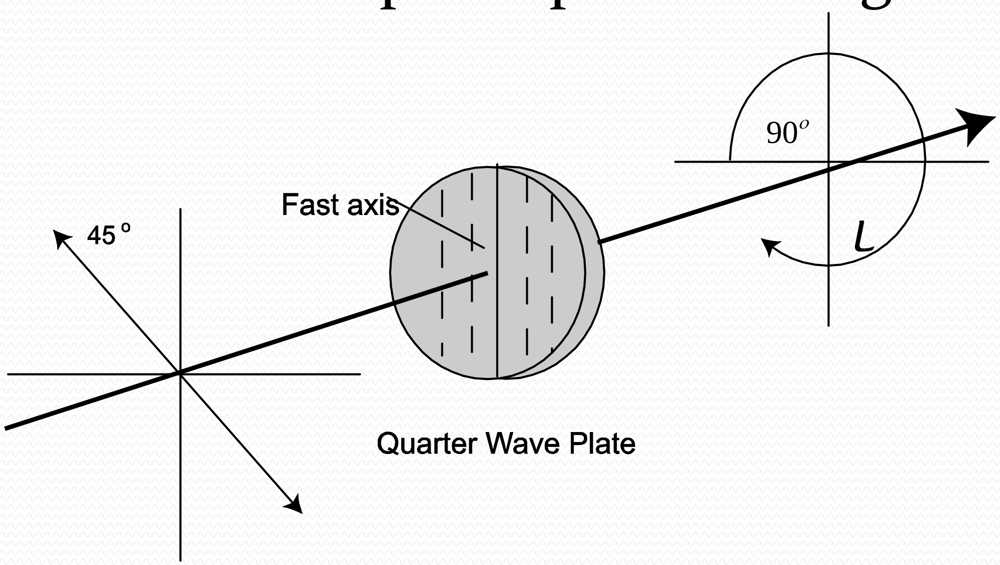
▪Wave Plates

- Objects made from birefringent crystals used to introduce predetermined phase shift in incident light beams.
- The emerging beams has a phase difference of

$$\Delta\varphi = \frac{2\pi}{\lambda_o} d(|n_o - n_e|)$$

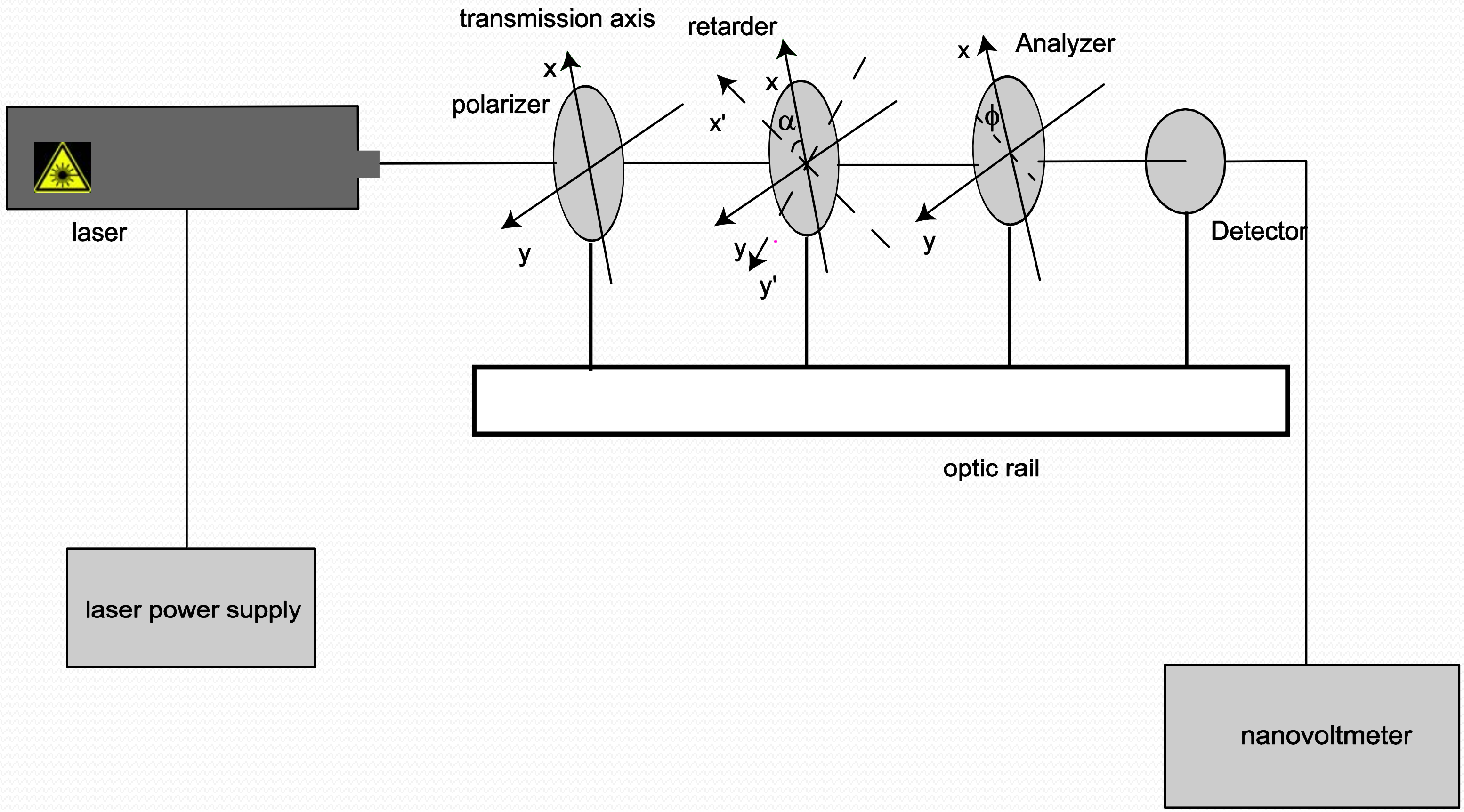
▪Quarter Wave Plate

- Introduces phase shift of 90^o
- Converts linearly polarized light into circular or elliptical polarized light



A quarter wave plate converts linearly polarized light into circular polarized light

Schematic of the experiment



Working Principle

- A system consisting of a polarizer, a quarter wave plate and an analyzer is represented by

$$H = T_3 T_2 T_1$$

where

$$T_1 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

$$T_2 = \begin{bmatrix} cos\alpha & sin\alpha \\ -sin\alpha & cos\alpha \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & e^{-i\Gamma} \end{bmatrix} \begin{bmatrix} cos\alpha & -sin\alpha \\ sin\alpha & cos\alpha \end{bmatrix}$$

$$T_3 = \begin{bmatrix} cos^2\varphi & sin\varphi cos\varphi \\ sin\varphi & sin^2\varphi \end{bmatrix}$$

The matrix can be solved and reduced to

$$H(\alpha, \varphi) = [cos\alpha \cos(\alpha + \varphi) + e^{-i\Gamma} sin\alpha \sin(\alpha + \varphi)] \begin{bmatrix} cos\varphi & 0 \\ sin\varphi & 0 \end{bmatrix}$$

Working Principle(cont…)

Transmitted intensity of the system is

$$I = I_o[\cos^2\alpha\cos^2(\alpha-\varphi)+\sin^2\alpha\sin^2(\alpha-\varphi)+\frac{\sin 2\alpha\sin(2\alpha-2\varphi)}{2}\cos\Gamma$$

By solving and manipulating

$$I(\alpha,\varphi)=\frac{1}{2}[1+(\cos^2 2\alpha+\cos\Gamma\sin^2 2\alpha)\cos 2\varphi+\sin^2\frac{\Gamma}{2}\sin 4\alpha\sin 2\varphi$$

By substituting $x=\cos 2\varphi, y=I, y_o=I_o$

$$x^2+4(y-\frac{y_o}{2})^2-4(\cos^2 2\alpha+\cos\Gamma\sin^2 2\alpha)x(y-\frac{y_o}{2})=\sin^4(\frac{\Gamma}{2})\sin^2 4\alpha$$

At $\alpha=45^o$

$$I=\frac{1}{2}I_o(1+\cos\Gamma\cos 2\varphi)$$

Working Principle(cont…)

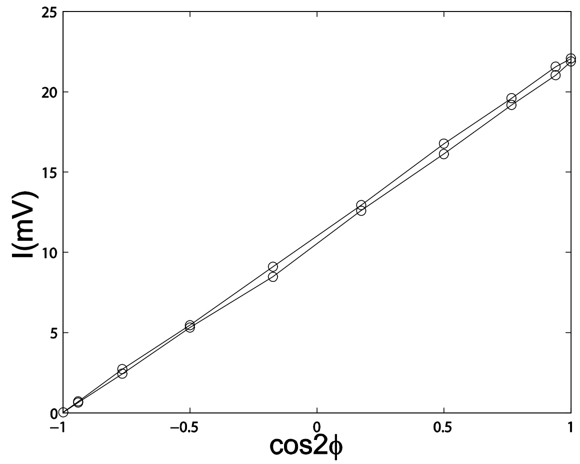
Equation of a straight line

$$y = mx + n$$

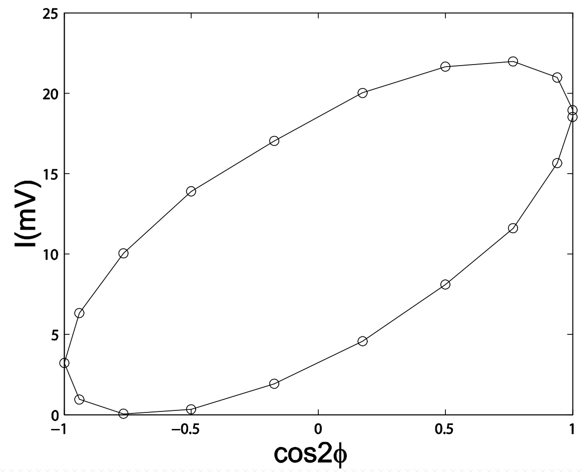
$$m = \frac{1}{2} I_o \cos\Gamma$$

$$n = \frac{1}{2} I_o$$

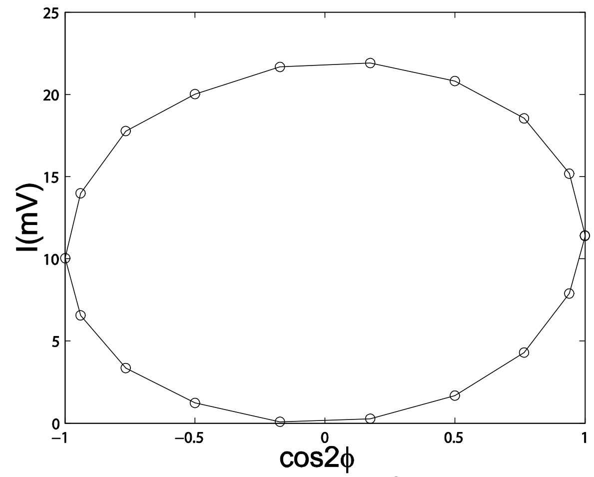
Results



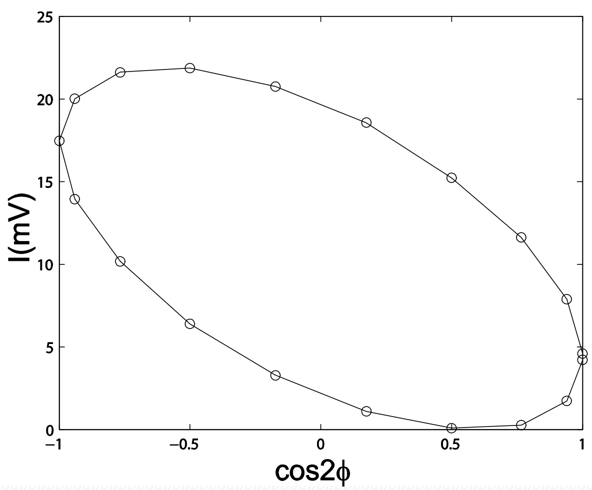
$\alpha = 0^o$



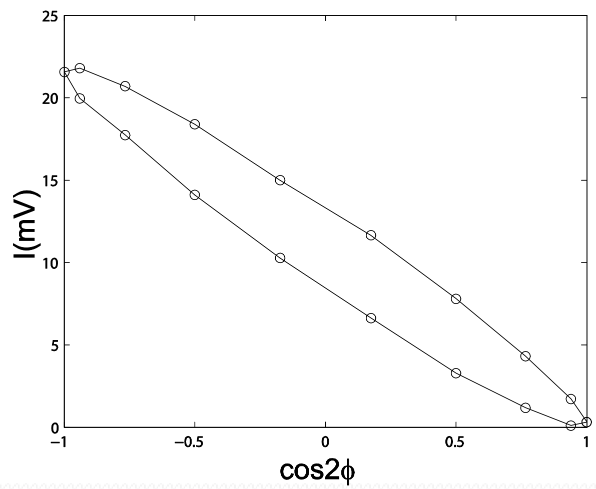
$\alpha = 10^o$



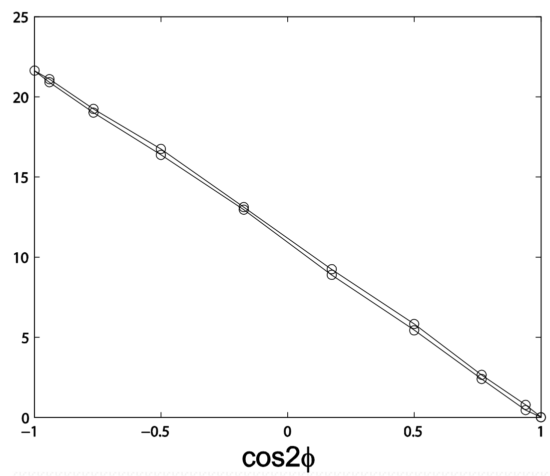
$\alpha = 20^o$



$\alpha = 30^o$

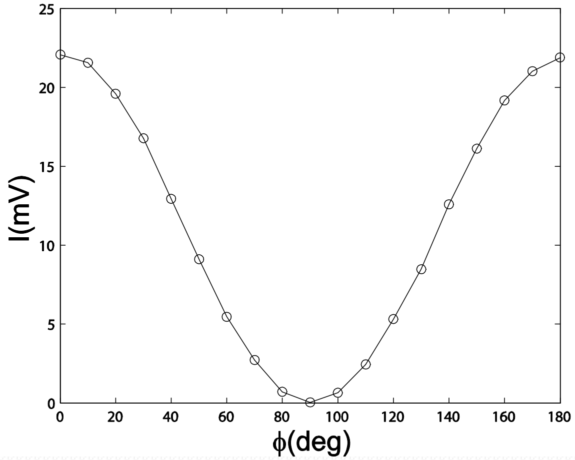


$\alpha = 40^o$

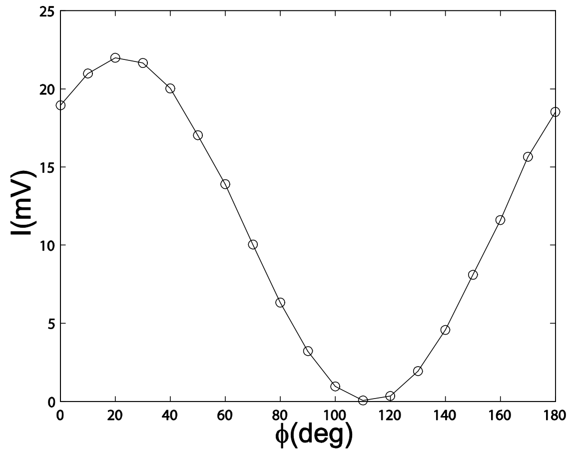


$\alpha = 45^o$

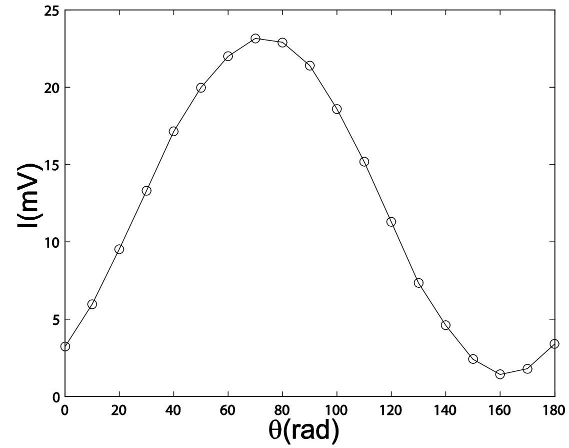
Results (cont…)



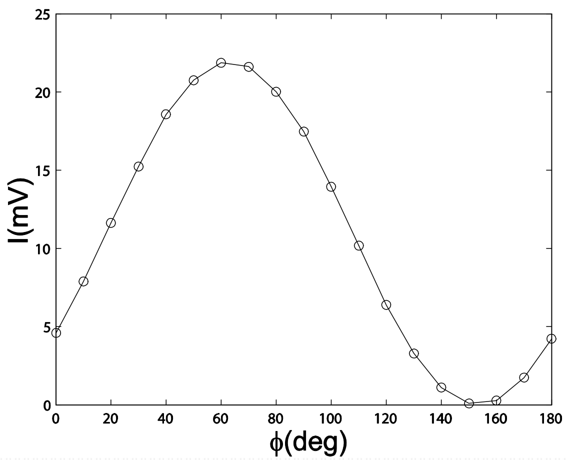
$\alpha = 0^o$



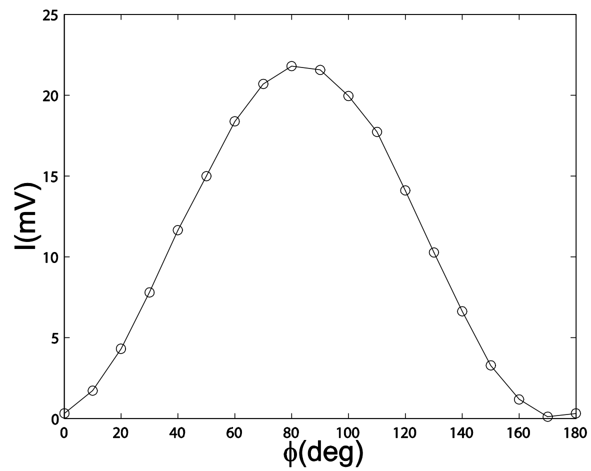
$\alpha = 10^o$



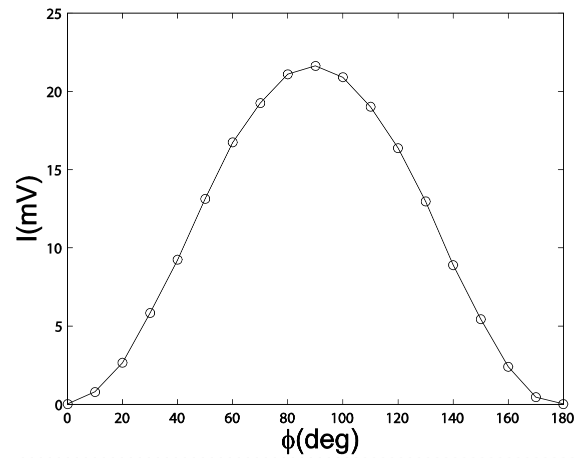
$\alpha = 20^o$



$\alpha = 30^o$



$\alpha = 40^o$



$\alpha = 45^o$

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Thanks