9/12/2023

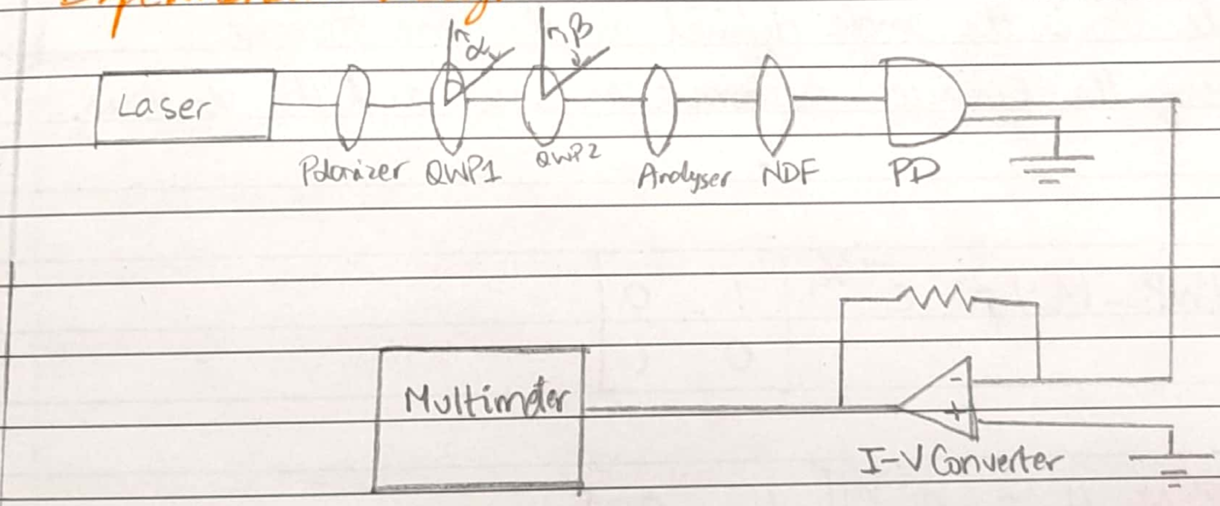
3·4:

9:43 am

Analysing the Polarization State of Light through the Fourier Series

We will be determining the polarization of light simply by using a polarizer and a quarter waveplate (QWP). We will develop a method to generate as well as analyze different polarization states of light. We will also be using matrix multiplications to solve systems of equations.

① Experimental Design:



② Theoretical Background:

Recall the Jones vector representation of a linear polarized light wave:

$$\vec{E} = \begin{pmatrix} \cos\alpha \\ \sin\alpha \end{pmatrix}$$

where α is the angle between the oscillation axis of the electric component and the x-axis.

A full derivation of Jones matrices for linear polarizers has been done on pages 47 — 50 of this notebook. Some key results that are relevant to our experiment once more are as follows:

1) $$M(\theta) = \begin{bmatrix} \cos^2\theta & \sin\theta\cos\theta \\ \sin\theta\cos\theta & \sin^2\theta \end{bmatrix}$$

Here the matrix M represents the Jones Matrix for a ~~linear~~ polarizer oriented at an angle θ where the angle θ is the angle defined as the one formed between the polarizer's transmission axis and the x-axis.

2) i) $$QWP\_M\_V = e^{-i\pi/4} \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$$

ii) $$QWP\_M\_H = e^{i\pi/4} \begin{bmatrix} 1 & 0 \\ 0 & -i \end{bmatrix}$$

iii) $$QWP\_M = e^{-i\pi/4} \begin{bmatrix} \cos^2\theta + i\sin^2\theta & \sin\theta\cos\theta(1-i) \\ \sin\theta\cos\theta(1-i) & \sin^2\theta + i\cos^2\theta \end{bmatrix}$$

i represents the action of a QWP at the vertical angle
ii represents the action of a QWP at the horizontal angle
iii represent the action of a QWP at any angle θ.

A detailed analysis of the working of a QWP, HWP, phase retarders, NDFs and analysers is presented on 48 - 52. The reader is advised to revisit the primer referenced before beginning this experiment.

3 Computational methods (LSE solutions):

For a highly constrained and overdetermined system of equations, it may become difficult to use ordinary least square error solutions or eigenvalue decomposition for calculating unknown coefficients. The problem may be simplified by computing few components of a matrix and then using the 'symmetric' and 'antisymmetric' decomposition to speed up computation.

However, the techniques mentioned above are only applicable to square matrix systems ($m \times m$). In an overdetermined system, the number of corresponding points or independent pairs of equations often exceeds the rank of the null space of the matrix. Is there a way to use this to our advantage? Hmm.

Introducing the pseudo inverse.
Mathematically, the following set of steps is guaranteed to procure a square matrix given any arbitrary $m \times n$ matrix.

$$A_{m\times n} X_{n\times 1} = B_{n\times 1}$$

where $m > n$.

$$X_{n\times 1} = (A^T_{n\times m} A_{m\times n})^{-1} A^T_{n\times m} B_{m\times 1}$$

where $(A^T_{n\times m} A_{m\times n})$ is a square (and thus, invertible) matrix.

This system can then be solved using LSE or SVD to compute the unknows in $X_{n\times 1}$.

4 Experimental Procedure:

Horizontally polarized light will be passed through 2 QWPs, an NDF and a polarizer after which the output voltage measured by the photodiode at the terminating end of the laser will be analyzed for different orientations of the 2 QWPs.

Three datasets are to be generated.

i) The first QWP will be oriented at 10° (α)
ii) The first QWP will be oriented at 20°
iii) The first QWP will be oriented at 30°

The second QWP will be cycled from 0° to 360° at increments of 20°. Special care must be taken here while recording the QWP angles. So calibrating your QWP initially is a good idea.

The polarizer will be oriented at 0° to produce a horizontally ~~procede~~ polarized wave.

The gain of the op-amp is set at 10K - tested to avoid saturating the DMM.

Lets take a look at the generated datasets under this configuration.

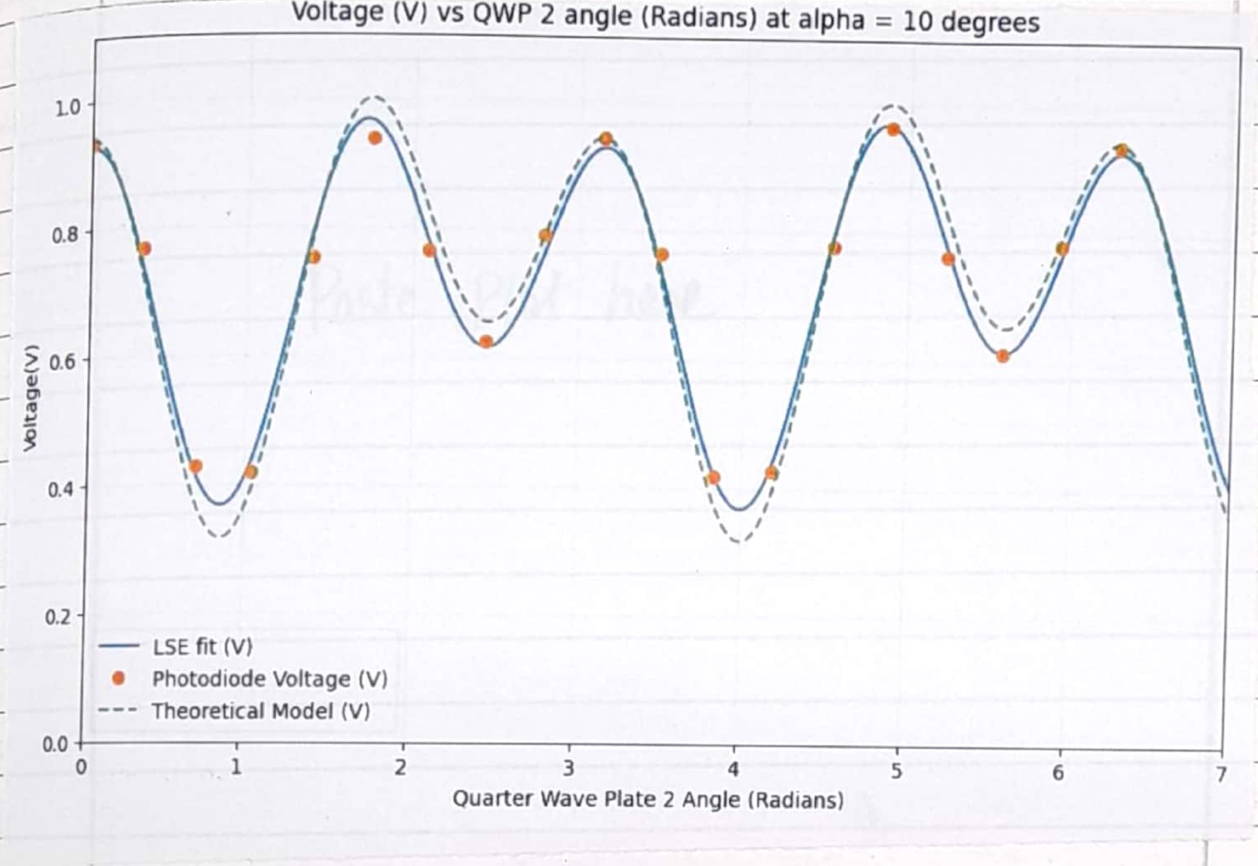
10/12/2023

7:00pm QWP 1 $\alpha = 10^\circ$

QWP 2 β (degrees)	PDout (V)
0	1.81
20	1.49
40	0.83
60	0.81
80	1.46
100	1.82
120	1.48
140	1.21
160	1.53
180	1.83
200	1.48
220	0.81
240	0.83
260	1.51
280	1.87
300	1.48
320	1.19
340	1.52
360	1.82

Uncertainty in β is $\pm 1^\circ$
Uncertainty in PDout is $\pm$ 0.005 V
→ These uncertainties are consistent for all datasets so refer to these values.

Photodiode Voltage v.s QW2 β (radians) at $\alpha = 10^\circ$



Alpha (degrees)	Beta (degrees)	PD out (V)
10	0	1.81
	20	1.49
	40	0.83
	60	0.81
	80	1.46
	100	1.82
	120	1.48
	140	1.21
	160	1.53
	180	1.83
	200	1.48
	220	0.81
	240	0.83
	260	1.51
	280	1.87
	300	1.48
	320	1.19
	340	1.52
	360	1.82

→ The 3 plots are

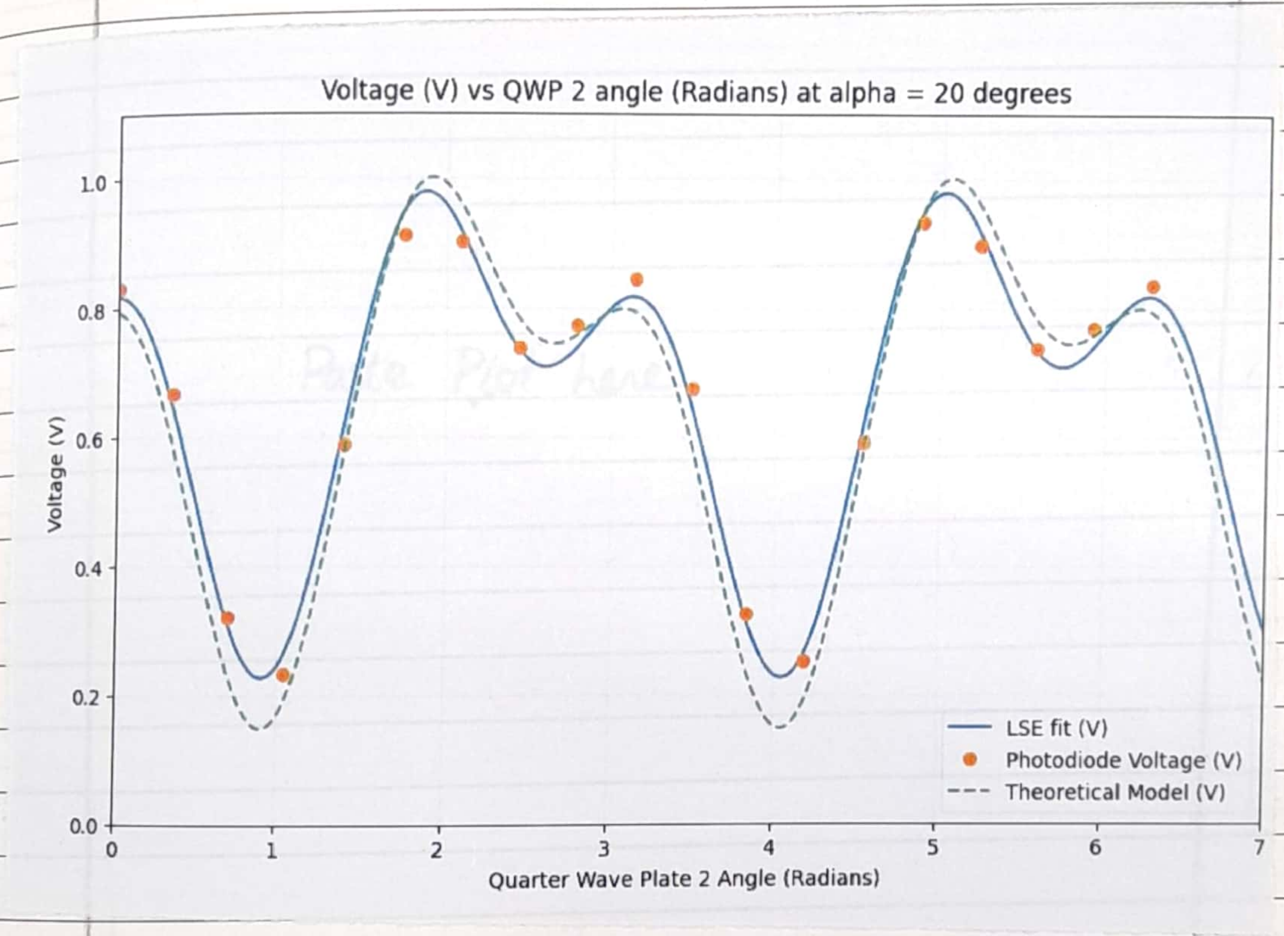
i) Experimental data
ii) Least Squares Solution
iii) Predicted model based on theory.

The next datasets are also presented in the same configuration. Refer to this page for legends if needed.

QWP 1 $\alpha = 20°$

QWP2 β (degrees)	PDout (V)
0	1.61
20	1.29
40	0.62
60	0.45
80	1.14
100	1.77
120	1.75
140	1.43
160	1.50
180	1.64
200	1.31
220	0.63
240	0.49
260	1.15
280	1.81
300	1.74
320	1.43
340	1.49
360	1.62

Photodiode Voltage vs QWP2 β (radians) at $\alpha = 20^\circ$

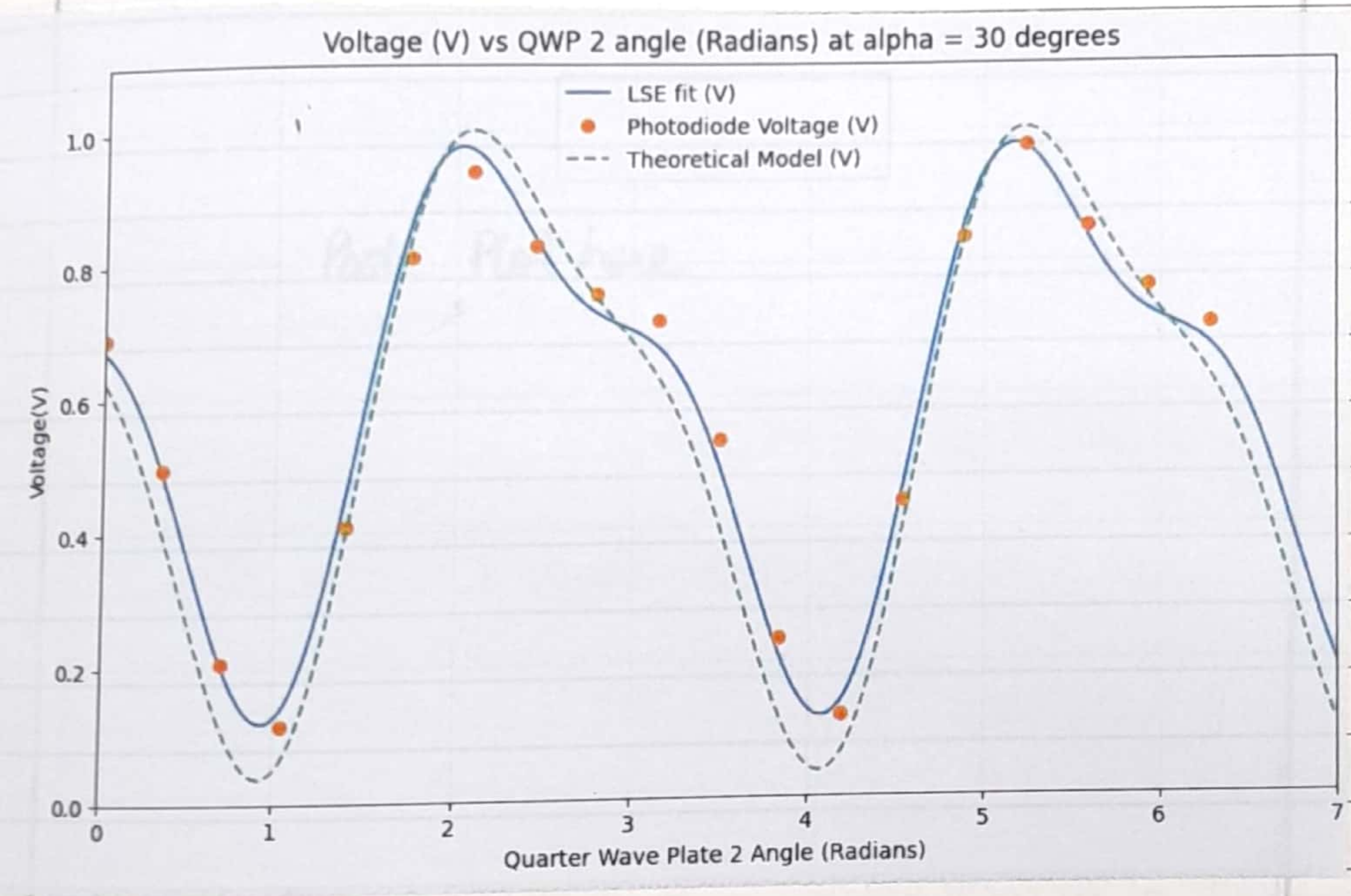


Alpha (degrees)	Beta (degrees)	PD out (V)
20	0	1.61
	20	1.29
	40	0.62
	60	0.45
	80	1.14
	100	1.77
	120	1.75
	140	1.43
	160	1.5
	180	1.64
	200	1.31
	220	0.63
	240	0.49
	260	1.15
	280	1.81
	300	1.74
	320	1.43
	340	1.49
	360	1.62

QWP1 $\alpha = 30^\circ$

QW2 β (degrees)	PDout (V)
0	1.34
20	0.96
40	0.40
60	0.22
80	0.79
100	1.57
120	1.82
140	1.60
160	1.46
180	1.38
200	1.03
220	0.45
240	0.23
260	0.85
280	1.62
300	1.87
320	1.65
340	1.47
360	1.36

Photodiode Voltage vs QWP 2 β (radians) at $\alpha = 30^\circ$



Alpha (degrees)	Beta (degrees)	PD out (V)
30	0	1.34
	20	0.96
	40	0.4
	60	0.22
	80	0.79
	100	1.57
	120	1.82
	140	1.6
	160	1.46
	180	1.38
	200	1.03
	220	0.45
	240	0.23
	260	0.85
	280	1.62
	300	1.87
	320	1.65
	340	1.47
	360	1.36

⑤ Analysis of Data:

Now that we have conducted our experiment and acquired the relevant datasets, it is time to see how well the experimental data matches up against our theoretical predictions.

Lets first revisit our theoretical model.

Theoretical Model of the Data.

For the transmitted beam recieved at the PD:

$$\vec{E} = -i\begin{bmatrix} (\cos^2\beta + i\sin^2\beta)(\cos^2\alpha + i\sin^2\alpha) - 2i\sin\beta\cos\beta\sin\alpha\cos\alpha \\ 0 \end{bmatrix}$$

$$I = A(\cos^4\beta + \sin^4\beta) + B^2[\sin(2\beta)]^2 - 2B[\sin(2\beta)][(C\cos\beta)^2 + (D\sin\beta)^2]$$

$$A = \frac{3 + \cos(4\alpha)}{4} \qquad B = \frac{\sin(2\alpha)}{2}$$

$$C = \sin\alpha \qquad D = \cos\alpha$$

The fourier series for the expression is:

$$I(\beta) = C_0 + S_2 \sin(2\beta) + S_4 \sin(4\beta) + C_4 \cos(4\beta)$$

$$C_0 = \frac{5 + \cos(4\alpha)}{8} \qquad S_2 = \frac{-\sin(2\alpha)}{2}$$

$$S_4 = \frac{\sin(4\alpha)}{8} \qquad C_4 = \frac{1 + \cos(4\alpha)}{8}$$

We will now use LSE to determine the Fourier coefficients as follows:

$$\begin{bmatrix} 1 & \sin(2\beta_1) & \sin(4\beta_1) & \cos(4\beta_1) \\ \vdots & \vdots & \vdots & \vdots \\ 1 & \sin(2\beta_{18}) & \sin(4\beta_{18}) & \cos(4\beta_{18}) \end{bmatrix} \begin{bmatrix} C_0 \\ S_2 \\ S_4 \\ C_4 \end{bmatrix} = \begin{bmatrix} I_1 \\ \vdots \\ \vdots \\ I_{18} \end{bmatrix}$$

where we use the pseudo inverse of the correspondence matrix to get a result for the coefficients for then calculating α. The value of α calculated here will then be compared to the QWP angle α for that dataset to analyse fit.

S_2 and S_4 give information of the handedness of polarization $-S_2 \cdot S_4$ traditionally suggests left handedness as was in our experiment. Make sure to use the correct expression based on how α is being measured with respect to clockwise or anticlockwise rotation of the QWP.

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4:15pm

Analysis of Fourier Coefficients and alpha ($\alpha°$)

$\alpha = 10°$

	Fourier Coefficient	Experimental α (°)	Averaged α (°)	% error
C_0	0.715	11.0	9.7	3.2%
S_2	-0.125	7.3		
S_4	0.073	8.9		
C_4	0.211	11.6		

$\alpha = 20°$

C_0	0.666	17.7	17.8	10.9%
S_2	-0.257	15.5		
S_4	0.120	18.6		
C_4	0.151	19.5		

$\alpha = 30°$

C_0	0.594	26.1	23.9	20.5%
S_2	-0.362	23.2		
S_4	0.119	17.9		
C_4	0.077	28.2		

Uncertainties: Fourier Coefficients : ± 0.0005
Experimental α : $\pm 0.05°$
Average α : $\pm 0.05°$
% error : $\pm 0.05\%$

6 Discussion:

It was observed that the average angle α was consistently calculated to be lower than the angle suggested by our theoretical model. A few reasons come to mind instantly.

i) The Fourier expansion is an inherent approximation of the underlying function that governs the data. This introduces a degree of freedom that was not available to the experimentalist.

ii) The pseudo inverse of the overdetermined system has a finite precision. The numpy.linalg library uses SVD calls and picks out the eigenvector corresponding to the lowest eigenvalue of the system. This is sometimes not the best way to go about it since degenerate eigenvalues may contribute and it is suspected that this led to the error propagating in α.

iii) Optical alignment, accurate ambience and complete darkness were observed to significantly improve results. The author advises future experimentalists to spend ample time in orienting the laser setup so as to achieve accurate results. The importance of this can not be stressed enough on! DO ALIGNMENT CAREFULLY.

iv) A systematic error appears to be in play with regard to the QWP orientation. Re-examining the QWP's calibration would immensely help future experimentalists in conducting their tasks.

7 Conclusion:

This experiment shed light on the study and calculation of polarization angle of light using QWPs and polarizers. Interesting relationships were discovered between the voltage output and polarization angle α. This experiment was conducted with a base case of horizontal polarization and the change in polarization as a function of variables expressed in a Fourier Series using Jones Calculus was successful in calculating the polarization angle of light to within 10% of its theoretical values. Future experiments would aim to study these deviations more closely to ascertain the handedness, ellipticity and polarization polar plots of light.