17/11/2023 3.3

9:58 am

Investigating Polarization of Light Through Jones Calculus

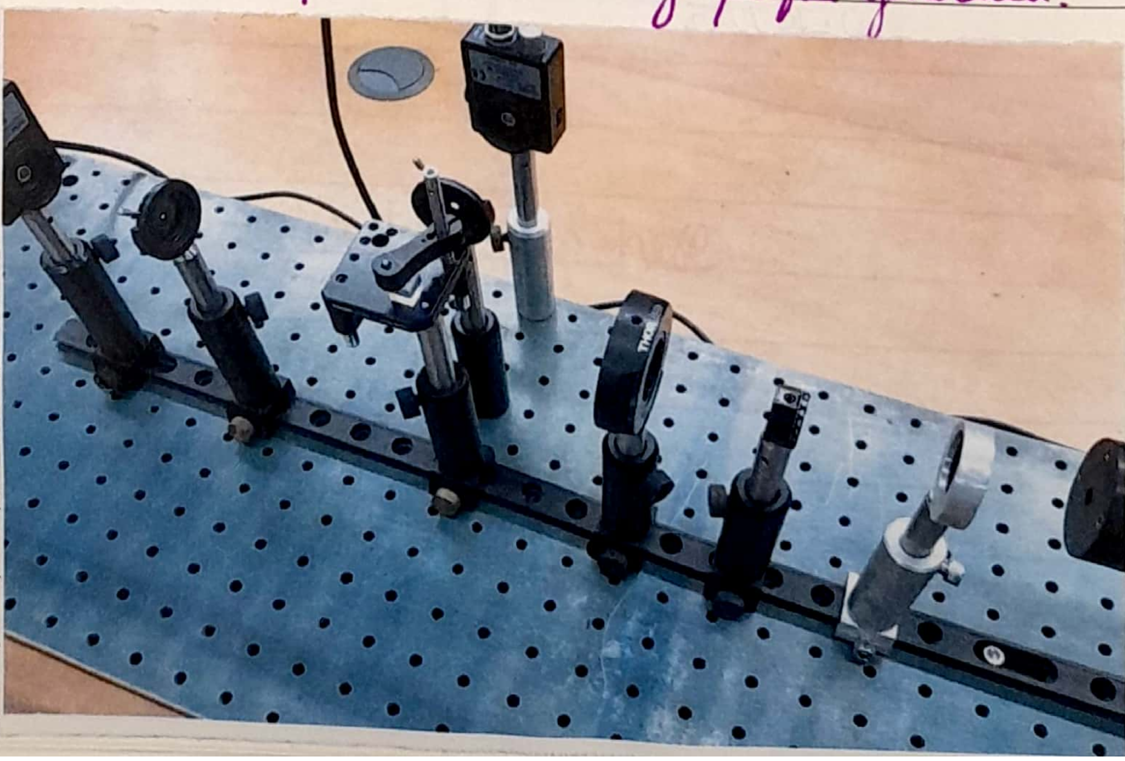
Objectives:-

1) Understand the working of half wave plate (HWP) and polarized beam splitter (PBS). What is the fast axis of a wave plate?

2) Derive Jones matrices for a linear polarizer and half wave plate placed at some arbitrary angle θ.

3) Using your understanding of the aforementioned optical elements and Jones calculus, derive methods to generate linear polarization at arbitrary angles.

4) Derive and plot the intensity profile generated.



* Matrix Study of Polarization

The direction of the electric field vector $\vec{E}$ is known as the polarization of the electromagnetic wave.

The electric field associated with a plane monochromatic wave is perpendicular to the direction of propagation of energy carried by the wave.

In general, plane monochromatic waves are elliptically polarized. Over time, the tip of the electric field vector in a given plane perpendicular to the direction of energy propagation traces out an ellipse.

→ Special cases of EM waves with elliptical propagation:

① Linearly Polarized waves:
The $\vec{E}$ vector always oscillates back and forth along a given direction

② Circularly Polarized waves:
The tip of $\vec{E}$ vector traces out a circle.

Any electromagnetic wave can be regarded as a Superposition of plane electromagnetic waves with various frequencies, amplitudes, phases and polarizations.

The polarization of light is proof of its transverse nature.

* Jones Vectors:

We write the complex field components for waves traveling in the +z direction with amplitudes E_{0x} and E_{0y} and phases φ_x and φ_y as:

$$\tilde{E}_x = E_{0x} e^{i(kz - \omega t + \varphi_x)}$$

$$\tilde{\vec{E}}_y = E_{0y} e^{i(kz - \omega t + \varphi_y)}$$

$$\Rightarrow \quad \tilde{E} = \left[E_{0x} e^{i\varphi_x} \hat{x} + E_{0y} e^{i\varphi_y} \hat{y} \right] e^{i(kz - \omega t)}$$

$$\tilde{E} = \tilde{E}_0 e^{i(kz - \omega t)}$$

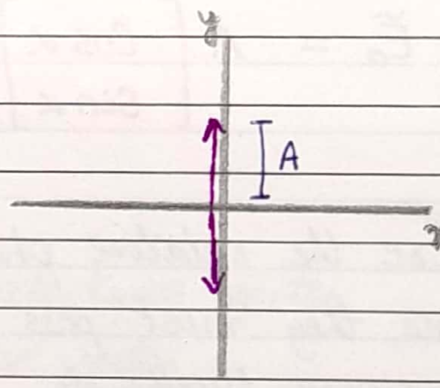
where $\tilde{E}_0$ is the complex amplitude vector for the polarized wave.

Written as a 2-element matrix (Jones Vector):

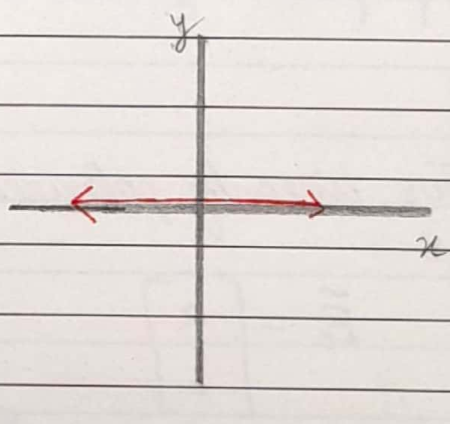
$$\tilde{E}_0 = \begin{bmatrix} \tilde{E}_{0x} \\ \tilde{E}_{0y} \end{bmatrix} = \begin{bmatrix} E_{0x} e^{i\varphi_x} \\ E_{0y} e^{i\varphi_y} \end{bmatrix}$$

Lets study some sample polarizations and their corresponding Jones vectors:

① Vertically Polarized Light:



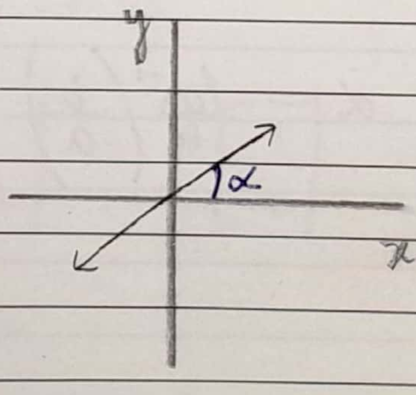
We see oscillations in the y direction. The field is propagating out of the page in the +z direction



now $\tilde{E}_0 = \begin{bmatrix} 0 \\ A \end{bmatrix} = A \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

normalizing $\tilde{E}_0$ sets $A \to 1$

$\therefore \quad \tilde{E}_0 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$



② Horizontally Polarized Light

Trivially by induction.

$$\breve{E}_0 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

③ Linearly Polarized Light at angle α wrt x

$$\breve{E}_0 = A \begin{bmatrix} \cos\alpha \\ \sin\alpha \end{bmatrix}$$

Here the relative phase between Ψ_x and $\Psi_y = 0$
Since they must pass the origin together, grow and
Superpose together etc.

→ For linearly polarized light:

$$\breve{E}_0 = \begin{bmatrix} a \\ b \end{bmatrix} \qquad \exists\, a \,\&\, b \geq 0$$

$$\alpha = \tan^{-1}\left(\frac{b}{a}\right) = \tan^{-1}\left(\frac{E_{0y}}{E_{0x}}\right)$$

→ In determining the resultant vibration due to 2 ⊥ components, we are determining the appropriate relevant

(Lee-suh-zhoo) Lissajous figure.

The Lissajous figure is a function of relative phase $\Delta\Psi = \Psi_y - \Psi_x$ for Ψ_y $E_{0x} \neq E_{0y}$

→ ① Straight lines: $\Delta\phi = 0, 2\pi \ldots$

→ ② Circles: $\Delta\phi = \frac{\pi}{2}, \frac{-\pi}{2}, \frac{3\pi}{2}$

→ ③ Ellipses $\Delta\phi = \pm\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

where if $\Delta\phi > 0$ → counterclockwise vector
$\Delta\phi < 0$ → clockwise vector

④ Circularly Polarized Light:

ⓘ Left Circularly Polarized (LCP):

$E_x = A \cos \omega t$ $\qquad E_y = A \sin \omega t$

For $\Psi_x = 0, \Psi_y = \frac{\pi}{2}$ $\quad \tilde{E}_0 = A\begin{bmatrix} 1 \\ e^{i\pi/2} \end{bmatrix} = A\begin{bmatrix} 1 \\ i \end{bmatrix}$

$\therefore \tilde{E}_0 = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ i \end{bmatrix}$

(ii) Right Circularly Polarized (RCP):

$$\tilde{E}_0 = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ -i \end{bmatrix}$$

The prefactor of a Jones vector may affect the amplitude, and hence, irradiation of light but not the polarization mode.

If $E_{0x} = A$, $E_{0y} = B$

then $\tilde{E}_0 = \begin{bmatrix} A \\ iB \end{bmatrix}$ for LCP

$\tilde{E}_0 = \begin{bmatrix} A \\ -iB \end{bmatrix}$ for RCP

$\therefore$ A Jones vector of unequal magnitude represents elliptically polarized light along the x, y axes.

The normalization constant now is:

$$\frac{1}{\sqrt{A^2+B^2}}$$

⑤ General Elliptical case:

when $\Delta\phi = \psi_y - \psi_x = \varepsilon$

$$\bar{E}_0 = \begin{bmatrix} A \\ be^{i\varepsilon} \end{bmatrix}$$

but $be^{i\varepsilon} = b(\cos\varepsilon + i\sin\varepsilon) = B + iC$

$$\bar{E}_0 = \begin{bmatrix} A \\ B + iC \end{bmatrix}$$

for a general counterclockwise rotation

$$E_0 = \frac{1}{\sqrt{A^2 + B^2 + C^2}}$$

$$\varepsilon = \tan^{-1}\left(\frac{C}{B}\right)$$

$$\alpha \Rightarrow \tan 2\alpha = \frac{2 E_{0x} E_{0y} \cos\varepsilon}{E_{0x}^2 - E_{0y}^2}$$

If A and C have the same sign → counterclockwise

* Superposition:

The superposition of two or more polarized modes of light is just the sum of their Jones vectors.

eg: Adding LCP and RCP:

$$\begin{bmatrix} 1 \\ i \end{bmatrix} + \begin{bmatrix} 1 \\ -i \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

$\therefore$ Linearly polarized light can be considered a composition of equal proportions of LCP & RCP!

→ The addition of orthogonal components of linearly polarized light is not unpolarized light, even though unpolarized light is often symbolized as such.

→ There is no Jones vector representing unpolarized or partially polarized light.

* Matrices of Jones Polarizers:

In general, polarization matrices can be represented as:

$$M = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

① Linear Polarizer:

It effectively removes all or most of the $\vec{E}$ vibrations in a given direction by absorption.

eg: For vertically linearized polarized light:

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\therefore \quad M = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$ linear polarizer, TA vertical

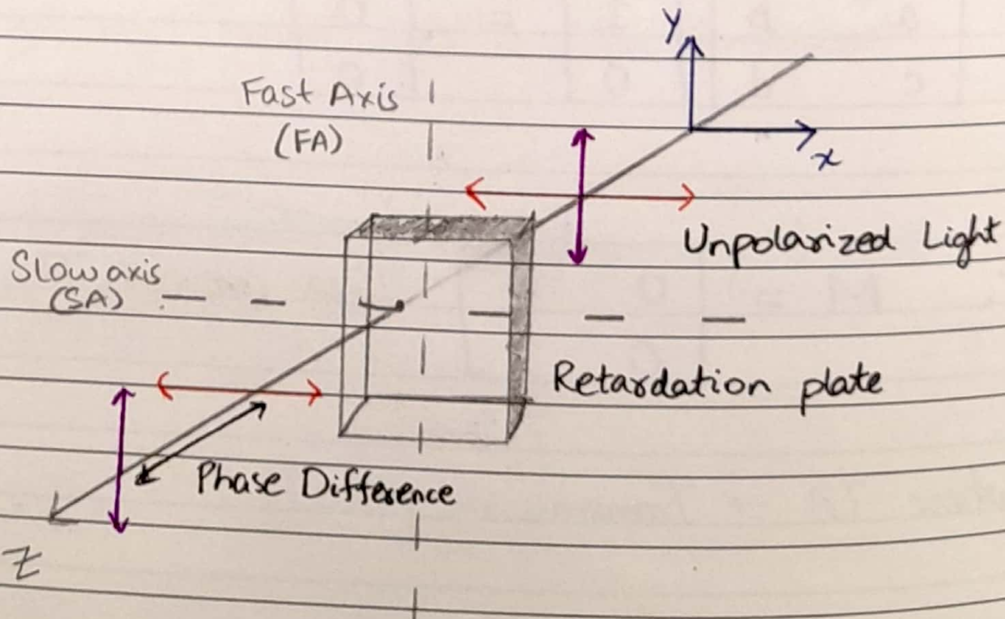
where TA → Transmission Axis

$$M = \frac{1}{2}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$ Linear Polarizer, TA 45°

$$M = \begin{bmatrix} \cos^2\theta & \sin\theta\cos\theta \\ \sin\theta\cos\theta & \sin^2\theta \end{bmatrix}$$ Linear Polarizer, TA θ°

② Phase Retarder:

A phase retarder does not remove $\vec{E}$ vibrations. It introduces a phase difference between them. It does this by passing light through a retardation plate where each orthogonal vibration travels at a different speed.



When the net phase difference $\Delta\phi = \frac{\pi}{2}$, the retardation plate is called a quarter wave plate.

At $\Delta\phi = \pi$, it is called a half wave plate

QWP, SA vertical $\quad M = e^{-i\pi/4} \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$

QWP, SA horizontal $\quad M = e^{i\pi/4} \begin{bmatrix} 1 & 0 \\ 0 & -i \end{bmatrix}$

HWP, SA vertical $\quad M = e^{-i\pi/2} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

HWP, SA horizontal $\quad M = e^{i\pi/2} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

In general:

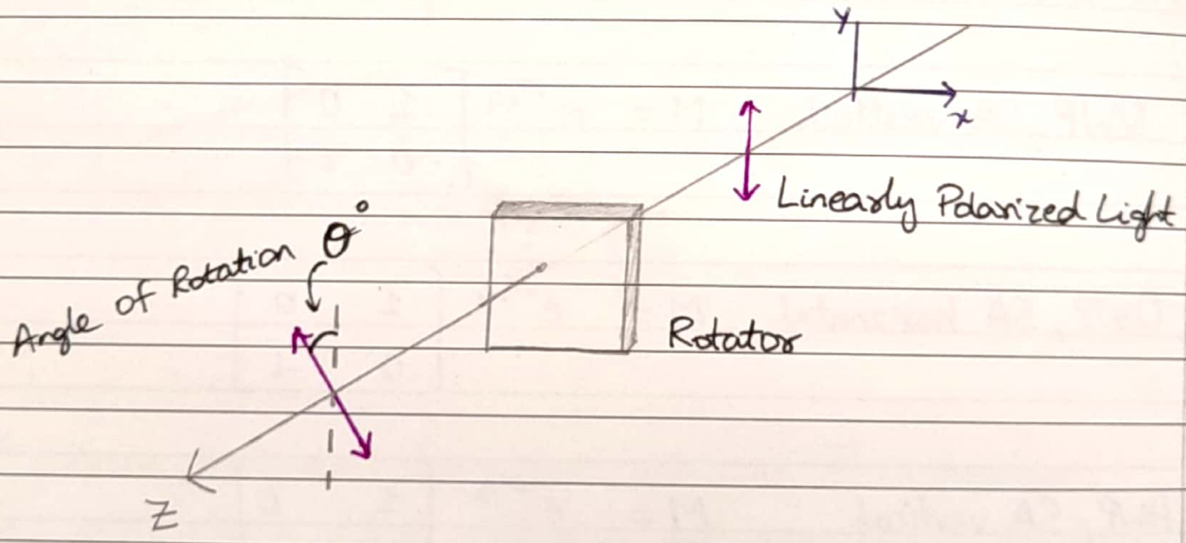
$$M = \begin{bmatrix} e^{i\varepsilon_x} & 0 \\ 0 & e^{i\varepsilon_y} \end{bmatrix}$$

Traditionally, let $\varepsilon_x = 0$

$$\varepsilon_y = \frac{2\pi}{\text{name}}$$

③ Rotator:

The rotator rotates the direction of linearly polarized light incident on it by some angle θ.



Rotation ($\beta \rightarrow \beta+\theta$)

$$M = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

* Polarized Beam Splitter (PBS):

A PBS is an optical device used to divide a beam of light based on its polarization state. (unlike a traditional beam splitter that does not care about the polarization).
It is made from birefringent materials of different refractive indices for light polarized in different directions.

The surface reflects one polarization while allowing the other to pass through.

Design imperfections may cause undesired mixing of polarizations or loss of intensity.

* Neutral Density Filter (NDF):
An NDF is a photographic filter that uniformly reduces the intensity of all wavelengths, or colors, of light equally.

This is usefull if using wider aperture or slower shutter speed

* What happens to linearly polarized light as it passes through a half-wave plate?

Essentially, the polarization direction of the light is rotated. The half wave plate is designed such that its optical thickness is half the wavelength of the incident light. The phase of one component of the EM wave along the optical axis of the plate is delayed by half a wavelength compared to the other component.

If the incident polarization is at an angle, it will be rotated by twice that angle.

A half-wave plate ensures that the light remains linearly polarized after rotation.

A quarter wave plate rotates the polarization to make it a circularly polarized light wave

A polarizing beam splitter allows p-polarized light to pass through and s-polarized light is reflected.

p - parallel # horizontal
s - perpendicular # vertical

8/12/2023

10:00pm

2.1 Rotating HWP keeping polarizer at a fixed angle:

We keep the polarizer at 0° to get horizontally polarized light. We now turn the HWP plate 360° to observe changes

HWP angle (°)	Photodiode A	Photodiode B
0	1.2	102
40	1.1	104
80	1	106
120	1.2	103
160	1.2	103
200	1.3	102
240	1.2	104
280	1.3	102
320	1.2	104
360	1.1	106

Looks wrong lets try again

At 0°, we get horizontally polarized light. P polarized light passes through the PBS and we get a high reading on photodiode A.

uncertainty
HWP → ± 2°
PDA → ± 0.05V
PDB → ± 0.5V

Bad Readings

Faulty Diode!

HWP angle ($^\circ$) Photodiode A Photodiode B

Plotted on Colab with graphs on page 56-57

Q4. It is in agreement with the intensity calculations done on Page # 42 of this notebook. We see that horizontally polarized light varies as $A\sin^2\alpha$ and vertically polarized light varies as $A\cos^2\alpha$. which is consistent with our data.

Q1: It is in agreement with our calculations where

$$M = \begin{bmatrix} \cos^2\theta & \sin\theta\cos\theta \\ \sin\theta\cos\theta & \sin^2\theta \end{bmatrix}$$ acting on our

horizontally polarized light gives us the same results.

Q2: Why do we need a neutral density filter?

The neutral density filter uniformly reduces the intensity of all wavelengths or colors that pass through it. This helps us normalize our intensity for our linearly polarized light.

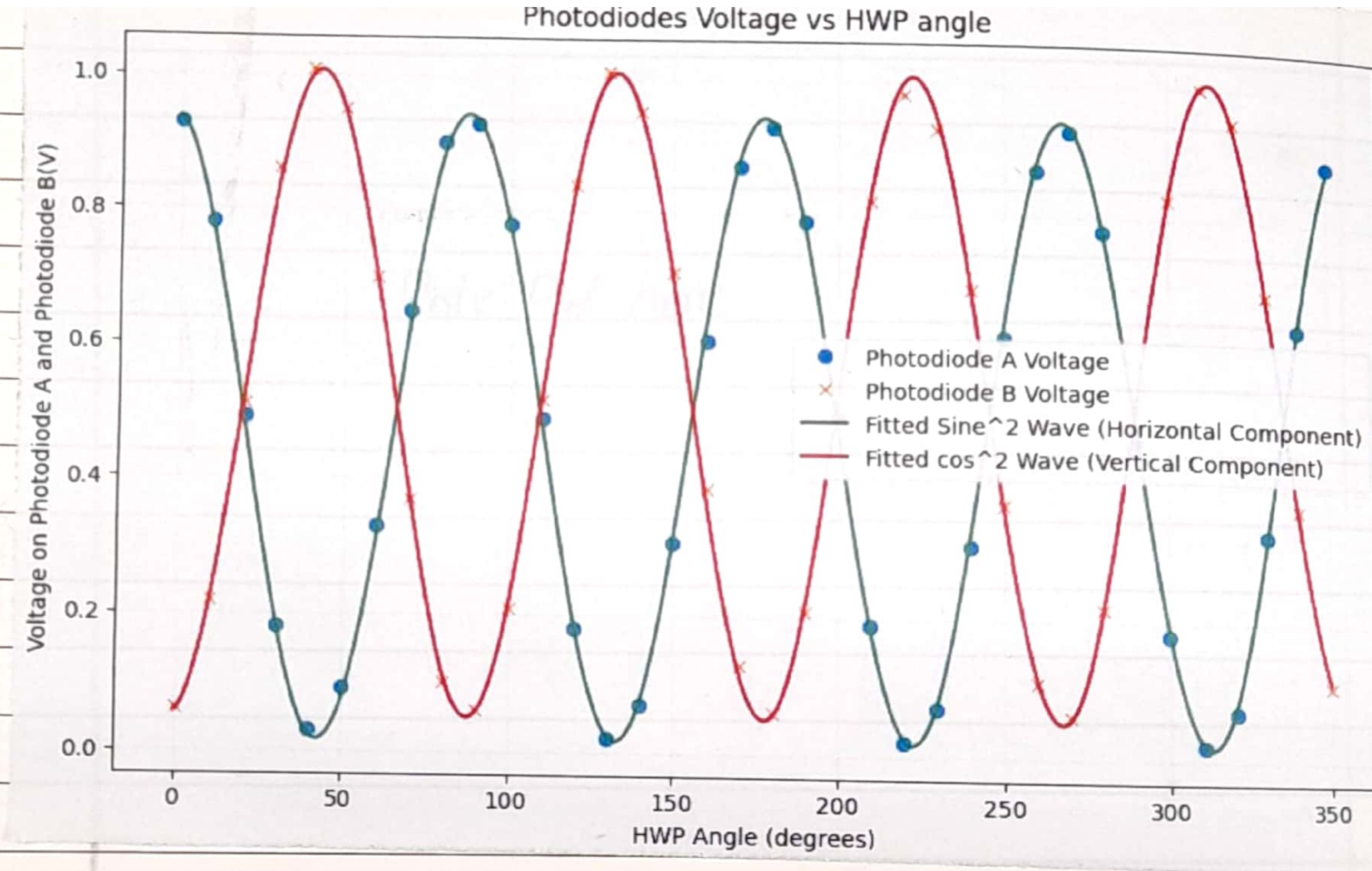
Q3: Why are we using an HWP for producing different linear polarizations? Describe the effect mathematically.

The half wave plate ensures that light remains linearly polarized after rotation.

Mathematically:

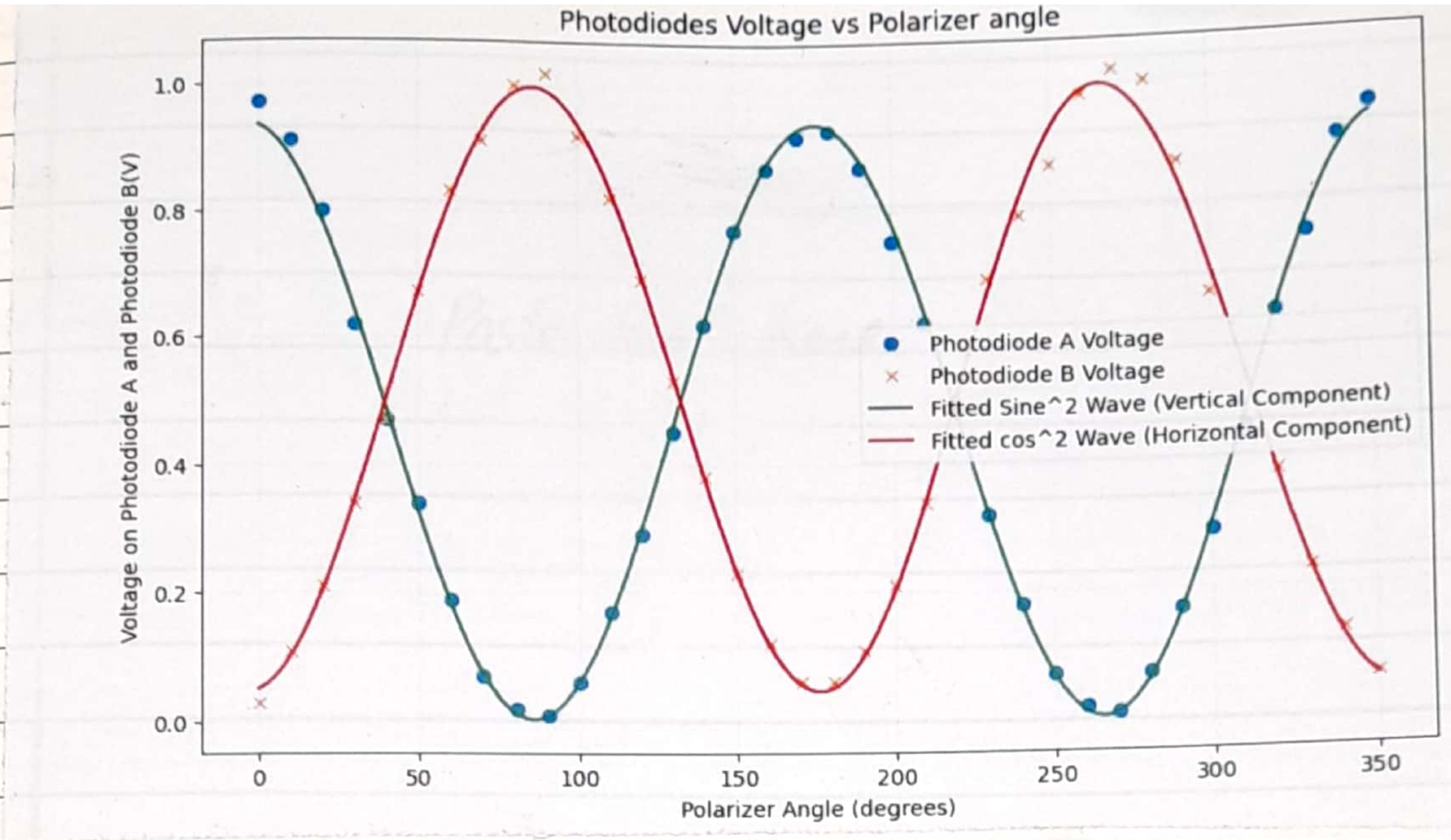
$$M = e^{i\pi/2}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

It introduces a $\Delta\phi = \pi$ in the given wave components.



The figure illustrates the experimental data as well as the fitted $\sin^2$ and $\cos^2$ functions for the horizontal and vertical components of the light.

We can see that as the half wave plate is rotated, the polarization of the light changes in a manner such that the vertical and horizontal components of the light vary sinusoidally in intensity.



[1] Error in angle reading of $\pm 1°$
[2] Error in multimeter reading of ± 0.005 V
[3] Values normalized by dividing by 1.75 (obtained by taking the sum of the detect
A and B readings for each angle and then calculating their average)

Note that the angle is measured with respect to the polarizer in this figure. This 'flips' the behavior of Photodiode A and B where each now reflects the opposite 'polarity' - vertical/horizontal component representation.

As expected, the intensity still varies sinusoidally as the polarizer angle is changed which is in agreement with our Jones formalism.

(6) Discussion:

The intensity of the horizontal and vertical components of the light vary sinusoidally as the halfwave plate or polarizer angle is altered. Following Jones calculus, the magnitude of the intensity is directly proportional to $\sin^2\theta$ where θ is the relative polarization angle.

(7) Conclusion:

In this experiment, we were introduced to Jones Calculus, a powerful technique for studying the polarization of light using matrices calculus. We learned the function of half wave plates, phase retarders, polarizers, NDFs and other lab equipment that would allow us to study the polarization of a source. We successfully setup a simple experiment to verify Jones formalism and understand the relationship between polarization angle and intensity of a given source. Future work would entail setting up arbitrary arrangements of phase retarders and polarizers to investigate, study and create elliptically or circularly polarized light from a linearly polarized source.